MATHEMATICS

Chapter 12: Areas Related to Circles

Areas Related to Circles

1. A **circle** is a set of points in a plane that are at an equal distance from a fixed point. The fixed point is called the centre of circle and equal distance is called the radius of the circle.
2. A line segment joining the centre of the circle to a point on the circle is called its **radius**.
3. A line segment joining any two points of a circle is called a **chord**. A chord passing through the centre
4. of circle is called its **diameter**.
5. The distance around the boundary of the circle is called **the perimeter or the circumference** of the circle.
6. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is he diameter, r is the radius of the circle and $\pi = \frac{22}{7}$
7. Perimeter of a semi circle or protractor = $\pi r + 2r$
8. Perimeter of a quadrant = $\frac{1}{4}$ Circumference $+ 2r = \frac{\pi r}{2} + 2r$
9. Distance moved by a wheel in 1 revolution = Circumference of the wheel.
   Number of revolutions in one minute = $\frac{Distance\ moved\ in\ 1\ minute}{Circumference}$
10. The region enclosed inside a circle is called its **area**.
11. Area of a circle = πr^2
12. Area of a semi circle= $\frac{1}{2}\pi r^2$
13. Area of a quadrant = $\frac{1}{4}$ Area of circle= $\frac{1}{4}\pi r^2$
14. Circles having the same centre but different radii are called **concentric circles**.
    Area enclosed by two concentric circles = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$
    Where, R and r are radii of two concentric circles
15. The part of the circumference between the two end points of the chord is called an **arc**. In the figure, arc $\widehat{AB}$ is shown.

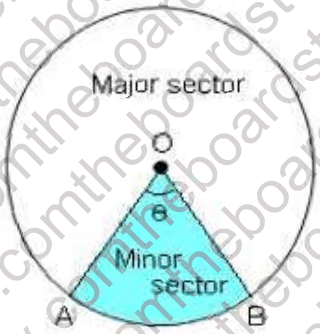
16. A diameter of circle divides a circle into two equal arcs, each known as a **semi-circle.**
17. An arc of a circle whose length is less than that of a semicircle of the same circle is called a **minor arc**.
18. An arc of a circle whose length is greater than that of a semicircle of the same circle is

called a **major arc**.

19. Length of an arc $= \frac{\pi r^2}{180^\circ}$

20. The region bounded by an arc of a circle and two radii at its end points is called a **sector**.

If the central angle of a sector is more than 180°, then the sector is called a **major sector** and if the central angle is less than 180°, then the sector is called a **minor sector**.

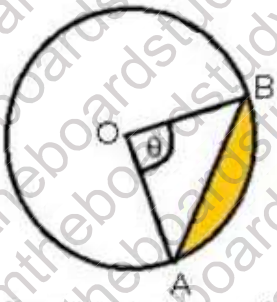


21. Perimeter of sector of angle $\theta = \frac{\pi r\theta}{180^\circ} + 2r$

22. Area of a sector of angle $= \frac{\pi r^2\theta}{360^\circ}$

23. Area of major sector $= \pi r^2 -$ Area of minor sector

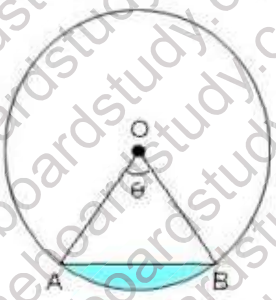
24. A chord divides the interior of a circle into two parts, each called a segment.



The segment which is smaller than the portion of semi-circle is called the **minor segment** and the segment which is larger than the portion of semi-circle is called the **major segment**. In the circle shown, the yellow portion is the minor segment while the non-shaded portion is the major segment.

25. Perimeter of segment of angle $\theta = \frac{2\pi r\theta}{360^\circ} + 2r \sin\frac{\theta}{2}$

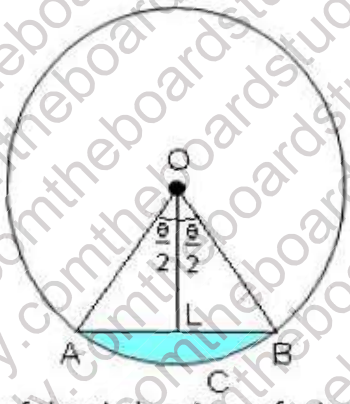
26. Area of minor segment = Area of sector - Area of Δ ABC



27. Area of minor segment can also be written as:

Area of the segment ACB = Area of sector OABC – Area of Δ OAB

Area of segment $ACB = \left\{\frac{\theta}{360^\circ} \times \pi r^2\right\} - \left\{\frac{\sin\theta}{2} + \frac{\cos\theta}{2}\right\}$



28. Area of major segment = Area of the circle – Area of minor segment

29. Area of a Circle

Area of a circle is πr^2, where π=22/7 or ≈3.14 (can be used interchangeably for problem-solving purposes) and r is the radius of the circle.

π is the ratio of the circumference of a circle to its diameter.

Circumference of a Circle

The perimeter of a circle is the distance covered by going around its boundary once. The perimeter of a circle has a special name: Circumference, which is π times the diameter which is given by the formula $2\pi r$

Segment of a Circle

A circular segment is a region of a circle that is "cut off" from the rest of the circle by a secant or a chord.

Sector of a Circle

A circle sector/ sector of a circle is defined as the region of a circle enclosed by an arc and two radii. The smaller area is called the minor sector and the larger area is called the major sector.

Angle of a Sector

The angle of a sector is the angle that is enclosed between the two radii of the sector.

Length of an arc of a sector

The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

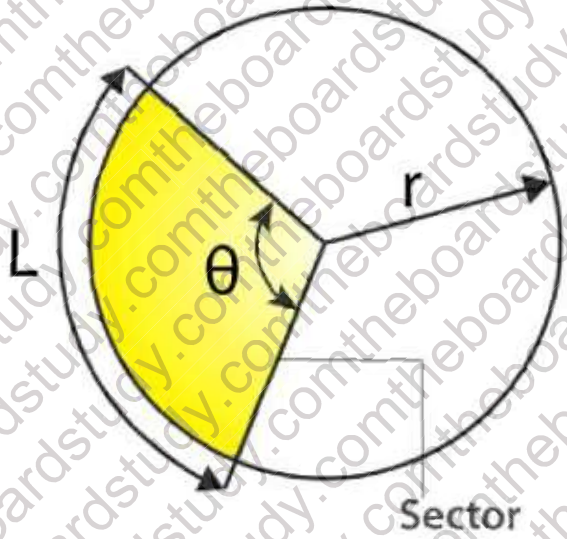
$L = (\theta/360^\circ) \times 2\pi r$

Length of an arc of a sector

The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

$L = (\theta/360°) \times 2\pi r$

where $\angle\theta$ is the angle of this sector (minor sector in the following case) and r is its radius



Area of a Triangle

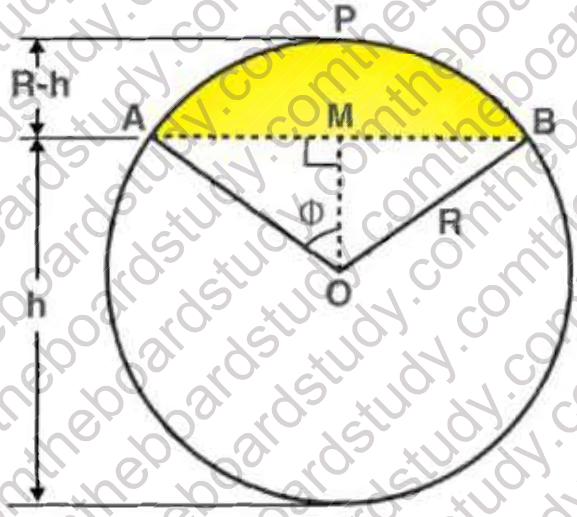
The Area of a triangle is,

Area = (1/2) × base × height

If the triangle is an equilateral then

Area = $(\sqrt{3}/4) \times a^2$ where "a" is the side length of the triangle.

Area of a Segment of a Circle



Area of segment APB (highlighted in yellow)

= (Area of sector OAPB) – (Area of triangle AOB)

$= [(Ø/360°) \times \pi r^2] - [(1/2) \times AB \times OM]$

[To find the area of triangle AOB, use trigonometric ratios to find OM (height) and AB (base)]

Also, the Area of segment APB can be calculated directly if the angle of the sector is known using the following formula.

$= [(\theta/360°) \times \pi r^2] - [r^2 \times \sin \theta/2 \times \cos\theta/2]$

Where θ is the angle of the sector and r is the radius of the circle

All these formulas are tabulated as given below for quick revision.

Parameters of Circles	Formulas
Area of the sector of angle θ	$(\theta/360°) \times \pi r^2$
Length of an arc of a sector of angle θ	$(\theta/360°) \times 2\pi r$
Area of major sector	$\pi r^2 - (\theta/360°) \times \pi r^2$
Area of a segment of a circle	Area of the corresponding sector – Area of the corresponding triangle
Area of the major segment	πr^2 – Area of segment (minor segment)

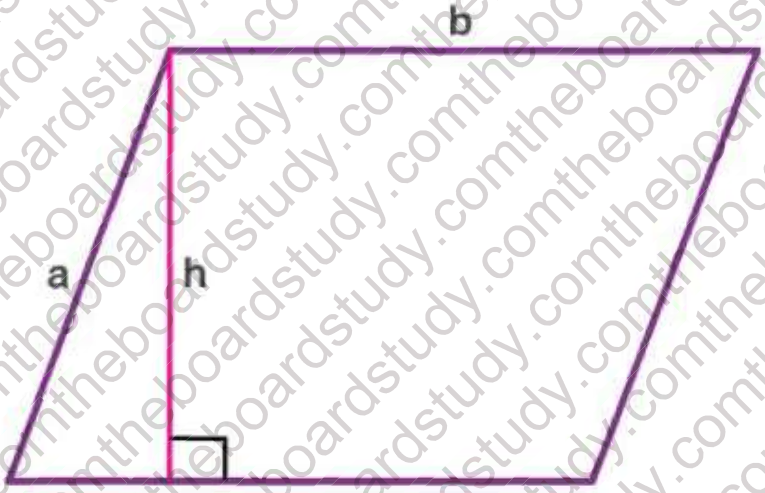
Visualizations

Areas of different plane figures

Area of a square (side l) = l^2

Area of a rectangle = l × b, where l and b are the length and breadth of the rectangle

Area of a parallelogram = b × h, where "b" is the base and "h" is the perpendicular height.



Parallelogram

Area of a trapezium = [(a + b) × h]/2,

where

a & b are the length of the parallel sides

h is the trapezium height

Area of a rhombus = pq/2, where p & q are the diagonals.

Area Of Shapes

In Geometry, a shape is defined as the figure closed by the boundary. The boundary is created by the combination of lines, points and curves. Basically, there are two different types of geometric shapes such as:

Two – Dimensional Shapes

Three – Dimensional Shapes

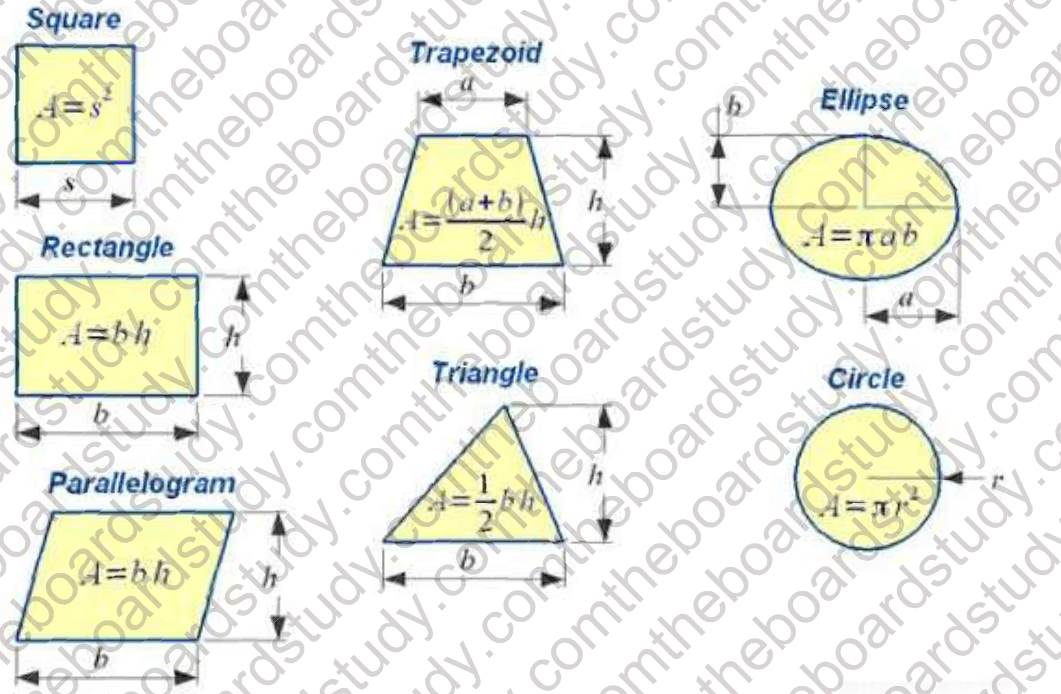
Each and every shape in the Geometry can be measured using different measures such as area, volume, surface area, perimeter and so on. In this article, let us discuss the area of shapes for 2D figures and 3D figures with formulas.

2D shapes

The two-dimensional shapes (2D shapes) are also known as flat shapes, are the shapes having two dimensions only. It has length and breadth. It does not have thickness. The two different measures used for measuring the flat shapes are area and the perimeter. Two-dimensional shapes are the shapes that can be drawn on the piece of paper. Some of the examples of 2D shapes are square, rectangle, circle, triangle and so on.

Area of 2D Shapes Formula

In general, the area of shapes can be defined as the amount of paint required to cover the surface with a single coat. Following are the ways to calculate area based on the number of sides that exist in the shape, as illustrated below in the fig.

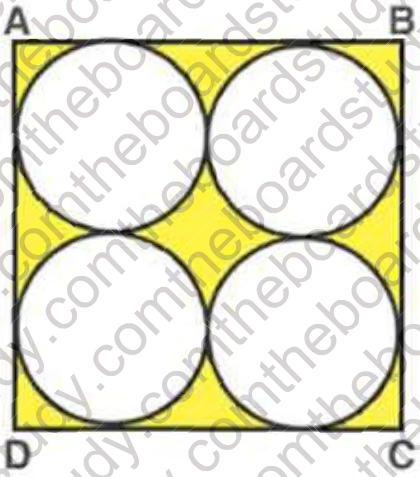


Let us write the formulas for all the different types of shapes in a tabular form.

Shape	Area	Terms
Circle	$\pi \times r^2$	r = radius of the circle
Triangle	½ × b × h	b = base h = height
Square	a^2	a = length of side
Rectangle	l × w	l = length w = width
Parallelogram	b × h	b=base h=vertical height
Trapezium	½(a+b) × h	a and b are the length of parallel sides h = height
Ellipse	πab	a = ½ minor axis b = ½ major axis

Areas of Combination of Plane figures

For example: Find the area of the shaded part in the following figure: Given the ABCD is a square of side 28 cm and has four equal circles enclosed within.



Area of the shaded region

Looking at the figure we can visualize that the required shaded area = A(square ABCD) – 4 × A(Circle).

Also, the diameter of each circle is 14 cm.

$= (l^2) - 4 \times (\pi r^2)$

$= (28^2) - [4 \times (\pi \times 49)]$

$= 784 - [4 \times 22/7 \times 49]$

$= 784 - 616$

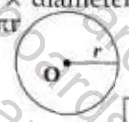
$= 168 cm^2$

MIND MAP : LEARNING MADE SIMPLE

Areas Related to Circles

Circle

- Circumference = $\pi \times$ diameter $= 2\pi r$
- Area = πr^2



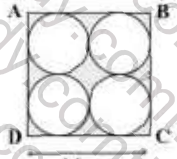
Area - Combination of figures

Meaning

Area of T = Area of P + Area of Q

Example

Find Area of shaded region



Area of square ABCD = 14×14 cm^2
$= 196$ cm^2

Diameter of each circle, D = $\frac{14}{2} = 7$ cm

For each circle, radius (r) = $\frac{7}{2}$ cm

Area of 1 circle = πr^2

Area of 4 circles = $4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$ cm^2

$= \frac{154}{4} \times 4$ cm^2

$= 154$ cm^2

Area of shaded region =
Area of ABCD – Area of 4 circles
$= (196 - 154) = 42$ cm^2

Sector

Meaning

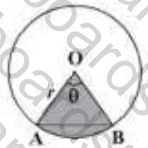
Portion of the circular region enclosed by two radius and the corresponding arc

Formula

Area

$A = \frac{\theta}{360^\circ} \times$ area of circle

$A = \frac{\theta}{360^\circ} \times \pi r^2$



Length of arc

$L = \frac{\theta}{360^\circ} \times$ circumference

$L = \frac{\theta}{360^\circ} \times 2\pi r$

Segment

Formula

Area = Area of the corresponding sector – Area of the corresponding triangle

$= \frac{\theta}{360^\circ} \times \pi r^2 -$ area of ΔOAB

Meaning

Portion of the circular region enclosed between a chord and the corresponding arc

Important Questions

Multiple Choice questions-

1. Perimeter of a sector of a circle whose central angle is 90° and radius 7 cm is

(a) 35 cm

(b) 25 cm

(c) 77 cm

(d) 7 cm

2. The area of a circle that can be inscribed in a square of side 10 cm is

(a) 40π cm^2

(b) 30π cm^2

(c) 100π cm^2

(d) 25π cm^2

3.The perimeter of a square circumscribing a circle of radius a units is

(a) 2 units

(b) 4α units

(c) 8α units

(d) 16α units

4. The perimeter of the sector with radius 10.5 cm and sector angle 60° is

(a) 32 cm

(b) 23 cm

(c) 41 cm

(d) 11 cm

5. In a circle of diameter 42 cm, if an arc subtends an angle of 60° at the centre, where $\pi = 227$ then length of arc is:

(a) 11 cm

(b) 227 cm

(c) 22 cm

(d) 44 cm

6. The perimeter of a sector of radius 5.2 cm is 16.4 cm, the area of the sector is

(a) 31.2 cm^2

(b) 15 cm^2

(c) 15.6 cm^2

(d) 16.6 cm^2

7. If the perimeter of a semicircular protractor is 72 cm where $\pi = 227$, then the diameter of protractor is:

(a) 14 cm

(b) 33 cm

(c) 28 cm

(d) 42 cm

8. If the radius of a circle is doubled, its area becomes

(a) 2 times

(b) 4 times

(c) 8 times

(d) 16 times

9. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to circumference of a circle of radius R, then

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) Can't say.

10. The perimeter of a circular and square fields are equal. If the area of the square field is 484 m^2 then the diameter of the circular field is

(a) 14 m

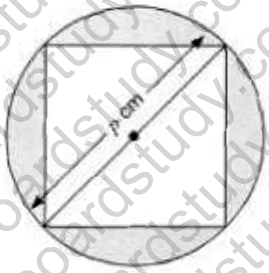
(b) 21 m

(c) 28 m

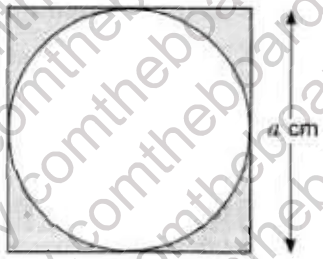
(d) 7 m

Very Short Questions:

1. Find the area of a square inscribed in a circle of diameter p cm.



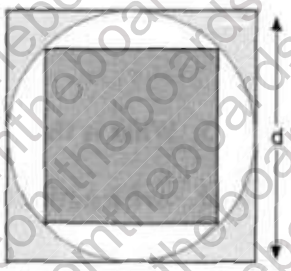
2. Find the area of the circle inscribed in a square of side a cm.



3. Find the area of a sector of a circle whose radius is and length of the arc is l.

4. Find the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal.

5. A square inscribed in a circle of diameter d and another square is circumscribing the circle. Show that the area of the outer square is twice the area of the inner square.



6. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

7. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.

8. If the perimeter of a semi-circular protractor is 36 cm, find its diameter.

9. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.

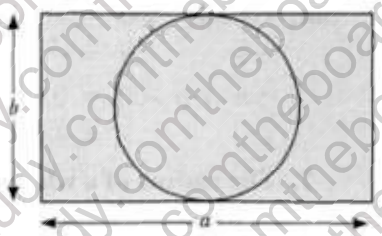
10. If a square is inscribed in a circle, what is the ratio of the areas of the circle and the square?

Short Questions :

1. What is the area of the largest triangle that is inscribed in a semi circle of radius r unit?

2. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?

3. What is the area of the largest circle that can be drawn inside a 4 rectangle of length a cm and breadth b cm (a > b)?



4. Difference between the circumference and radius of a circle is 37 cm. Find the area of circle.

5. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

6. If the perimeter of a semicircular protractor is 66 cm, find the diameter of the protractor. (Take $\pi = \frac{22}{7}$).

7. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

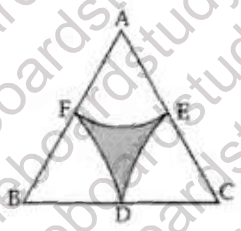
8. A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

9. The inner circumference of a circular track [Fig. 12.10] is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹2 per metre.

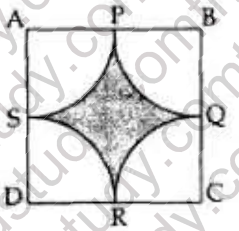
10. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Long Questions :

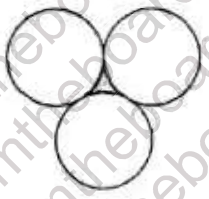
1. In Figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle ABC of side 14 cm as centres to intersect the sides BC, CA and AB at BZ their respective mid-points D, E and F. Find the area of the shaded region. [Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$]



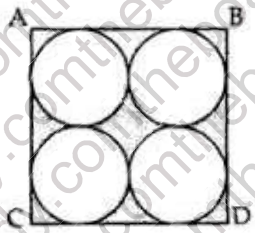
2. Find the area of the shaded region in Figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD, where the length of each side of square is 14 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$



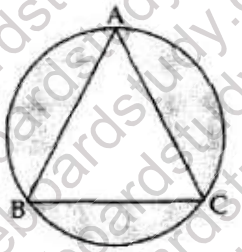
3. In Figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these three circles (shaded region). $\left[\text{Use } \pi = \frac{22}{7}\right]$



4. Find the area of the shaded region in Figure, where ABCD is a square of side 28 cm.



5. In Figure, an equilateral triangle has been inscribed in a circle of radius 6 cm. Find the area of the shaded region. [Use $\pi = 3.14$]



Assertion Reason Questions-

1. Principle of a school decided to give badges to students who are chosen for the post of Head boy, Head girl, Prefect, and Vice Prefect. Badges are circular in shape with two color area, red and silver, as shown in figure. The diameter of the region representing red color is 22cm and silver color is filled in l0.5 cm wide ring. Based on the above information, answer the following questions.

i. The radius of circle representing the red region is:

a. 9cm
b. 10cm
c. 11cm
d. 12cm

ii. Find the area of the red region.

a. 380.28cm²
b. 382.28cm²
c. 384.28cm²
d. 378.28cm²

iii. Find the radius of the circle formed by combining the red and silver region.

a. 20.5cm
b. 21.5cm
c. 22.5cm
d. 23.5cm

iv. Find the area of the silver region.

a. 172.50cm²
b. 1062.50cm²
c. 1172.50cm²
d. 1072.50cm²

v. Area of the circular path formed by two concentric circles of radii r_1 and r_2 ($r_1 > r_2$) =

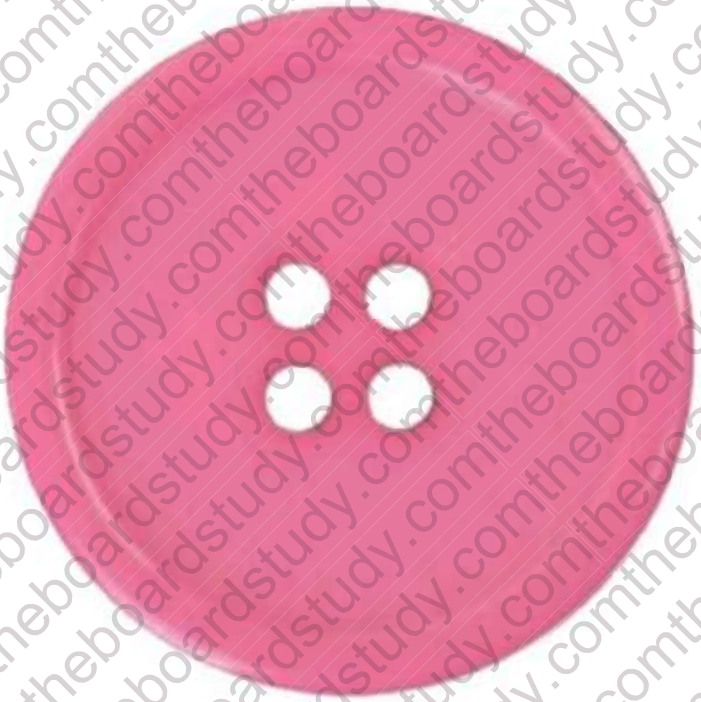
a. $\pi(r_1^2 + r_1^2)$sq.units

b. $\pi(r_1^2 - r_1^2)$sq.units

c. $2\pi(r_1 - r_1)$sq.units

d. $2\pi(r_1 + r_1)$sq.units

2. While doing dusting, a maid found a button whose upper face is of black color, as shown in the figure. The diameter of each of the smaller identical circles is 1414 of the diameter of the larger circle, whose radius is 16cm. Based on the above information, answer the following questions.



i. The area of each of the smaller circle is:

a. 40.28cm^2
b. 46.39cm^2
c. 50.28cm^2
d. 52.3cm^2

ii. The area of the larger circle is:

a. 804.57cm^2
b. 704.57cm^2
c. 855.57cm^2
d. 990.57cm^2

iii. The area of the black color region is:

a. 600.45cm^2
b. 603.45cm^2
c. 610.45cm^2
d. 623.45cm^2

iv. The area of a quadrant of a smaller circle is:

a. 11.57cm^2
b. 13.68cm^2
c. 12cm^2
d. 12.57cm^2

v. If two concentric circles are of radii 2cm and 5cm, then the area between them is:

a. $60cm^2$
b. $63cm^2$
c. $66cm^2$
d. $68cm^2$

Case Study Answers:

1. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

a. A is true, R is true; R is a correct explanation for A.

b. A is true, R is true; R is not a correct explanation for A.

c. A is true; R is false.

d. A is false; R is true.

Assertion: If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason: Circumference = $2\pi \times$ radius

2. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

a. A is true, R is true; R is a correct explanation for A.

b. A is true, R is true; R is not a correct explanation for A.

c. A is true; R is false.

d. A is false; R is true.

Assertion: If a wire of length 22 cm is bent is the shape of a circle, then area of the circle so formed is 40 cm.

Reason: Circumference of the circle = length of the wire.

Answer Key-

Multiple Choice questions-

1. (b) 25 cm
2. (d) 25π cm^2
3. (c) 8α units
4. (a) 32 cm
5. (c) 22 cm
6. (c) 15.6 cm^2
7. (c) 28 cm
8. (b) 4 times
9. (a) $R_1 + R_2 = R$
10. (c) 28 m

Very Short Answer :

1. Diagonal of the square = p cm

   $\therefore p^2 = side^2 + side^2$

   $\Rightarrow p^2 = 2side^2$

   or $side^2 = \frac{p^2}{2}$ cm^2 = area of the square

2. Diameter of the circle = a

   $\Rightarrow$ Radius $= \frac{a}{2}$ $\Rightarrow$ Area $= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$ cm^2

3. Area ola sector ola circle with radius r

   $= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{\theta}{360^\circ} \times 2\pi r \frac{r}{2} = \frac{1}{2} lr$ sq. units $\left(\because l = \frac{2\pi r\theta}{360^\circ}\right)$

4.

Given, $2r = a \Rightarrow \frac{r}{a} = \frac{1}{2}$

$$\frac{\text{Area of circle}}{\text{Area of equilateral triangle}} = \frac{\pi r^2}{\frac{\sqrt{3}}{4}a^2} = \frac{4\pi}{\sqrt{3}}\left(\frac{r}{a}\right)^2 = \frac{4\pi}{\sqrt{3}} \times \frac{1}{4} = \frac{\pi}{\sqrt{3}}$$

5. Side of outer square = d

$\therefore$ Its area = d

Diagonal of inner square = d

$\therefore$ Side = $\frac{d}{\sqrt{2}}$

$\Rightarrow$ Area = $\frac{d^2}{2}$

Area of outer square = 2 × Area of inner square.

6. Given, $2\pi r = \pi r^2$

$\Rightarrow 2r = r^2$

$\Rightarrow r(r-2) = 0$ or $r = 2$

i.e. d = 4 units

7.

$$\text{Number of revolutions} = \frac{11 \times 1000}{2 \times \frac{22}{7} \times 0.25} = 7000.$$

8. Perimeter of a semicircular protractor = Perimeter of a semicircle

= (2r + πr) cm

$\Rightarrow 2r + \pi r = 36$

$\Rightarrow r\left(2 + \frac{22}{7}\right) = 36$

$\Rightarrow$ r = 7cm

Diameter 2r = 2 × 7 = 14 cm.

9. Perimeter of a semicircle = πr + 2r

$= \frac{22}{7} \times 7 + 2 \times 7 = 22 + 14 = 36$cm

10. Let radius of the circle be r units.

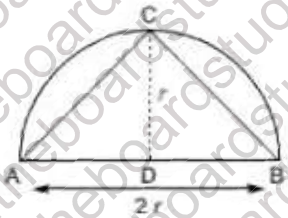
Then, diagonal of the square = 2r

$$\Rightarrow \quad \text{Side of the square} = \frac{2r}{\sqrt{2}} = \sqrt{2}r$$

$$\therefore \quad \frac{\text{Area of the circle}}{\text{Area of the square}} = \frac{\pi r^2}{(\sqrt{2}r)^2} = \frac{\pi r^2}{2r^2} = \pi : 2$$

Short Answer :

1.



Area of largest $\Delta ABC = \frac{1}{2} \times AB \times CD$

$\frac{1}{2} \times 2r \times r = r^2$ sq. units

2.

$$\text{Arc length of a circle of radius } r = \frac{\theta}{360^\circ} \times 2\pi r$$

$$\Rightarrow \quad 5\pi = \frac{360^\circ}{4} \times 2\pi \times 10 \quad \text{or} \quad \frac{360^\circ}{4} = \frac{5\pi}{20\pi} = \frac{1}{4} \Rightarrow \theta = \frac{360^\circ}{4} = 90^\circ$$

3. Diameter of the largest circle that can be inscribed in the given b

rectangle = b cm

$\therefore$ Radius = $\frac{b}{2}$ cm

$$\Rightarrow \text{ Area of required circle} = \pi\left(\frac{b}{2}\right)^2 = \frac{\pi b^2}{4} \text{ cm}^2$$

4. Given $2\pi r - r = 37$

or $r(2\pi - 1) = 37$

$$r = \frac{37}{2\pi - 1} = \frac{37}{2 \times \frac{22}{7} - 1} = \frac{37 \times 7}{37} = 7$$

So area of circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

5. Let r be the radius of required circle. Then, we have

$\pi r^2 = p(8)2 + p(6)^2$

$\Rightarrow \pi r^2 = 64p + 36p$

$\Rightarrow pr^2 = 100p$

$\therefore r^2 = 100pp = 100$

$\Rightarrow r = 10\text{cm}$

Hence, radius of required circle is 10 cm.

6. Let the radius of the protractor be r cm. Then,

Perimeter = 66 cm

$= \pi r + 2r = 66$ [$\because$ Perimeter of a semicircle $= \pi r + 2r$]

$$\Rightarrow \quad r\left(\frac{22}{7} + 2\right) = 66 \quad \Rightarrow \quad \frac{36}{7}r = 66$$

$$\Rightarrow \quad r = \frac{66 \times 7}{36} = \frac{77}{6} \text{ cm}$$

$$\therefore \quad \text{Diameter of the protractor} = 2r = 2 \times \frac{77}{6} = \frac{77}{3} = 25\frac{2}{3} \text{ cm}$$

7. Let the radius of the circle be r cm. Then,

Diameter = 2r cm and Circumference = $2\pi r$ cm

According to question,

Circumference = Diameter + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2 \times \frac{22}{7} \times r = 2r + 16.8$

$\Rightarrow 44r = 14r + 16.8 \times 7$

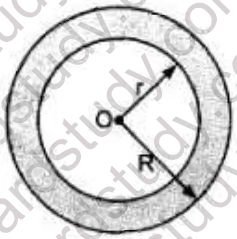
$\Rightarrow 44r - 14r = 117.6$ or $30r = 117.6$

$\Rightarrow r = \frac{117.6}{30} = 3.92$

Hence, radius = 3.92 cm.

8. Let the outer and inner radii of the ring be R m and r m respectively. Then,

$2\pi R = 396$ and $2\pi r = 352$



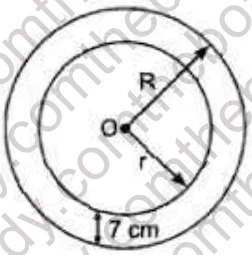
$$\Rightarrow \quad 2 \times \frac{22}{7} \times R = 396 \quad \text{and} \quad 2 \times \frac{22}{7} \times r = 352$$

$$\Rightarrow \quad R = 396 \times \frac{7}{22} \times \frac{1}{2} \quad \text{and} \quad r = 352 \times \frac{7}{22} \times \frac{1}{2}$$

$$\Rightarrow \quad R = 63 \text{ m} \quad \text{and} \quad r = 56 \text{ m}$$

Hence, width of the track = (R – r) m = (63 – 56) m = 7 m

9.



Let the inner and outer radii of the circular track berm and R m respectively. Then,

Inner circumference = $2\pi r$ = 220 m

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220 \quad \Rightarrow \quad r = \frac{220 \times 7}{2 \times 22} = 35 \text{ m}$$

Since the track is 7 m wide everywhere. Therefore,

R = Outer radius = r + 7 = (35 + 7)m = 42 m

$\therefore$ Outer circumference = $2\pi R = 2 \times \frac{22}{7} \times 42\text{m} = 264\text{m}$

Rate of fencing = ₹ 2 per metre

$\therefore$ Total cost of fencing = (Circumference × Rate) = ₹(264 × 2) = ₹ 528

10. The diameter of a wheel = 80 cm.

radius of the wheel = 40 cm.

Now, distance travelled in one complete revolution of wheel = $2\pi \times 40 = 80\pi$

Since, speed of the car is 66 km/h

So, distance travelled in 10 minutes = $\frac{66\times100000\times10}{60}$

= 11 × 100000 cm = 1100000 cm.

So, Number of complete revolutions in 10 minutes

$$= \frac{1100000}{80\pi} = \frac{1100000}{8\times\frac{22}{7}}$$

$$= \frac{110000\times7}{8\times22} = \frac{70000}{16} = 4375$$

Long Answer :

1. $\angle ABC = \angle BAC = \angle ACB = 60°$ ... [equilateral Δ]

Let $\theta = 60°$, $r = \frac{14}{2} = 7$ cm

Area of shaded region

= ar(ΔABC) – 3 (ar of sector)

$$= \frac{\sqrt{3}}{4}(\text{side})^2 - 3 \cdot \frac{\theta}{360}\pi r^2$$

...[**Area of equilateral Δ = $\frac{\sqrt{3}}{4}$ side²**]

$$= \frac{1.73}{4} \times 14 \times 14 - 3 \times \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= 84.77 - 77 = \mathbf{7.77\ cm^2}$$

2. Side = 14 cm, radius, $r = \frac{14}{2} = 7$ cm

Area of the shaded region

= ar (square) – 4 (ar of quadrant)

$$= (\text{side})^2 - 4\left(\frac{1}{4}\pi r^2\right)$$

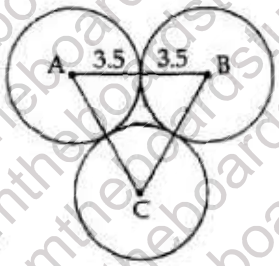
$$= (14)^2 - \frac{22}{7} \times 7 \times 7$$

$$= 196 - 154 = \mathbf{42\ cm^2}$$

3. AB = BC = CA

= 2(3.5) = 7 cm

∴ ΔABC is an equilateral Δ



$$\angle A = \angle B = \angle C = \frac{180^\circ}{3}$$
$$= 60^\circ$$

$\theta = 60^\circ$, $r = 3.5 = \frac{7}{2}$ cm

Shaded area = area of ΔABC – 3(area of sector)

$$= \frac{\sqrt{3}}{4}(\text{side})^2 - 3 \times \frac{\theta}{360}\pi r^2$$

$$= \frac{\sqrt{3}}{4}(7)^2 - 3 \times \frac{60}{360} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{49\sqrt{3}}{4} - \frac{77}{4}$$

$$= \mathbf{\frac{1}{4}(49\sqrt{3} - 77)\ cm^2}$$

4. Here $r = \frac{28}{4} = 7$ cm

Area of the shaded region

= ar(square) – 4(circle)

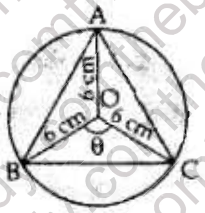
$= (\text{side})^2 - 4(\pi r^2)$

$= (28)^2 - 4 \times \frac{22}{7} \times 7 \times 7 = 784 - 616 = 168\ cm^2$

5. Here $\theta = \frac{360}{3} = 120^\circ$, r = 6 cm

Area of shaded region

= 3(ar of minor segment) = 3[ar(minor sector) – ar(ΔABC)]



$$= 3\left[\frac{\theta}{360^\circ}\pi r^2 - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

$$= 3\left[\frac{120^\circ}{360^\circ}(3.14)\times 6^2 - 6^2 \sin\left(\frac{120^\circ}{2}\right)\cos\left(\frac{120^\circ}{2}\right)\right]$$

$$= 3 \times 6^2\left[\frac{3.14}{3} - \sin 60^\circ \cos 60^\circ\right]$$

$$= 3(36)\left[\frac{3.14}{3} - \frac{\sqrt{3}}{2}\times\frac{1}{2}\right]$$

$$= 108\left[\frac{12.56 - 3(1.73)}{12}\right] \quad \text{...[} \sqrt{3} = 1.73$$

$$= 9\ (12.56 - 5.19) = 9\ (7.37) = \mathbf{66.33\ cm^2}$$

Case Study Answers:

1. Answer :

i. (c) 11cm

Solution:

Radius of circle representing red region

$= \frac{22}{2} = 11\text{cm}$ [∵ Diameter = 22cm (Given)]

ii. (a) 380.28cm²

Solution:

Area of red region πr^2

$= \frac{22}{7} \times 11 \times 11 = 380.28\text{cm}^2$

iii. (b) 21.5cm

Solution:

Radius of circle formed by combining red and silver region = Radius of red region + width of silver sign.

= (11+ 10.5)cm = 21.5cm

iv. (d) 1072.50cm²

Solution:

Area of silver region = Area of combined region - Area of red region.

$= \frac{22}{7} \times 21.5 \times 21.5 - 380.28$

= 1452.78 - 380.28 = 1072.50cm²

v. (b) $\pi(r_1^2 - r_1^2)$sq.units

Solution:

Area of circular path formed by two concentric circles $= \pi(r_1^2 - r_1^2)$sq.units.

2. Answer :

Let r and R be the radii of each smaller circle and larger circle, respectively.

We have, $d = \frac{1}{4}D$

$\Rightarrow r = \frac{1}{4}R \Rightarrow r = \frac{1}{4} \times 16 \Rightarrow r = 4cm.$

i. (c) 50.28cm²

Solution:

Area of smaller circle πr^2

$= \frac{22}{7} \times 4 \times 4 = 50.28cm^2$

ii. (a) 804.57cm²

Solution:

Area of larger circle πR^2

$= \frac{22}{7} \times 16 \times 16 = \frac{5632}{7} = 804.57cm^2$

iii. (b) 603.45cm²

Solution:

Area of the black color region = Area of larger circle - Area of 4 smaller circles.

= 804.57 - 4 × 50.28 = 603.45cm²

iv. (d) 12.57cm²

Solution:

Area of quadrant of a smaller circle

$$= \frac{1}{4} \times 450.2 = 12.57\text{cm}^2$$

v. (c) 66cm²

Solution:

Area between two concentric circles

$$= \pi(R^2 - r^2) = \frac{22}{7}(5^2 - 2^2)$$

$$= \frac{22}{7}(25 - 4) = \frac{22}{7} \times 21 = 66\text{cm}^2$$

Assertion Reason Answer-

1. (a) A is true, R is true; R is a correct explanation for A.
2. (d) A is false; R is true.