

MATHEMATICS

Chapter 4: Quadratic Equations



Quadratic Equations

1. Introduction to Quadratic equation

If $p(x)$ is a quadratic polynomial, then $p(x) = 0$ is called a **quadratic equation**.

The general or standard form of a quadratic equation, in the variable x , is given by $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

2. Roots of the quadratic equation

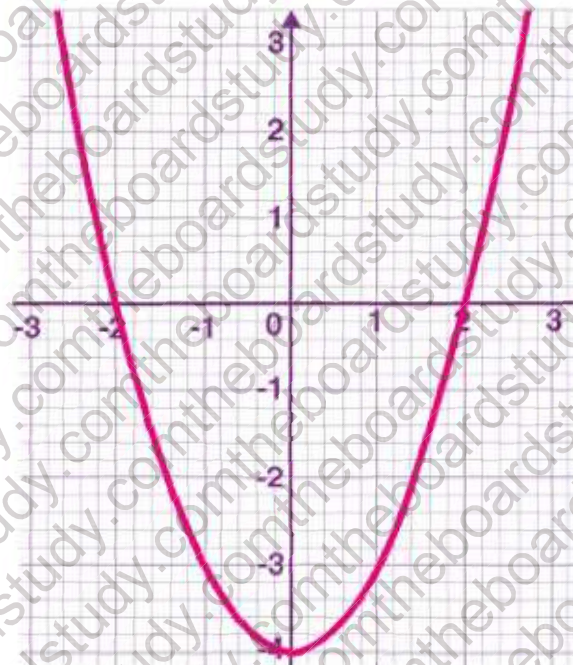
The value of x that satisfies an equation is called the **zeroes** or **roots** of the equation.

A real number α is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.

A quadratic equation has **at most two roots**.

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x -axis.

Consider the graph of a quadratic equation $x^2 - 4 = 0$:



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation $x^2 - 4 = 0$.

Note:

- If the graph of the quadratic polynomial cuts the x -axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x -axis, then it has real and equal roots.

- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

3. The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a , b and c are real numbers and $a \neq 0$.

' a ' is the coefficient of x^2 . It is called the quadratic coefficient. ' b ' is the coefficient of x . It is called the linear coefficient. ' c ' is the constant term.

4. A quadratic equation can be solved by following algebraic methods:

- i. Splitting the middle term (factorization)
- ii. Completing squares
- iii. Quadratic formula

5. Splitting the middle term (or factorization) method

- If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Steps involved in solving quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) by **splitting the middle term** (or factorization) method:

Step 1: Find the product ac .

Step 2: Find the factors of ' ac ' that add to up to b , using the following criteria:

- i. If $ac > 0$ and $b > 0$, then both the factors are positive.
- ii. If $ac > 0$ and $b < 0$, then both the factors are negative.
- iii. If $ac < 0$ and $b > 0$, then larger factor is positive and smaller factor is negative.
- iv. If $ac < 0$ and $b < 0$, then larger factor is negative and smaller factor is positive.

Step 3: Split the middle term into two parts using the factors obtained in the above step.

Step 4: Factorize the quadratic equation obtained in the above step by grouping method. Two factors will be obtained.

Step 5: Equate each of the linear factors to zero to get the value of x .

6. Completing the square method

- Any quadratic equation can be converted to the form $(x + a)^2 - b^2 = 0$ or $(x - a)^2 + b^2 = 0$ by adding and subtracting the constant term. This method of finding the roots of quadratic equation is called the method of completing the square.
- The steps involved in solving a quadratic equation by **completing the square**, are as follows:

Step 1: Make the coefficient of x^2 unity.

Step 2: Express the coefficient of x in the form $2 \times x \times p$.

Step 3: Add and subtract the square of p .

Step 4: Use the square identity $(a + b)^2$ or $(a - b)^2$ to obtain the quadratic equation in the required form $(x + a)^2 - b^2 = 0$ or $(x - a)^2 + b^2 = 0$.

Step 5: Take the constant term to the other side of the equation.

Step 6: Take the square root on both the sides of the obtained equation to get the roots of the given quadratic equation.

7. Quadratic formula

The roots of a quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) can be calculated by using the **quadratic formula**:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ where } b^2 - 4ac \geq 0$$

If $b^2 - 4ac < 0$, then equation does not have real roots.

The quadratic formula is used to find the roots of a quadratic equation. This formula helps to evaluate the solution of quadratic equations replacing the factorization method. If a quadratic equation does not contain real roots, then the quadratic formula helps to find the imaginary roots of that equation. The quadratic formula is also known as Shreedhara Acharya's formula. In this article, you will learn the quadratic formula, derivation and proof of the quadratic formula, along with a video lesson and solved examples.

An algebraic expression of degree 2 is called the **quadratic equation**. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a , b and c are real numbers, also called "numeric coefficients" and $a \neq 0$. Here, x is an unknown variable for which we need to find the solution. We know that the **quadratic formula** used to find the solutions (or roots) of the quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here,

a , b , c = Constants (real numbers)

$a \neq 0$

x = Unknown, i.e. variable

The above formula can also be written as:

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

or

$$x = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

What is the Quadratic Formula used for?

The quadratic formula is used to find the roots of a quadratic equation and these roots are called the solutions of the quadratic equation. However, there are several methods of solving quadratic equations such as factoring, completing the square, graphing, etc.

Roots of Quadratic Equation by Quadratic Formula

We know that a second-degree polynomial will have at most two zeros, and therefore a quadratic equation will have at most two roots.

In general, if α is a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$; then, $a\alpha^2 + b\alpha + c = 0$. We can also say that $x = \alpha$ is a solution of the quadratic equation or α satisfies the equation, $ax^2 + bx + c = 0$.

Note: Roots of the quadratic equation $ax^2 + bx + c = 0$ are the same as zeros of the polynomial $ax^2 + bx + c$.

One of the easiest ways to find the roots of a quadratic equation is to apply the quadratic formula.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $b^2 - 4ac$ is called the discriminant and is denoted by D .

The sign of plus (+) and minus (-) in the quadratic formula represents that there are two solutions for quadratic equations and are called the roots of the quadratic equation.

Root 1:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

And

Root 2:

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

8. Derivation of Quadratic Formula

We can derive the quadratic formula in different ways using various techniques.

Derivation Using Completing the Square Technique

Let us write the standard form of a quadratic equation.

$$ax^2 + bx + c = 0$$

Divide the equation by the coefficient of x^2 , i.e., a ,

$$x^2 + (b/a)x + (c/a) = 0$$

Subtract c/a from both sides of this equation,

$$x^2 + (b/a)x = -c/a$$

Now, apply the method of completing the square.

Add a constant to both sides of the equation to make the LHS of the equation as complete square.

Adding $(b/2a)^2$ on both sides,

$$x^2 + (b/a)x + (b/2a)^2 = (-c/a) + (b/2a)^2$$

Using the identity $a^2 + 2ab + b^2 = (a + b)^2$,

$$[x + (b/2a)]^2 = (-c/a) + (b^2/4a^2)$$

$$[x + (b/2a)]^2 = (b^2 - 4ac)/4a^2$$

Take the square root on both sides,

Shortcut Method of Derivation

Write the standard form of a quadratic equation,

$$ax^2 + bx + c = 0$$

Multiply both sides of the equation by $4a$,

$$4a(ax^2 + bx + c) = 4a(0)$$

$$4a^2x^2 + 4abx + 4ac = 0$$

$$4a^2x^2 + 4abx = -4ac$$

Add a constant on sides such that LHS will become a complete square.

Adding b^2 on both sides,

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

$$(2ax)^2 + 2(2ax)(b) + b^2 = b^2 - 4ac$$

Using algebraic identity $a^2 + 2ab + b^2 = (a + b)^2$,

$$(2ax + b)^2 = b^2 - 4ac$$

Taking square root on both sides,

$$2ax + b = \pm \sqrt{b^2 - 4ac}$$

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$

$$x = [-b \pm \sqrt{b^2 - 4ac}]/2a$$

9. Nature of Roots

Based on the value of the discriminant, $D = b^2 - 4ac$, the roots of a quadratic equation can be of three types.

Case 1: If $D > 0$, the equation has two distinct real roots.

Case 2: If $D = 0$, the equation has two equal real roots.

Case 3: If $D < 0$, the equation has no real roots.

The number of roots of a polynomial equation is equal to its degree. So, a quadratic equation has two roots. Some methods for finding the roots are:

Factorization method

Quadratic Formula

Completing the square method

All the quadratic equations with real roots can be factorized. The physical significance of the roots is that at the roots of an equation, the graph of the equation intersects x-axis. The x-axis represents the real line in the Cartesian plane. This means that if the equation has unreal roots, it won't intersect x-axis and hence it cannot be written in factorized form. Let us now go ahead and learn how to determine whether a quadratic equation will have real roots or not.

Nature Of Roots Of Quadratic Equation

Value of Discriminant	Nature of Roots				
$D > 0$	Real, Distinct <table border="1"> <tr> <td>Dis a perfect square</td><td>Rational roots</td></tr> <tr> <td>Dis not a perfect square</td><td>Irrational roots</td></tr> </table>	Dis a perfect square	Rational roots	Dis not a perfect square	Irrational roots
Dis a perfect square	Rational roots				
Dis not a perfect square	Irrational roots				
$D = 0$	Real, Equal				
$D < 0$	Complex, Distinct (A pair of complex conjugates)				

10. Graphical Representation of a Quadratic Equation

The graph of a quadratic polynomial is a parabola. The roots of a quadratic equation are the points where the parabola cuts the x-axis i.e, the points where the value of the quadratic polynomial is zero.

Now, the graph of $x^2 + 5x + 6 = 0$ is:



In the above figure, -2 and -3 are the roots of the quadratic equation $x^2 + 5x + 6 = 0$.

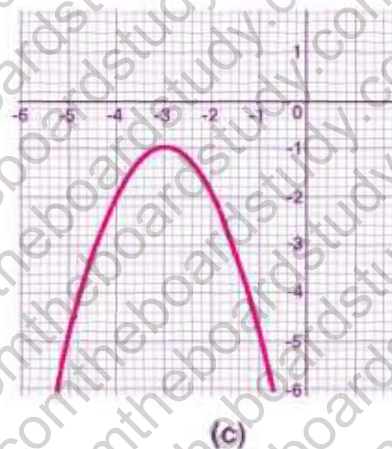
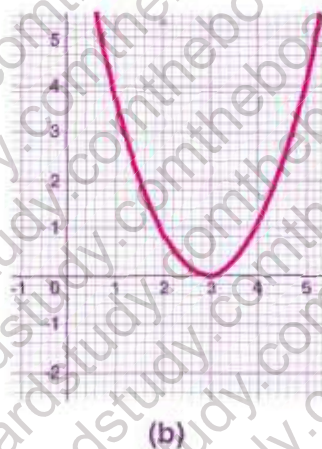
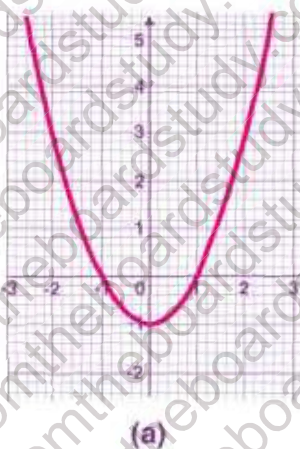
For a quadratic polynomial $ax^2 + bx + c$,

If $a > 0$, the parabola opens upwards.

If $a < 0$, the parabola opens downwards.

If $a = 0$, the polynomial will become a first-degree polynomial and its graph is linear.

The discriminant, $D = b^2 - 4ac$



Nature of graph for different values of D .

If $D > 0$, the parabola cuts the x-axis at exactly two distinct points. The roots are distinct. This case is shown in the above figure in a, where the quadratic polynomial cuts the x-axis at two distinct points.

If $D = 0$, the parabola just touches the x-axis at one point and the rest of the parabola lies above or below the x-axis. In this case, the roots are equal.

This case is shown in the above figure in b, where the quadratic polynomial touches the x-axis at only one point.

If $D < 0$, the parabola lies entirely above or below the x-axis and there is no point of contact with the x-axis. In this case, there are no real roots.

This case is shown in the above figure in c, where the quadratic polynomial neither cuts nor touch the x-axis.

11. Discriminant of a quadratic equation

For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the expression $b^2 - 4ac$ is known as **discriminant**.

12. Nature of the roots of a quadratic equation:

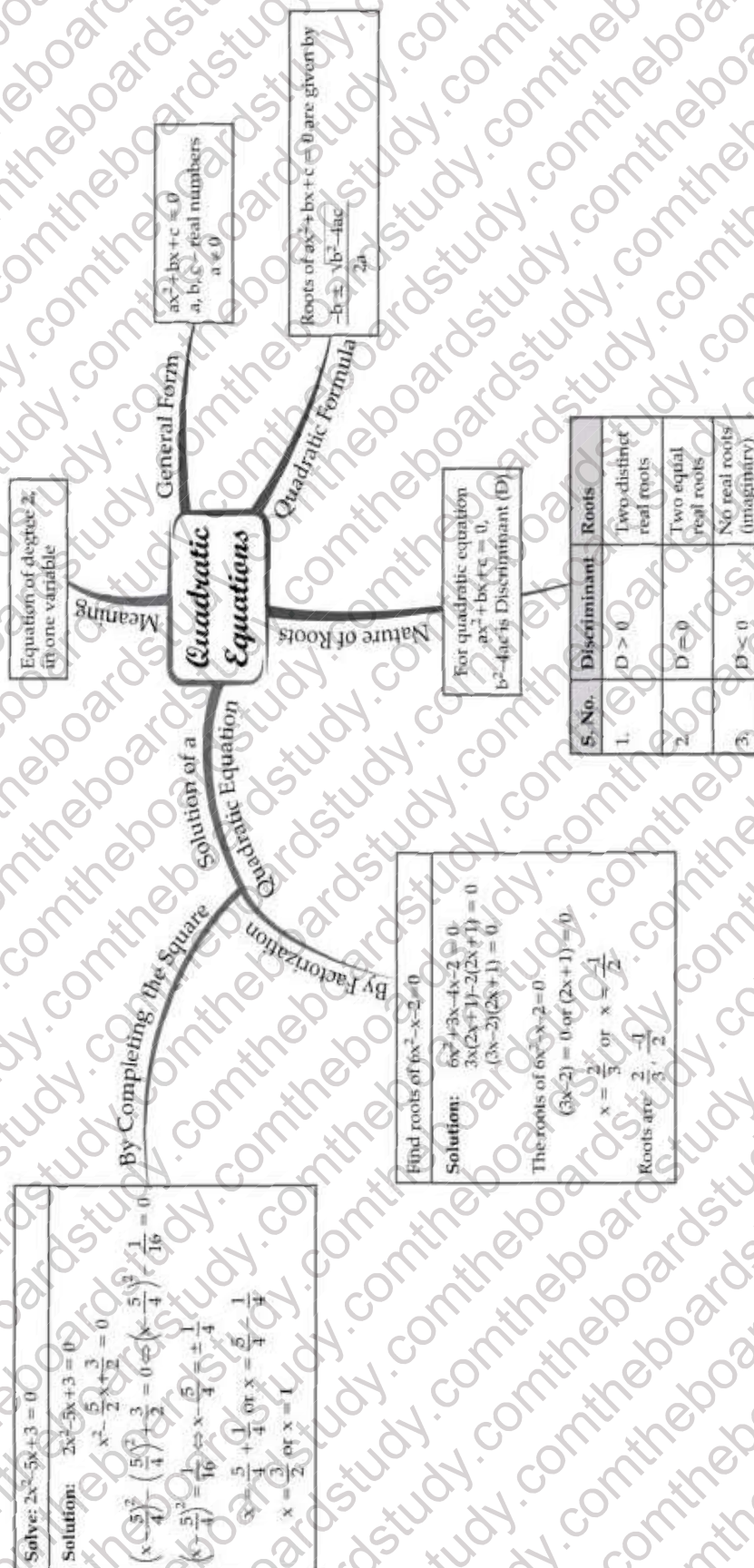
- i. If $b^2 - 4ac > 0$, the quadratic equation has **two distinct real roots**.
- ii. If $b^2 - 4ac = 0$, the quadratic equation has **two equal real roots**.
- iii. If $b^2 - 4ac < 0$, the quadratic equation has **no real roots**.

13. There are many equations which are not in the quadratic form but can be reduced to the quadratic form by simplifications.

14. Application of quadratic equations

- The applications of quadratic equation can be utilized in solving real life problems.
- Following points can be helpful in solving word problems:
 - i. Every two digit number 'xy' where x is a ten's place and y is a unit's place can be expressed as $xy = 10x + y$.
 - ii. Downstream: It means that the boat is running in the direction of the stream
Upstream: It means that the boat is running in the opposite direction of the stream
Thus, if Speed of boat in still water is x km/h
And the speed of stream is y km/h
Then the speed of boat downstream will be $(x + y)$ km/h and in upstream it will be $(x - y)$ km/h.
 - iii. If a person takes x days to finish a work, then his one day's work $= \frac{1}{x}$

MIND MAP : LEARNING MADE SIMPLE



Important Questions

Multiple Choice questions-

1. Which of the following is not a quadratic equation

(a) $x^2 + 3x - 5 = 0$

(b) $x^2 + x^3 + 2 = 0$

(c) $3 + x + x^2 = 0$

(d) $x^2 - 9 = 0$

2. The quadratic equation has degree

(a) 0

(b) 1

(c) 2

(d) 3

3. The cubic equation has degree

(a) 1

(b) 2

(c) 3

(d) 4

4. A bi-quadratic equation has degree

(a) 1

(b) 2

(c) 3

(d) 4

5. The polynomial equation $x(x+1)+8=(x+2)(x-2)$ is

(a) linear equation

(b) quadratic equation

(c) cubic equation

(d) bi-quadratic equation

6. The equation $(x - 2)^2 + 1 = 2x - 3$ is a

(a) linear equation

(b) quadratic equation

(c) cubic equation

(d) bi-quadratic equation

7. The quadratic equation whose roots are 1 and

(a) $2x^2 + x - 1 = 0$

(b) $2x^2 - x - 1 = 0$

(c) $2x^2 + x + 1 = 0$

(d) $2x^2 - x + 1 = 0$

8. The quadratic equation whose one rational root is $3 + \sqrt{2}$ is

(a) $x^2 - 7x + 5 = 0$

(b) $x^2 + 7x + 6 = 0$

(c) $x^2 - 7x + 6 = 0$

(d) $x^2 - 6x + 7 = 0$

9. The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is

(a) $\pm\sqrt{6}$

(b) ± 4

(c) $\pm 3\sqrt{2}$

(d) $\pm 2\sqrt{6}$

10. The sum of the roots of the quadratic equation $3x^2 - 9x + 5 = 0$ is

(a) 3

(b) 6

(c) -3

(d) 2

Very Short Questions:

1. What will be the nature of roots of quadratic equation $2x^2 + 4x - n = 0$?
2. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - 54 = 0$, then find the value of k .
3. If $ax^2 + bx + c = 0$ has equal roots, find the value of c .
4. If a and b are the roots of the equation $x^2 + ax - b = 0$, then find a and b .
5. Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.
6. Find the discriminant of the quadratic equation $4\sqrt{2}x^2 + 8x + 2\sqrt{2} = 0$.
7. State whether the equation $(x + 1)(x - 2) + x = 0$ has two distinct real roots or not. Justify your answer.
8. Is 0.3 a root of the equation $x^2 - 0.9 = 0$? Justify.
9. For what value of k , is 3 a root of the equation $2x^2 + x + k = 0$?
10. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

Short Questions :

1. Find the roots of the following quadratic equations by factorisation:
(i) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ (ii) $2x^2 - x + \frac{1}{8} = 0$
2. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:
(i) $2x^2 + x - 4 = 0$
(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$
3. Find the roots of the following quadratic equations by applying the quadratic formula.
(i) $2x^2 - 7x + 3 = 0$
(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$
4. Using quadratic formula solve the following quadratic equation:
 $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

5. Find the roots of the following equation:

$$\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}; \quad x \neq -3, 6$$

6. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $3x^2 - 4\sqrt{3}x + 4 = 0$ (ii) $2x^2 - 6x + 3 = 0$

7. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x-2) + 6 = 0$

8. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

9. If the equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1+m^2)$.

10. If $\sin \theta$ and $\cos \theta$ are roots of the equation $ax^2 + bx + c = 0$, prove that $a^2 - b^2 + 2ac = 0$.

Long Questions :

1. Using quadratic formula, solve the following equation for x :

$$abx^2 + (b^2 - ac)x - bc = 0$$

2. Find the value of p for which the quadratic equation

$$(2p+1)x^2 - (7p+2)x + (7p-3) = 0$$
 has equal roots. Also find these roots.

3. Solve for

$$x: \frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}; \quad x \neq 5, 7$$

4. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is Find his present age.

5. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

6. The sum of the squares of two consecutive odd numbers is 394. Find the

numbers.

7. The sum of two numbers is 15 and the sum of their reciprocals is 3. Find the numbers.
8. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.
9. A train travels 360 km at a uniform speed. If the speed has been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
10. The sum of the areas of two squares is 468 m^2 . If the difference of their perimeters is 24 m, find the sides of the two squares.

Case Study Question:

1. If $p(x)$ is a quadratic polynomial i.e., $p(x) = ax^2 + bx + c$, $a \neq 0$ then $p(x) = 0$ is called a quadratic equation. Now, answer the following questions.

i. Which of the following is correct about the quadratic equation $ax^2 + bx + c = 0$?

- a. a, b and c are real numbers $c \neq 0$
- b. a, b and c are rational numbers, $a \neq 0$
- c. a, b and c are integers, a, b and $c \neq 0$
- d. a, b and c are real numbers $a \neq 0$

ii. The degree of a quadratic equation is:

- a. 1
- b. 2
- c. 3
- d. Other than 1

iii. Which of the following is a quadratic equation?

- a. $x(x + 3) + 7 = 5x - 11$
- b. $(x - 1)^2 - 9 = (x - 4)(x + 3)$
- c. $x^2(2x + 1) - 4 = 5x^2 - 10$
- d. $x(x - 1)(x + 7) = x(6x - 9)$

iv. Which of the following is incorrect about the quadratic equation $ax^2 + bx + c = 0$?

- a. If $a\alpha^2 + b\alpha + c = 0$ then $x = -\alpha$ is the solution of the given quadratic equation.
- b. The additive inverse of zeroes of the polynomial $ax^2 + bx + c$ is the roots of the given equation.

- c. If α is a root of the given quadratic equation, then its other root is $-\alpha$
 d. All of these.

V. Which of the following is not a method of finding solutions of the given quadratic equation:

- a. Factorisation method
 b. Completing the square method
 c. Formula method
 d. None of these

2. Quadratic equations started around 3000 B.C. with the Babylonians. They were one of the world's first civilisation, and came up with some great ideas like agriculture, irrigation and writing. There were many reasons why Babylonians needed to solve quadratic equations. for example to know what amount of crop you can grow on the square field. Based on the above information, represent the following questions in the form of quadratic equation.

i. The sum of squares of two consecutive integers is 650.

- a. $x^2 + 2x - 650 = 0$
 b. $2x^2 + 2x - 649 = 0$
 c. $x^2 - 2x - 650 = 0$
 d. $2x^2 + 6x - 550 = 0$

ii. The sum of two numbers is 15 and the sum of their reciprocals is $\frac{1}{3} + \frac{1}{10}$.

- a. $x^2 + 10x - 150 = 0$
 b. $15x^2 - x + 150 = 0$
 c. $x^2 - 15x + 50 = 0$
 d. $3x^2 - 10x + 15 = 0$

iii. Two numbers differ by 3 and their product is 504.

- a. $3x^2 - 504 = 0$
 b. $x^2 - 504x + 3 = 0$
 c. $504x^2 + 3 = x$
 d. $x^2 + 3x - 504 = 0$

iv. A natural number whose square diminished by 84 is thrice of 8 more of given number.

- a. $x^2 + 8x - 84 = 0$
 b. $3x^2 - 84x + 3 = 0$
 c. $x^2 - 3x - 108 = 0$
 d. $x^2 - 11x + 60 = 0$

v. A natural number when increased by 12, equals 160 times its reciprocal.

- a. $x^2 - 12x + 160 = 0$
 b. $x^2 - 160x + 12 = 0$
 c. $12x^2 - x - 160 = 0$
 d. $x^2 + 12x - 160 = 0$

Assertion Reason Questions-

1. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- a. Both A and R are true and R is the correct explanation for A.
- b. Both A and R are true and R is the correct explanation for A.
- c. A is true but R is false.
- d. A is false but R is true.

Assertion: The product of two successive positive integral multiples of 5 is 300, then the two numbers are 15 and 20.

Reason: The product of two consecutive integers is a multiple of 2.

2. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- a. Both A and R are true and R is the correct explanation for A.
- b. Both A and R are true and R is the correct explanation for A.
- c. A is true but R is false.
- d. A is false but R is true.

Assertion: The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.

Reason: If discriminant $D = b^2 - 4ac < 0$ then the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary.

Answer Key-

Multiple Choice questions-

1. (b) -10
2. (b) $x^2 + x + 2 = 0$
3. (c) 2
4. (c) 3
5. (d) 4
6. (a) linear equation
7. (b) quadratic equation
8. (b) $2x^2 - x - 1 = 0$
9. (d) $x^2 - 6x + 7 = 0$
10. (d) $\pm 2\sqrt{6}$
11. (c) -3

Very Short Answer :

1. $D = b^2 - 4ac$

$$\Rightarrow 42 - 4 \times 2 \times (-7)$$

$$\Rightarrow 16 + 56 = 72 > 0$$

Hence, roots of quadratic equation are real and unequal.

2. $\therefore \frac{1}{2}$ is a root of quadratic equation.

$\therefore$ It must satisfy the quadratic equation.

$$x^2 + kx - \frac{5}{4} = 0$$

$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \Rightarrow \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0 \Rightarrow 2k - 4 = 0$$

$$\Rightarrow k = 2$$

3. For equal roots $D = 0$

$$\text{i.e., } b^2 - 4ac = 0$$

$$\Rightarrow b^2 = 4ac$$

$$\Rightarrow c = \frac{b^2}{4a}$$

$$4. \text{ Sum of the roots} = a + b = -\frac{B}{A} = -a$$

$$\text{Product of the roots} = ab = \frac{B}{A} = -b$$

$$= a + b = -a \text{ and } ab = -b$$

$$\Rightarrow 2a = -b \text{ and } a = -1$$

$$\Rightarrow b = 2 \text{ and } a = -1$$

$$5. \text{ Put the value of } x \text{ in the quadratic equation,}$$

$$\Rightarrow \text{LHS} = 3x^2 + 13x + 14$$

$$\Rightarrow 3(-2)^2 + 13(-2) + 14$$

$$\Rightarrow 12 - 26 + 14 = 0$$

$$\Rightarrow \text{RHS Hence, } x = -2 \text{ is a solution.}$$

$$6. D = 62 - 4ac = (8)^2 - 4(4\sqrt{2})(2\sqrt{2})$$

$$\Rightarrow 64 - 64 = 0$$

$$7. (x+1)(x-2) + x = 0$$

$$\Rightarrow x^2 - x - 2 + x = 0$$

$$\Rightarrow x^2 - 2 = 0$$

$$D = b^2 - 4ac$$

$$\Rightarrow (-4)(1)(-2) = 8 > 0$$

$\therefore$ Given equation has two distinct real roots.

$$8. \therefore 0.3 \text{ is a root of the equation } x^2 - 0.9 = 0$$

$$\therefore x^2 - 0.9 = (0.3)^2 - 0.9 = 0.09 - 0.9 \neq 0$$

Hence, 0.3 is not a root of given equation.

$$9. 3 \text{ is a root of } 2x^2 + x + k = 0, \text{ when}$$

$$\Rightarrow 2(3)^2 + 3 + k = 0$$

$$\Rightarrow 18 + 3 + k = 0$$

$$\Rightarrow k = -21$$

10. For equal roots:

$$D = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (-3k)^2 - 4 \times 9 \times k = 0$$

$$\Rightarrow 9k^2 = 36k$$

$$\Rightarrow k = 4$$

Short Answer :

1. (i) We have, $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

$$= (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

$$\therefore \text{Either } \sqrt{2}x + 5 = 0 \text{ or } x + \sqrt{2} = 0$$

$$\therefore x = -\frac{5}{\sqrt{2}} \text{ or } x = -\sqrt{2}$$

$$\text{Hence, the roots are } -\frac{5}{\sqrt{2}} \text{ and } -\sqrt{2}.$$

(ii) We have, $2x^2 - x + 18 = 0$

$$\Rightarrow \frac{16x^2 - 8x + 1}{8} = 0 \Rightarrow 16x^2 - 8x + 1 = 0$$

$$\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$$

$$\Rightarrow (4x - 1)(4x - 1) = 0$$

$$\text{So, either } 4x - 1 = 0 \text{ or } 4x - 1 = 0$$

$$x = \frac{1}{4} \text{ or } x = \frac{1}{4}$$

$$\text{Hence, the roots of the given equation are } \frac{1}{4} \text{ and } \frac{1}{4}.$$

2. (i) We have, $2x^2 + x - 4 = 0$

On dividing both sides by 2, we have

$$\begin{aligned} x^2 + \frac{x}{2} - 2 &= 0 \\ \Rightarrow x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 &= 0 & \left[b = \frac{1}{2} \text{ (coefficient of } x) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \right] \\ \Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 &= 0 \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + 2 = \frac{1+32}{16} = \frac{33}{16} > 0 \\ \Rightarrow \text{Roots exist.} \\ \therefore x + \frac{1}{4} &= \pm \sqrt{\frac{33}{16}} \Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{4} \\ \Rightarrow x + \frac{1}{4} &= \frac{\sqrt{33}}{4} \quad \text{or} \quad x + \frac{1}{4} = -\frac{\sqrt{33}}{4} \\ \therefore x &= -\frac{1}{4} + \frac{\sqrt{33}}{4} \quad \text{or} \quad x = -\frac{1}{4} - \frac{\sqrt{33}}{4} \\ \Rightarrow x &= \frac{\sqrt{33}-1}{4} \quad \text{or} \quad x = \frac{-(\sqrt{33}+1)}{4} \end{aligned}$$

Hence, roots of given equation are $\frac{\sqrt{33}-1}{4}$ and $\frac{-(\sqrt{33}+1)}{4}$.

- (ii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

On dividing both sides by 4, we have

$$\begin{aligned} x^2 + \sqrt{3}x + \frac{3}{4} &= 0 \Rightarrow x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0 \\ \Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} &= 0 \Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0 \quad \dots(i) \\ \Rightarrow \text{Roots exist.} \therefore (i) &\Rightarrow x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2} \end{aligned}$$

Hence, roots of given equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

3. (i) We have, $2x^2 - 7x + 3 = 0$

Here, $a = 2$, $b = -7$ and $c = 3$

Therefore, $D = b^2 - 4ac$

$$\Rightarrow D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$$

$\therefore D > 0$, $\therefore$ roots exist.

$$\text{Thus, } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-7) \pm \sqrt{25}}{2 \times 2} = \frac{7 \pm 5}{4}$$

$$x = \frac{7+5}{4} \quad \text{or} \quad \frac{7-5}{4}$$

$$= 3 \quad \text{or} \quad \frac{1}{2}$$

So, the roots of given equation are 3 and $\frac{1}{2}$.

(ii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

Here, $a = 4$, $b = 4\sqrt{3}$ and $c = 3$

$$\text{Therefore, } D = b^2 - 4ac = (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$$

$\therefore D = 0$, roots exist and are equal.

$$\text{Thus, } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-4\sqrt{3} \pm 0}{2 \times 4} = \frac{-\sqrt{3}}{2}$$

Hence, the roots of given equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

4. We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$$a = p^2, b = p^2 - q^2 \text{ and } c = -q^2$$

$$\therefore D = b^2 - 4ac$$

$$\Rightarrow (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$$

$$\Rightarrow (p^2 - q^2)^2 + 4p^2q^2$$

$$\Rightarrow (p^2 + q^2)^2 > 0$$

So, the given equation has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$

Hence, roots are $\frac{q^2}{p^2}$ and -1 .

5.

Given, $\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}; x \neq -3, 6$

$$\Rightarrow \frac{(x-6) - (x+3)}{(x+3)(x-6)} = \frac{9}{20} \Rightarrow \frac{-9}{(x+3)(x-6)} = \frac{9}{20}$$

$$\Rightarrow (x+3)(x-6)$$

$$\Rightarrow -20 \text{ or } x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-1)(x-2) = 0$$

$$\Rightarrow x = 1 \text{ or } x = 2$$

Both $x = 1$ and $x = 2$ are satisfying the given equation. Hence, $x = 1, 2$ are the solutions of the equation.

6. (i) We have, $3x^2 - 4\sqrt{3}x + 4 = 1$

Here, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$

Therefore,

$$D = b^2 - 4ac$$

$$\Rightarrow (-4\sqrt{3})^2 - 4 \times 3 \times 4$$

$$\Rightarrow 48 - 48 = 0$$

Hence, the given quadratic equation has real and equal roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4\sqrt{3}) \pm \sqrt{0}}{2 \times 3} = \frac{2\sqrt{3}}{3}$

Hence, equal roots of given equation are $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

(ii) We have, $2x^2 - 6x + 3 = 0$

Here, $a = 2, b = -6, c = 3$

Therefore, $D = b^2 - 4ac$

$= (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

Hence, given quadratic equation has real and distinct roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence, roots of given equation are $\frac{3 + \sqrt{3}}{2}$ and $\frac{3 - \sqrt{3}}{2}$

7. (i) We have, $2x^2 + kx + 3 = 0$

Here, $a = 2, b = k, c = 3$

$D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$ For equal roots

$D = 0$

i.e., $k^2 - 24 = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \pm \sqrt{24}$

$\Rightarrow k = \pm 2\sqrt{6}$

(ii) We have, $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

Here, $a = k, b = -2k, c = 6$

For equal roots, we have

$D = 0$

i.e., $b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4 \times k \times 6 = 0$

$$\Rightarrow 4k^2 - 24k = 0$$

$$\Rightarrow 4k(k - 6) = 0$$

Either $4k = 0$ or $k - 6 = 0$

$$\Rightarrow k = 0 \text{ or } k = 6$$

But $k = 0$ because if $k = 0$ then given equation will not be a quadratic equation).

So, $k = 6$.

8. Since the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ has equal roots, therefore discriminant

$$D = (b - c)^2 - 4(a - b)(c - a) = 0$$

$$\Rightarrow b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab)$$

$$\Rightarrow b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$

$$\Rightarrow 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$$

$$\Rightarrow (2a)^2 + (-b)^2 + (-c)^2 + 2(2a)(-b) + 2(-b)(-c) + 2(-c)(2a) = 0$$

$$\Rightarrow (2a - b - c)^2 = 0$$

$$\Rightarrow 2a - b - c = 0$$

$$\Rightarrow 2a = b + c. \text{ Hence Proved.}$$

9. The given equation is $(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore $D = 0 = B^2 - 4AC = 0$.

$$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$

$$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$

$$\Rightarrow m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0. \text{ [Dividing throughout by 4]}$$

$$\Rightarrow -c^2 + a^2(1 + m^2) = 0$$

$$\Rightarrow c^2 = a(1 + m^2) \text{ Hence Proved}$$

10.

$$\text{Sum of the roots} = \frac{-B}{A} \Rightarrow \sin \theta + \cos \theta = \frac{-b}{a} \dots (i)$$

$$\text{Product of the roots} = \frac{C}{A} \Rightarrow \sin \theta \cdot \cos \theta = \frac{c}{a} \dots (ii)$$

$$\text{Now, we have, } \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow (\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta = 1 \Rightarrow \left(\frac{-b}{a} \right)^2 - 2 \cdot \frac{c}{a} = 1$$

$$\Rightarrow \frac{b^2}{a^2} - \frac{2c}{a} = 1 \quad \text{or} \quad b^2 - 2ac = a^2$$

$$\Rightarrow a^2 - b^2 + 2ac = 0$$

Long Answer :

1. We have, $abx^2 + (b^2 - ac)x - bc = 0$

Here, $A = ab$, $B = b^2 - ac$, $C = -bc$

$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 - 4(ab)(-bc)}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 + 4ab^2c}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^4 - 2ab^2c + a^2c^2 + 4ab^2c)}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 + ac)^2}}{2ab} \Rightarrow x = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} \quad \text{or} \quad x = \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$x = \frac{2ac}{2ab} \quad \text{or} \quad x = \frac{-2b^2}{2ab} \Rightarrow x = \frac{c}{b} \quad \text{or} \quad x = -\frac{b}{a}$$

2. Since the quadratic equation has equal roots, $D = 0$

i.e., $b^2 - 4ac = 0$

$$\text{In } (2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$$

Here, $a = (2p + 1)$, $b = -(7p + 2)$, $c = (7p - 3)$

For $p = -\frac{4}{7}$

$$\left(2 \times \left(-\frac{4}{7}\right) + 1\right)x^2 - \left(7 \times \left(-\frac{4}{7}\right) + 2\right)x + \left(7 \times \left(-\frac{4}{7}\right) - 3\right) = 0$$

$$\Rightarrow \frac{-1}{7}x^2 + 2x - 7 = 0 \Rightarrow x^2 - 14x + 49 = 0$$

$$\Rightarrow x^2 - 7x - 7x + 49 = 0 \Rightarrow x(x - 7) - 7(x - 7) = 0$$

$$\Rightarrow (x - 7)^2 = 0 \Rightarrow x = 7, 7$$

For $p = 4$,

$$(2 \times 4 + 1)x^2 - (7 \times 4 + 2)x + (7 \times 4 - 3) = 0$$

$$\Rightarrow 9x^2 - 30x + 25 = 0 \Rightarrow 9x^2 - 15x - 15x + 25 = 0$$

$$\Rightarrow 3x(3x - 5) - 5(3x - 5) = 0 \Rightarrow (3x - 5)(3x - 5) = 0$$

$$\Rightarrow x = \frac{5}{3}, \frac{5}{3}$$

3.

$$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3} \Rightarrow \frac{(x-4)(x-7) + (x-6)(x-5)}{(x-5)(x-7)} = \frac{10}{3}$$

$$\Rightarrow \frac{x^2 - 7x - 4x + 28 + x^2 - 5x - 6x + 30}{x^2 - 7x - 5x + 35} = \frac{10}{3}$$

$$\Rightarrow \frac{2x^2 - 22x + 58}{x^2 - 12x + 35} = \frac{10}{3} \Rightarrow \frac{x^2 - 11x + 29}{x^2 - 12x + 35} = \frac{5}{3}$$

$$\Rightarrow 3x^2 - 33x + 87 = 5x^2 - 60x + 175 \Rightarrow 2x^2 - 27x + 88 = 0$$

$$\Rightarrow 2x^2 - 16x - 11x + 88 = 0 \Rightarrow 2x(x - 8) - 11(x - 8) = 0$$

$$\Rightarrow (2x - 11)(x - 8) = 0 \Rightarrow 2x - 11 = 0 \text{ or } x - 8 = 0$$

$$\Rightarrow x = \frac{11}{2} \text{ or } x = 8$$

4. Let the present age of Rehman be x years.

So, 3 years ago, Rehman's age = $(x - 3)$ years

And 5 years from now, Rehman's age = $(x + 5)$ years

Now, according to question, we have

$$\begin{aligned}
 \frac{1}{x-3} + \frac{1}{x+5} &= \frac{1}{3} \\
 \Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} &= \frac{1}{3} & \Rightarrow \frac{2x+2}{(x-3)(x+5)} &= \frac{1}{3} \\
 \Rightarrow 6x+6 &= (x-3)(x+5) & \Rightarrow 6x+6 &= x^2+5x-3x-15 \\
 \Rightarrow x^2+2x-15-6x-6 &= 0 & \Rightarrow x^2-4x-21 &= 0 \\
 \Rightarrow x^2-7x+3x-21 &= 0 & \Rightarrow x(x-7)+3(x-7) &= 0 \\
 \Rightarrow (x-7)(x+3) &= 0 & \Rightarrow x &= 7 \text{ or } x = -3
 \end{aligned}$$

But $x \neq -3$ (age cannot be negative)

Therefore, present age of Rehman = 7 years.

5. Let the two natural numbers be x and y such that $x > y$.

According to the question,

Difference of numbers, $x - y = 5 \Rightarrow x = 5 + y$ (i)

Difference of the reciprocals,

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{10} \quad \dots(ii)$$

Putting the value of (i) in (ii)

$$\begin{aligned}
 \frac{1}{y} - \frac{1}{5+y} &= \frac{1}{10} & \Rightarrow \frac{5+y-y}{y(5+y)} &= \frac{1}{10} \\
 \Rightarrow 50 &= 5y + y^2 & \Rightarrow y^2 + 5y - 50 &= 0 \\
 \Rightarrow y^2 + 10y - 5y - 50 &= 0 & \Rightarrow y(y+10) - 5(y+10) &= 0 \\
 \Rightarrow (y-5)(y+10) &= 0
 \end{aligned}$$

$\therefore y$ is a natural number.

$\therefore y = 5$

Putting the value of y in (i), we have

$$\Rightarrow x = 5 + 5$$

$$\Rightarrow x = 10$$

The required numbers are 10 and 5.

6. Let the two consecutive odd numbers be x and $x + 2$.

$$\begin{aligned} \Rightarrow x^2 + (x+2)^2 &= 394 & \Rightarrow x^2 + x^2 + 4 + 4x &= 394 \\ \Rightarrow 2x^2 + 4x + 4 &= 394 & \Rightarrow 2x^2 + 4x - 390 &= 0 \\ \Rightarrow x^2 + 2x - 195 &= 0 & \Rightarrow x^2 + 15x - 13x - 195 &= 0 \\ \Rightarrow x(x+15) - 13(x+15) &= 0 & \Rightarrow (x-13)(x+15) &= 0 \\ \Rightarrow x-13=0 \text{ or } x+15=0 & \Rightarrow x=13 \text{ or } x=-15 \end{aligned}$$

Hence, the numbers are 13 and 15 or -15 and -13.

7. Let the numbers be x and $15-x$.

According to given condition,

$$\begin{aligned} \frac{1}{x} + \frac{1}{15-x} &= \frac{3}{10} & \Rightarrow \frac{15-x+x}{x(15-x)} &= \frac{3}{10} \\ \Rightarrow 150 &= 3x(15-x) \\ \Rightarrow 50 &= 15x - x^2 \\ \Rightarrow x^2 - 15x + 50 &= 0 \\ \Rightarrow x^2 - 5x - 10x + 50 &= 0 \\ \Rightarrow x(x-5) - 10(x-5) &= 0 \\ \Rightarrow (x-5)(x-10) &= 0 \\ \Rightarrow x &= 5 \text{ or } 10. \end{aligned}$$

When $x = 5$, then $15 - x = 15 - 5 = 10$

When $x = 10$, then $15 - x = 15 - 10 = 5$

Hence, the two numbers are 5 and 10.

8. Let Shefali's marks in Mathematics be x .

Therefore, Shefali's marks in English is $(30 - x)$.

Now, according to question,

$$\begin{aligned} \Rightarrow (x+2)(30-x-3) &= 210 \\ \Rightarrow (x+2)(27-x) &= 210 \\ \Rightarrow 27x - x^2 + 54 - 2x &= 210 \\ \Rightarrow 25x - x^2 + 54 - 210 &= 0 \end{aligned}$$

$$\Rightarrow 25x - x^2 - 156 = 0$$

$$\Rightarrow -(x^2 - 25x + 156) = 0$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

$$= x^2 - 13x - 12x + 156 = 0$$

$$\Rightarrow x(x - 13) - 12(x - 13) = 0$$

$$\Rightarrow (x - 13)(x - 12) = 0$$

$$\text{Either } x - 13 \text{ or } x - 12 = 0$$

$$\therefore x = 13 \text{ or } x = 12$$

Therefore, Shefali's marks in Mathematics = 13

$$\text{Marks in English} = 30 - 13 = 17$$

or Shefali's marks in Mathematics = 12

$$\text{marks in English} = 30 - 12 = 18.$$

9. Let the uniform speed of the train be x km/h.

$$\text{Then, time taken to cover 360 km} = \frac{360}{x} \text{ h}$$

$$\text{Now, new increased speed} = (x + 5) \text{ km/h}$$

$$\text{So, time taken to cover 360 km} = \frac{360}{x+5} \text{ h}$$

$$\text{According to question, } \frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow \frac{360}{x} \left(\frac{1}{x} - \frac{1}{x+5} \right) = 1 \quad \Rightarrow \frac{360(x+5-x)}{x(x+5)} = 1$$

$$\Rightarrow \frac{360 \times 5}{x(x+5)} = 1 \quad \Rightarrow 1800 = x^2 + 5x$$

$$\therefore x^2 + 5x - 1800 = 0 \quad \Rightarrow x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow x(x + 45) - 40(x + 45) = 0 \quad \Rightarrow (x + 45)(x - 40) = 0$$

$$\text{Either } x + 45 = 0 \text{ or } x - 40 = 0$$

$$\therefore x = -45 \text{ or } x = 40$$

But x cannot be negative, so $x \neq -45$

therefore, $x = 40$

Hence, the uniform speed of train is 40 km/h

10. Let x be the length of the side of first square and y be the length of side of the second square.

$$\text{Then, } x^2 + y^2 = 468 \dots (i)$$

Let x be the length of the side of the bigger square.

$$4x - 4y = 24$$

$$\Rightarrow x - y = 6 \text{ or } x = y + 6 \dots (ii)$$

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$$(y + 6)^2 + y^2 = 468$$

$$\Rightarrow y^2 + 12y + 36 + y^2 = 468 \text{ or } 232 + 12y - 432 = 0$$

$$\Rightarrow y^2 + 6y - 216 = 0$$

$$\Rightarrow y^2 + 18y - 12y - 216 = 0$$

$$\Rightarrow y(y + 18) - 12(y + 18) = 0$$

$$\Rightarrow (y + 18)(y - 12) = 0$$

$$\text{Either } y + 18 = 0 \text{ or } y - 12 = 0$$

$$\Rightarrow y = -18 \text{ or } y = 12$$

But, sides cannot be negative, so $y = 12$

$$\text{Therefore, } x = 12 + 6 = 18$$

Hence, sides of two squares are 18 m and 12 m.

Case Study Answers:

1. Answer :

- i. (d) a, b and c are real numbers $a \neq 0$
- ii. (b) 2
- iii. (a) $x(x + 3) + 7 = 5x - 11$

Solution:

$$\text{a. } x(x + 3) + 7 = 5x - 11$$

$$\Rightarrow x^2 + 3x + 7 = 5x - 11$$

$$\Rightarrow x^2 - 2x + 18 = 0 \text{ is a quadratic equation.}$$

b. $(x - 1)^2 - 9 = (x - 4)(x + 3)$

$$\Rightarrow x^2 - 2x - 8 = x^2 - x - 12$$

$$\Rightarrow x - 4 = 0 \text{ is not a quadratic equation.}$$

c. $x^2(2x + 1) - 4 = 5x^2 - 10$

$$\Rightarrow 2x^3 + x^2 - 4 = 5x^2 - 10$$

$$\Rightarrow 2x^3 - 4x^2 + 6 = 0 \text{ is not a quadratic equation.}$$

d. $x(x - 1)(x + 7) = x(6x - 9)$

$$x^3 + 6x^2 - 7x = 6x^2 - 9x$$

$$x^3 + 2x = 0 \text{ is not a quadratic equation.}$$

iv. (d) All of these.

v. (d) None of these

2. Answer :

i. (b) $2x^2 + 2x - 649 = 0$

Solution:

Let two consecutive integers be $x, x + 1$. Given, $x^2 + (x + 1)^2 = 650$.

$$\Rightarrow 2x^2 + 2x + 1 - 650 = 0$$

$$\Rightarrow 2x^2 + 2x - 649 = 0$$

ii. (c) $x^2 - 15x + 50 = 0$

Solution:

Let the two numbers be x and $15 - x$. Given $1x + 115 - x = 310$

$$\Rightarrow 10(15 - x + x) = 3x(15 - x)$$

$$\Rightarrow 50 = 15x - x^2$$

$$\Rightarrow x^2 - 15x + 50 = 0$$

iii. (d) $x^2 + 3x - 504 = 0$

Solution:

Let the numbers be x and $x + 3$. Given, $x(x + 3) = 504$

$$\Rightarrow x^2 + 3x - 504 = 0$$

iv. (c) $x^2 - 3x - 108 = 0$

Solution:

Let the number be x . According to question, $x^2 - 84 = 3(x + 8)$

$$\Rightarrow x^2 - 84 = 3x + 24$$

$$\Rightarrow x^2 - 3x - 108 = 0$$

v. (d) $x^2 + 12x - 160 = 0$

Solution:

Let the number be x . According to question $x+12=160$
 $xx+12=160x$

$$\Rightarrow x^2 + 12x - 160 = 0$$

Assertion Reason Answer-

1. (b) Both A and R are true and R is the correct explanation for A.
2. (c) A is true but R is false.