EXERCISE 2.1

1. If (x/3 + 1, y - 2/3) = (5/3, 1/3), find the values of x and y

Solution:

It is given that (x/3 + 1, y - 2/3) = (5/3, 1/3)

Since the two ordered pairs are equal, the corresponding elements will also be equal

Therefore,

x/3 + 1 = 5/3 and y - 2/3 = 1/3

x/3 + 1 = 5/3 and y - 2/3 = 1/3

⇒ x/3 = 5/3 - 1 and ⇒ y = 1/3 + 2/3

⇒ x/3 = 2/3 and ⇒ y = 1

⇒ x = 2

Hence,

x = 2, y = 1

2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A×B)

Solution:

It is given that set A has 3 elements and the set B = {3, 4, 5}

Number of elements in set A, n (A) = 3

Number of elements in set B, n (B) = 3

Number of elements in set A, n (A) = 3

Number of elements in set B, n (B) = 3

Number of elements in (A x B) = (Number of elements in A)´(Number of elements in B)

n (A x B) = n (A) x n (B)

= 3 x 3

= 9

Thus, the number of elements in (A x B) is 9.

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G

Solution:

It is given that G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P x Q of two non-empty sets P and Q is defined as

P x Q = {(p, q) : p ∈ P, q ∈ Q}

Therefore,

G x H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H x G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

4. State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly
(i) If P = {m, n} and Q = { n, m}, then P × Q = {(m, n),(n, m)}
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B
(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ φ) = φ

Solution:

(i) False,

⇒ P x Q = {(m, n), (m, m), (n, n), (n, m)}

(ii) True

(iii) True

5. If A = {- 1, 1}, find A x A x A

Solution:

It is known that for any non-empty set A,

A x A x A is defined as

A x A x A = {(a, b, c) : a, b, c ∈ A}

It is given that A = {- 1, 1}

Therefore,

$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

6. If $A \times B = \{(a, x),(a , y), (b, x), (b, y)\}$. Find A and B

Solution:

It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

Therefore, A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

7. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that
(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(ii) $A \times C$ is a subset of $B \times D$

Solution:

(i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

LHS = $A \times (B \cap C)$

$= A \times \Phi$

$= \Phi$

Now,

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2,1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

Therefore,

RHS = $(A \times B) \cap (A \times C)$

$= \Phi$

Therefore, L.H.S. = R.H.S

Hence, $A \times (B \cap C) = (A \cap B) \cap (A \cap C)$

(ii) To verify: $A \times C$ is a subset of $B \times D$.

We have

A x C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B x D = {(1, 5),(1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A x C are the elements of set B x D .

Therefore, A x C is a subset of B x D

8.Let A = {1, 2} and B = {3, 4} . Write A x B . How many subsets will A x B have? List them

Solution:

A = {1, 2} and B = {3, 4}

Therefore, A x B = {(1, 3), (1, 4), (2, 3), (2, 4)}

Hence, n (A x B) = 4

We know that if C is a set with n (C) = m , then n [P (C)] = 2^m

Therefore, the set A x B has 2^4 = 16 subsets.

These are

{Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)},

{(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)},

{(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},

{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)},

{(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)},{(1, 3), (1, 4), (2, 3), (2, 4)}}

9. Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements

Solution:

It is given that n (A) = 3 and n (B) = 2 .

If (x, 1), (y, 2), (z, 1) are in A x B.

We know that

A = Set of first elements of the ordered pair elements of A x B.

B = Set of second elements of the ordered pair elements of A x B.

Therefore, x, y, and z are the elements of A;

1 and 2 are the elements of B.

Since n (A) = 3 and n (B) = 2,

It is clear that A = {x, y, z} and B = {1, 2}

10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0,1). Find the set A and the remaining elements of A × A.

Solution:

We know that if n (A) = p and n (B) = q ,

then n (A x B) = pq

Therefore, n (A x A) = n (A) x n (A)

It is given that n (A x A) = 9

n (A) x n (A) = 9

Hence,

n (A) = 3

The ordered pairs (- 1, 0) and (0, 1) are two of the nine elements of A x A.

We know that A x A = {(a, a) : a ∈ A} .

Therefore, - 1, 0, and 1 are elements of A.

Since n (A) = 3 , it is clear that A = {- 1, 0, 1}

The remaining elements of set A´ A are (- 1, - 1), (- 1, 1), (0, - 1), (0, 0), (1, - 1), (1, 0), and (1, 1)