EXERCISE 2.2

1. Let A = {1, 2, 3,...,14}. Define a relation R from A to A by R = {(x, y) : 3x – y = 0, where x, y ∈ A}. Write down its domain, codomain and range

Solution:

The relation R from A to A is given as R = {(x, y), 3x - y = 0; x, y ∈ A}.

Thus, R = {(x, y), 3x = y; x, y ∈ A}.

Therefore,

R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of R = {1, 2, 3, 4}

The whole set A is the co-domain of the relation R.

Therefore, Co-domain of R = A = {1, 2, 3,...., 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of R = {3, 6, 9, 12}

2. Define a relation R on the set N of natural numbers by
R = {(x, y) : y = x + 5, x is a natural number less than 4; x, y ∈ N}
Depict this relationship using roster form. Write down the domain and the range

Solution:

R = {(x, y) : y = x + 5, x is a natural number less than 4; x, y ∈ N}

The natural numbers less than 4 are 1, 2, and 3.

Therefore, R = {(1, 6), (2, 7), (3, 8)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of R = {1, 2, 3}.

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, range of R = {6, 7, 8}

3. A = {1, 2, 3, 5} and B = {4, 6, 9} . Define a relation R from A to B by R = {(x, y) : the difference between x and y is odd; x ∈ A, y ∈ B}
Write R in roster form

Solution:

From the given data in the problem, we have

A = {1, 2, 3, 5},

B = {4, 6, 9} and

The relation from A to B is given as:

R = {(x, y) : the difference between x and y is odd; x ∈ A, y ∈ B}

Therefore,

R in roster form can be written as

R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

4. The Fig2.7 shows a relationship between the sets P and Q. Write this relation (i) in set-builder form (ii) in roster form

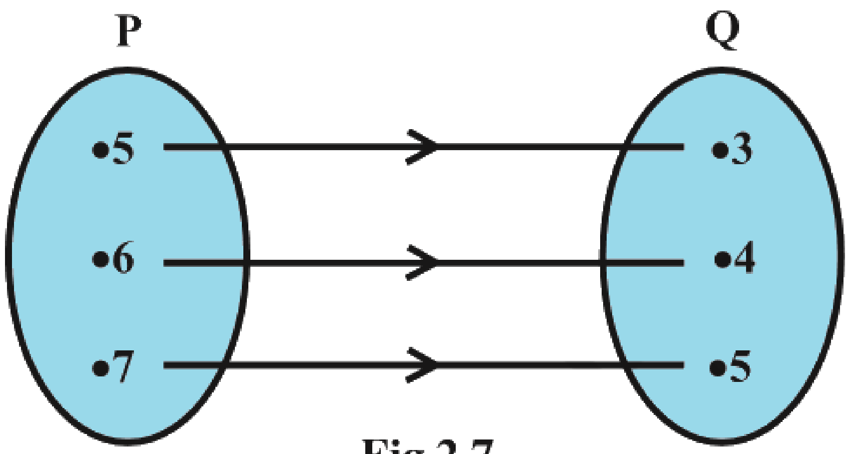

Fig 2.7

What is its domain and range?

As per the Fig 2.7

P = {5, 6, 7} and Q = {3, 4, 5}

(i) R = {(x, y) : y = x - 2; x ∈ P}

or R = {(x, y) : y = x - 2, for x = 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

The domain of a function is the set of all possible inputs for the function.

The range of a function is the set of all its outputs.

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by{(a, b): a, b ∈ A, b is exactly divisible by a}
(i) Write R in roster form (ii) Find the domain of R (iii) Find the range of R

Solution:

It is given that A = {1, 2, 3, 4, 6} and

R = {(a, b) : a, b ∈ A, b is exactly divisible by a}

The domain of a function is the set of all possible inputs for the function.

The range of a function is the set of all its outputs.

(i) R

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R

= {1, 2, 3, 4, 6}

(iii) Range of R

= {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by R = {(x, x + 5) : x ∈ {0,1, 2, 3, 4, 5}}

Solution:

It is given that

R = {(x, x + 5) : x ∈ {0,1, 2, 3, 4, 5}}

Therefore, R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

The domain of a function is the set of all possible inputs for the function.

The range of a function is the set of all its outputs.

Hence,

Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

7. Write the relation R = {(x, x^3) : x is a prime number less than 10} in roster form

Solution:

It is given that

R = {(x, x^3) : x is a prime number less than 10}.

The prime numbers less than 10 are given by

2, 3, 5, and 7.

In roster notation,

the elements of a set are represented in a row surrounded by curly brackets.

and if the set contains more than one element then every two elements are separated by commas.

Therefore,

R in roster form can be written as :

R = {(2, 8), (3, 27), (5, 125), (7, 343)}

8. Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B

Solution:

It is given that

A = {x, y, z}

and B = {1, 2}

Therefore,

A x B = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}

Since n (A x B) = 6,

the number of subsets of (A x B)

$= 2^6$.

Hence, the number of relations from A to B is 2^6.

9. Let R be the relation on Z defined by R = {(a,b): a, b ∈ Z, a – b is an integer}. Find the domain and range of R

Solution:

It is given that

R = {(a, b) : a, b ∈ Z, a - b is an integer}.

It is known that the difference between any two integers is always an integer.

The domain of a function is the set of all possible inputs for the function.

The range of a function is the set of all its outputs.

Therefore,

The domain of R = Z

The Range of R = Z

Here Z is denoted as an integer