EXERCISE 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range
(i) {(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)}
(ii) {(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)}
(iii) {(1,3), (1,5), (2,5)}

Solution:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12 and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and

range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element that is, 1 corresponds to two different images that are, 3 and 5, this relation is not a function.

2. Find the domain and range of the following real functions: (i) f (x) = - |x| (ii) f (x) = √(9 - x^2)

Solution:

(i) f (x) = - |x| x ∈R

We know that, |x| = {x, if x ≥ 0; - x if x < 0}

Therefore, f (x) = |- x| = {- x, if x ≥ 0; x if x < 0}

Since f (x) is defined for x Î R, the domain of f = R

It can be observed that the range of f (x) = - |x| is all real numbers except positive real numbers.

Therefore,

the range of is f = (- ∞, 0)

(ii) f (x) = √(9 - x^2)

Since√(9 - x^2) is defined for all real numbers that are greater than or equal to - 3 and less than or equal to 3,

the domain of f(x) is {x : - 3 ≤ x ≤ 3} or [- 3, 3].

For any value of x such that - 3 ≤ x ≤ 3, the value of f (x) will lie between 0 and 3.

Therefore, the range of f (x) is {x : 0 ≤ x ≤ 3} or [0, 3]

3. A function f is defined by f (x) = 2x - 5. Write down the values of:
(i) f (0) (ii) f (7) (iii) f (- 3)

Solution:

In mathematics, a function means a correspondence from one value x of the first set to another value y of the second set.

From the given function:

f(x) = 2x - 5

(i) f(0) = 2 x 0 - 5

= 0 - 5 = - 5

(ii) f(7) = 2 x 7 - 5

= 14 - 5 = 9

(iii) f(- 3) = 2 x (- 3) - 5

= - 6 - 5 = - 11

4. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by t(C) = 9C/5 + 32.
Find (i) t(0) (ii) t(28) (iii) t(–10) (iv) The value of C, when t(C) = 212

Solution:

The given function of temperature is t (C) = 9C/5 + 32

(i) t (0) = = (9 x 0) / 5 + 32 = 0 + 32 = 32

(ii) t (28) = (9 x 28) / 5 + 32

= (252 + 160) / 5

= 412 / 5 = 82.4

(iii) t (- 10) = 9 x (- 10) + 32

= 9 x (- 2) + 32

= - 18 + 32 = 14

(iv) It is given that

t (C) = 212

⇒ 9C/5 + 32 = 212

⇒ 9C/5 = 212 - 32

⇒ C = (180 x 5)/9

= 100

Thus, the value of ‘t’, when t (C) = 212 is 100

5. Find the range of each of the following functions
(i) f (x) = 2 – 3x, x ∈ R, x > 0
(ii) f (x) = x^2 + 2, x is a real number
(iii) f (x) = x, x is a real number

Solution:

(i) f (x) = 2 - 3x, x ∈ R, x > 0

The values of f (x) for various values of real numbers x > 0 can be written in the tabular form as:

X	0.01	0.1	0.9	1	2	2.5	4	5	…….
F(x)	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	…….

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.

that is range of f = (- ∞, 2)

Alternative Method:

Let $x > 0$

⇒ $3x > 0$

⇒ $2 - 3x < 2$

⇒ $f(x) < 2$

Therefore, Range of f = (- ∞, 2)

(ii) $f(x) = x^2 + 2$, x is a real number.

The values of f(x) for various values of real numbers x can be written in the tabular form as:

X	0	± 0.3	± 0.8	± 1	± 2	± 3	…….
F(x)	2	2.09	2.64	3	6	11	…….

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

that is range of f = [2, ∞]

Alternative Method:

Let x be any real number i.e., $x^2 \geq 0$

Accordingly,

⇒ $x^2 + 2 \geq 0 + 2$

⇒ $x^2 + 2 \geq 2$

⇒ $x^2 \geq 0$.

⇒ $f(x) \geq 2$

Therefore, Range of f = [2, ∞)

(iii) $f(x) = x$, x is a real number.

It is clear that the range of f is the set of all real numbers. Therefore, the Range of f = R.