

EXERCISE 1.3

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces :

(i) $\{2, 3, 4\} \dots \{1, 2, 3, 4, 5\}$

Solution:-(i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \dots \{b, c, d\}$

Solution:- (ii) $\{a, b, c\} \not\subset \{b, c, d\}$

(iii) $\{x : x \text{ is a student of Class XI of your school}\} \dots \{x : x \text{ student of your school}\}$

Solution:- (iii) $\{x : x \text{ is a student of class XI of your school}\} \subset \{x : x \text{ is student of your school}\}$

(iv) $\{x : x \text{ is a circle in the plane}\} \dots \{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

Solution:- (iv) $\{x : x \text{ is a circle in the plane}\} \not\subset \{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

(v) $\{x : x \text{ is a triangle in a plane}\} \dots \{x : x \text{ is a rectangle in the plane}\}$

Solution:- (v) $\{x : x \text{ is a triangle in a plane}\} \not\subset \{x : x \text{ is a rectangle in the plane}\}$

(vi) $\{x : x \text{ is an equilateral triangle in a plane}\} \dots \{x : x \text{ is a triangle in the same plane}\}$

Solution:- (vi) $\{x : x \text{ is an equilateral triangle in a plane}\} \subset \{x : x \text{ in a triangle in the same plane}\}$

(vii) $\{x : x \text{ is an even natural number}\} \dots \{x : x \text{ is an integer}\}$

Solution:-(vii) $\{x: x \text{ is an even natural number}\} \subset \{x: x \text{ is an integer}\}$

Examine whether the following statements are true or false:

- (i) $\{a, b\} \not\subset \{b, c, a\}$
- (ii) $\{a, e\} \subset \{x : x \text{ is a vowel in the English alphabet}\}$
- (iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$
- (iv) $\{a\} \subset \{a, b, c\}$
- (v) $\{a\} \in \{a, b, c\}$
- (vi) $\{x : x \text{ is an even natural number less than 6}\} \subset \{x : x \text{ is a natural number which divides 36}\}$

Solution:

- (i) $\{a, b\} \not\subset \{b, c, a\}$ is FALSE as $\{a, b\} \subset \{b, c, a\}$
- (ii) $\{a, e\} \subset \{x : x \text{ is a vowel in the English alphabet}\}$ is TRUE.
- (iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$ is FALSE as $\{1, 2, 3\} \not\subset \{1, 3, 5\}$
- (iv) $\{a\} \subset \{a, b, c\}$ is TRUE.
- (v) $\{a\} \in \{a, b, c\}$ is FALSE.
- (vi) $\{x : x \text{ is an even natural number less than 6}\} \subset \{x : x \text{ is a natural number which divides 36}\}$ is TRUE as

$$\{x: x \text{ is an even natural number less than 6}\} = \{2, 4\}$$

$$\{x: x \text{ is a natural number which divides 36}\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\} \text{ and}$$

$$\{2, 4\} \subset \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$$

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

- (i) $\{3, 4\} \subset A$
(i) Incorrect as $3 \in \{3, 4\}$ but $3 \notin A$

- (ii) $\{3, 4\} \in A$

(ii) Correct because $\{3, 4\}$ is an element of A .

(iii) $\{\{3, 4\}\} \subset A$

(iii) Correct because $\{3, 4\} \in \{\{3, 4\}\}$ and $\{3, 4\} \in A$.

(iv) $1 \in A$

(iv) Correct because 1 is an element of A .

(v) $1 \subset A$

(v) Incorrect because an element of a set can never be a set and hence it cannot be a subset.

(vi) $\{1, 2, 5\} \subset A$

(vi) Correct because each element of $\{1, 2, 5\}$ is also an element of A .

(vii) $\{1, 2, 5\} \in A$

(vii) Incorrect because $\{1, 2, 5\}$ is not an element of A .

(viii) $\{1, 2, 3\} \subset A$

(viii) Incorrect because $3 \in \{1, 2, 3\}$; however, $3 \notin A$.

(ix) $\varnothing \in A$

(ix) Incorrect because $\varnothing$ is not an element of A .

(x) $\varnothing \subset A$

(x) Correct because $\varnothing$ is a subset of every set.

(xi) $\{\varnothing\} \subset A$

(xi) Incorrect because of $\Phi = \{ \}$ and $\{\Phi\} = \{ \{ \} \}$ which is NOT present in A .

4. Write down all the subsets of the following sets

(i) $\{a\}$

(i) The subsets of $\{a\}$ are Φ and $\{a\}$.

(ii) $\{a, b\}$

(ii) The subsets of $\{a, b\}$ are Φ , $\{a\}$, $\{b\}$ and $\{a, b\}$.

(iii) $\{1, 2, 3\}$

(iii) The subsets of $\{1, 2, 3\}$ are Φ , $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$ and $\{1, 2, 3\}$

(iv) $\varnothing$

(iv) The only subset of Φ is Φ .

5. Write the following as intervals :

(i) $\{x : x \in \mathbb{R}, -4 < x \leq 6\}$

(ii) $\{x : x \in \mathbb{R}, -12 < x < -10\}$

(iii) $\{x : x \in \mathbb{R}, 0 \leq x < 7\}$

(iv) $\{x : x \in \mathbb{R}, 3 \leq x \leq 4\}$

Solution:-

(i) $\{x : x \in \mathbb{R}, -4 < x \leq 6\} = (-4, 6]$

(ii) $\{x : x \in \mathbb{R}, -12 < x < -10\} = (-12, -10)$

(iii) $\{x : x \in \mathbb{R}, 0 \leq x < 7\} = [0, 7)$

(iv) $\{x : x \in \mathbb{R}, 3 \leq x \leq 4\} = [3, 4]$

6. Write the following intervals in set-builder form :

(i) $(-3, 0)$

(ii) $[6, 12]$

(iii) $(6, 12]$

(iv) $[-23, 5)$

Solution:-

(i) $(-3, 0) = \{x : x \in \mathbb{R}, -3 < x < 0\}$

(ii) $[6, 12] = \{x : x \in \mathbb{R}, 6 \leq x \leq 12\}$

(iii) $(6, 12] = \{x : x \in \mathbb{R}, 6 < x \leq 12\}$

(iv) $[-23, 5) = \{x : x \in \mathbb{R}, -23 \leq x < 5\}$

7. What universal set(s) would you propose for each of the following :

- (i) The set of right triangles.
- (ii) The set of isosceles triangles

Solution:-

(i) For the set of right triangles, the universal set can be the set of triangles or the set of polygons.

(ii) For the set of isosceles triangles,

8. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C

- (i) $\{0, 1, 2, 3, 4, 5, 6\}$

Solution:- (i) We see that

$$A \subset \{0, 1, 2, 3, 4, 5, 6\}$$

$$B \subset \{0, 1, 2, 3, 4, 5, 6\}$$

$$C \not\subset \{0, 1, 2, 3, 4, 5, 6\}$$

Therefore, the set $\{0, 1, 2, 3, 4, 5, 6\}$ cannot be the universal set for the sets A, B, and C.

- (ii) ϕ

Solution:- (ii) $A \not\subset \phi$, $B \not\subset \phi$, $C \not\subset \phi$

Therefore, ϕ cannot be the universal set for sets A, B, and C.

- (iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Solution:- (iii) We see that

$$A \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$B \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$C \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Therefore, the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is a universal set for the sets A, B, and C.

(iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Solution:- (iv) We see that

$$A \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$B \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$C \not\subset \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Therefore, the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ cannot be the universal set for the sets A, B, and C