

EXERCISE 1.4

1. Find the union of each of the following pairs of sets :

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

Solution:- (i) When $X = \{1, 3, 5\}$ and $Y = \{1, 2, 3\}$,

$$X \cup Y = \{1, 2, 3, 5\}$$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

Solution:- (ii) When $A = \{a, e, i, o, u\}$ and $B = \{a, b, c\}$,

$$A \cup B = \{a, b, c, e, i, o, u\}$$

(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$ $B = \{x : x \text{ is a natural number less than } 6\}$

Solution:- (iii) $A = \{x : x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9, \dots\}$

$$B = \{x : x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5\}$$

$$\text{Then } A \cup B = \{1, 2, 4, 5, 3, 6, 9, 12, \dots\}$$

Therefore, $A \cup B = \{x : x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$ $B = \{x : x \text{ is a natural number and } 6 < x < 10\}$

Solution:- (iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$$B = \{x : x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$$

$$\text{Then } A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

Therefore,

$$A \cup B = \{x : x \in \mathbb{N} \text{ and } 1 < x < 10\}$$

(v) $A = \{1, 2, 3\}$, $B = \varnothing$

Solution:- (v) When $A = \{1, 2, 3\}$ and $B = \Phi$,

$$A \cup B = \{1, 2, 3\}$$

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution:- We say that $A \subset B$ (which is read as "A is a subset of B") if every element of A is present in B.

Here,

$$A = \{a, b\} \text{ and } B = \{a, b, c\}$$

We see that every element of A is present in B.

Hence, $A \subset B$.

$A \cup B$ is read as the "union of A and B" and it is the set formed with all elements of both A and B.

Thus,

$$A \cup B = \{a, b, c\}$$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution:- A set is a well-defined collection of numbers, alphabets, objects, or any items.

A subset is a part of the set.

Set A is a subset of set B if all the elements in set A are present in set B.

Here,

A is called the subset, and B is called the superset.

If A and B are two sets such that $A \subset B$,

it means B consists all the elements of A.

Thus,

$$A \cup B = B.$$

4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cap B$

Solution:- (i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C$

Solution:- (ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C$

Solution:- (iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D$

Solution:- (iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C$

Solution:- (v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D$

Solution:- (vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D$

Solution:- (vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5. Find the intersection of each pair of sets of question 1 above

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$;

(i) $A \cap B$

(i) $A \cap B = \{3, 4\}$

(ii) $A \cap C$

Solution:- (ii) $A \cap C = \emptyset$

(iii) $B \cap C$

Solution:- (iii) $B \cap C = \{5, 6\}$

(iv) $B \cap D$

Solution:- (iv) $B \cap D = \emptyset$

(v) $A \cap B \cap C$

Solution:- (v) $A \cap B \cap C = \emptyset$

(vi) $A \cap B \cap D$

Solution:- (vi) $A \cap B \cap D = \emptyset$

(vii) $B \cap C \cap D$

Solution:- (vii) $B \cap C \cap D = \emptyset$

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$ (ii) $B \cap C$ (iii) $A \cap C \cap D$ (iv) $A \cap C$ (v) $B \cap D$

(vi) $A \cap (B \cup C)$ (vii) $A \cap D$ (viii) $A \cap (B \cup D)$ (ix) $(A \cap B) \cap (B \cup C)$

(x) $(A \cup D) \cap (B \cup C)$

Solution:-

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = \{A \cap C\} \cap D$

$$= \{11\} \cap \{15, 17\}$$

$$= \emptyset$$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \emptyset$

$$(vi) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$= \{7, 9, 11\} \cup \{11\}$$

$$= \{7, 9, 11\}$$

$$(vii) A \cap D = \Phi$$

$$(viii) A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$$

$$= \{7, 9, 11\} \cup \Phi$$

$$= \{7, 9, 11\}$$

$$(ix) (A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$$

$$= \{7, 9, 11\}$$

$$(x) (A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$$

$$= \{7, 9, 11, 15\}$$

7. If $A = \{x : x \text{ is a natural number}\}$, $B = \{x : x \text{ is an even natural number}\}$

$C = \{x : x \text{ is an odd natural number}\}$ and $D = \{x : x \text{ is a prime number}\}$, find

$$(i) A \cap B \quad (ii) A \cap C \quad (iii) A \cap D \quad (iv) B \cap C \quad (v) B \cap D \quad (vi) C \cap D$$

8. Which of the following pairs of sets are disjoint

$$(i) \{1, 2, 3, 4\} \text{ and } \{x : x \text{ is a natural number and } 4 \leq x \leq 6\}$$

Solution:-

$$(i) \text{ The first set is } \{1, 2, 3, 4\}$$

$$\text{The second set is } \{x : x \text{ is a natural number and } 4 \leq x \leq 6\}$$

$$= \{4, 5, 6\}$$

Now,

$$\{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\} \neq \Phi$$

Therefore, this pair of sets is NOT disjoint.

(ii) { a, e, i, o, u } and { c, d, e, f }

Solution:-

(ii) $\{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\} \neq \Phi$

Therefore, this pair of sets are NOT disjoint.

(iii) {x : x is an even integer } and {x : x is an odd integer}

Solution:-

(iii) $\{x : x \text{ is an even integer}\} \cap \{x : x \text{ is an odd integer}\} = \Phi$ because there is no integer which can be both even and odd.

Therefore, this pair of sets is disjoint

9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$; find

- (i) $A - B$ (ii) $A - C$ (iii) $A - D$ (iv) $B - A$ (v) $C - A$ (vi) $D - A$
 (vii) $B - C$ (viii) $B - D$ (ix) $C - B$ (x) $D - B$ (xi) $C - D$ (xii) $D - C$

Solution:- The given sets are:

$A = \{3, 6, 9, 12, 15, 18, 21\}$

$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$D = \{5, 10, 15, 20\}$

Thus,

(i) $A - B = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{3, 6, 9, 12, 18, 21\}$

(iv) $B - A = \{4, 8, 16, 20\}$

(v) $C - A = \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{5, 10, 20\}$

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$

$$(ix) C - B = \{2, 6, 10, 14\}$$

$$(x) D - B = \{5, 10, 15\}$$

$$(xi) C - D = \{2, 4, 6, 8, 12, 14, 16\}$$

$$(xii) D - C = \{5, 15, 20\}$$

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

$$(i) X - Y \quad (ii) Y - X \quad (iii) X \cap Y$$

Solution:- The given sets are:

$$X = \{a, b, c, d\} \text{ and } Y = \{f, b, d, g\}.$$

Thus,

$$(i) X - Y = \{a, c\}$$

$$(ii) Y - X = \{f, g\}$$

$$(ii) X \cap Y = \{b, d\}$$

11. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Solution:-It is given that

R : a set of real numbers

Q : a set of rational numbers

The difference between two sets A and B is a set denoted by $A - B$ and is obtained by writing the elements of A that are NOT in B in a set.

Therefore,

$R - Q$ is a set consisting of real numbers that are NOT rational.

i.e., $R - Q$ is a set of irrational numbers

12. State whether each of the following statement is true or false. Justify your answer.

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

Solution:-(i) We have $3 \in \{2, 3, 4, 5\}$ and $3 \in \{3, 6\}$

Therefore, $\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$ and hence they are NOT disjoint.

Thus, the given statement is FALSE.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

Solution:- (ii) We have $a \in \{a, e, i, o, u\}$ and $a \in \{a, b, c, d\}$

Therefore, $\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$ and hence they are NOT disjoint.

Thus, the given statement is FALSE.

(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

Solution:- (iii) We have $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \Phi$ and hence they are disjoint.

Thus, the given statement is TRUE.

(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

Solution: (iv) We have $\{2, 6, 10\} \cap \{3, 7, 11\} = \Phi$ and hence they are disjoint.

Thus, the given statement is TRUE

theboard