EXERCISE 1.5

1. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = { 1, 2, 3, 4}, B = { 2, 4, 6, 8 } and C = { 3, 4, 5, 6 }. Find

(i) A'

(ii) B'

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $(A')'$

(vi) $(B - C)'$

Solution:

The given sets are as follows:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

A = {1, 2, 3, 4},

B = {2, 4, 6, 8} and C = {3, 4, 5, 6}

The complement of a set A is U - A, where U is the universal set.

Thus,

(i) A' = U - A = {5, 6, 7, 8, 9}

(ii) B' = U - B = {1, 3, 5, 7, 9}

(iii) A ∪ C = {1, 2, 3, 4, 5, 6}

Therefore,

(A ∪ C)' = U - (A ∪ C) = {7, 8, 9}

(iv) A ∪ B = {1, 2, 3, 4, 6, 8}

Therefore,

(A ∪ B)' = U - (A ∪ B)

= {5, 7, 9}

(v) (A')' = A = {1, 2, 3, 4}

(vi) The difference between two sets B and C is a set denoted by B - C and is obtained by writing the elements of B that are NOT in C in a set.

Thus,

B - C = {2, 8}

Therefore,

(B - C)' = U - (B - C) = {1, 3, 4, 5, 6, 7, 9}

2. If U = { a, b, c, d, e, f, g, h}, find the complements of the following sets :

(i) A = {a, b, c}

(ii) B = {d, e, f, g}

(iii) C = {a, c, e, g}

(iv) D = { f, g, h, a}

Solutions:

The universal set is,

U = { a, b, c, d, e, f, g, h}.

The complement of a set A is U - A,

where U is the universal set.

Thus,

(i) A' = U - A

= { a, b, c, d, e, f, g, h} - {a, b, c}

= {d , e, f , g, h}

(ii) B' = U - B

= { a, b, c, d, e, f, g, h} - {d, e, f, g}

= {a, b, c, h}

(iii) C' = U - C

= { a, b, c, d, e, f, g, h} - {a, c, e, g}

= {b, d , f, h}

(iv) D = U - D

= { a, b, c, d, e, f, g, h} - {f, g, h, a}

= {b, c, d, e}

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {x : x is an even natural number}

(ii) { x : x is an odd natural number }

(iii) {x : x is a positive multiple of 3}

(iv) { x : x is a prime number }

(v){x : x is a natural number divisible by 3 and 5}

(vi) { x : x is a perfect square } (

(vii) { x : x is a perfect cube}

(viii) { x : x + 5 = 8 }

(ix) { x : 2x + 5 = 9}

(x) { x : x ≥ 7 }

(xi) { x : x ∈ N and 2x + 1 > 10 }

Solution:-

It is given that the universal set is,

U = N = Set of natural numbers

We know that the complement of a set A is denoted by A' and it is equal to U - A, where U is the universal set. Thus,

(i){x : x is an even natural number}

(i) {x : x is an even natural number}´ = {x : x is an odd natural number}

(ii) { x : x is an odd natural number }

(ii) {x : x is an odd natural number}´ = {x : x is an even natural number}

(iii) {x : x is a positive multiple of 3}

(iii) {x : x is a positive multiple of 3}´ = {x : x ∈ N and x is not a multiple of 3}

(iv) { x : x is a prime number }

(iv) {x : x is a prime number}´ = { x : x is a positive composite number or x = 1}

(v){x : x is a natural number divisible by 3 and 5}

(v) {x : x is a natural number divisible by 3 and 5}´ = {x : x is a natural number that is not divisible by 3 or not divisible by 5}

(vi) { x : x is a perfect square }

(vi) {x : x is a perfect square}´ = {x : x ∈ N and x is not a perfect square}

(vii) { x : x is a perfect cube}

(vii) { x : x is a perfect cube}´ = {x : x Î N and x is not a perfect cube}

(viii) { x : x + 5 = 8 }

(viii) We have {x: x + 5 = 8} = { x: x = 3}.

Thus, its complement is

{x: x + 5 = 8}' = {x : x ∈ N and x ≠ 3}

(ix) { x : 2x + 5 = 9}

(ix) We have {x : 2x + 5 = 9} = { x : 2x = 4} = {x : x = 2}.

Thus, its complement is

{x : 2x + 5 = 9}´ = {x : x ∈ N and x ≠ 2}

(x) { x : x ≥ 7 }

(x) {x : x ≥ 7}´ = {x : x ∈ N and x < 7}

(xi) { x : x ∈ N and 2x + 1 > 10 }

(xi) We have {x : x ∈ N and 2x + 1 > 10} = {x : x ∈ N and 2x > 9} = {x : x ∈ N and x > 9/2}.

Thus, its complement is

$\{x : x \in N \text{ and } 2x + 1 > 10\}' = \{x : x \in N \text{ and } x \leq 9/2\}$

4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9 }, A = {2, 4, 6, 8} and B = { 2, 3, 5, 7}. Verify that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Solution:- The given sets are as follows:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}

(i) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

We know that the complement of a set A is denoted by A' and it is equal to U - A,

where U is the universal set.

Thus,

$(A \cup B)' = U - (A \cup B) = \{1, 9\}$ ....(1)

$A' = U - A = \{1, 3, 5, 7, 9\}$

$B' = U - B = \{1, 4, 6, 8, 9\}$

The intersection of two sets is obtained by taking their common elements.

Thus,

$A' \cap B' = \{1, 9\}$ .... (2)

From (1) and (2),

$\Rightarrow (A \cup B)' = A' \cap B'$

(ii) U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

A = {2, 4, 6, 8} and B = {2, 3, 5, 7}

$A \cap B = \{2\}$

$(A \cap B)' = U - (A \cap B) = \{1, 3, 4, 5, 6, 7, 8, 9\}$ .... (3)

We know that the union of two sets is obtained writing all the elements of both sets in a set by removing the duplicates.

A' ∪ B' = {1, 3, 4, 5, 6, 7, 8, 9} ... (4)

From (3) and (4),

⇒ (A ∩ B)' = A' ∪ B'

5. Draw appropriate Venn diagram for each of the following :

(i) (A ∪ B)′,

(ii) A′ ∩ B′,

(iii) (A ∩ B)′,

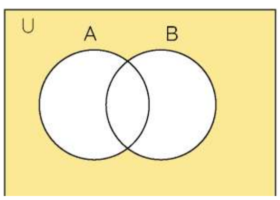
(iv) A′ ∪ B′

Solution:

(i) We know that (A ∪ B)' = U - (A ∪ B).

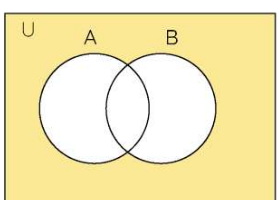
Thus, in the Venn diagram of (A ∪ B)',

we have to shade the entire region of U excluding A ∪ B.

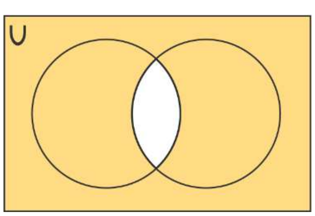


(ii) A' ∩ B'

By a Demorgan law of sets, A' ∩ B' = (A ∪ B)′. Thus, its Venn diagram is as same as that of **(i)**.

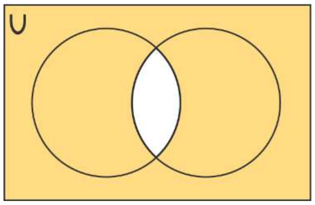


(iii) We know that (A ∩ B)' = U - (A ∩ B). Thus, in the Venn diagram of (A ∩ B)', we have to shade the entire region of U excluding A ∩ B.



(iv) A' ∪ B'

By a Demorgan law of sets, A' ∪ B' = (A ∩ B)′. Thus, its Venn diagram is as same as that of **(iii)**.



6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A′?

Solution:

It is given that U is the set of all triangles in a plane.

Also, A is the set of all triangles with at least one angle different from 60°.

Then A', which is the complement of A, is

A' = U - A

= {All triangles} - {all triangles with at least one angle different from 60°}

= {All triangles in which all angles are equal to 60°}

= {All equilateral triangles}

Thus, A′ is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement :

(i) A ∪ A′ = . . . (ii) ϕ′ ∩ A = . . . (iii) A ∩ A′ = . . . (iv) U′ ∩ A = . . .

Solution:

We will fill in the blanks to make each of the following a true statement.

Let us assume that U is the universal set.

We know that the complement of a set A is denoted by A′ and A′ contains all the elements of U that are NOT in A.

Then

(i) A ∪ A' = U

(ii) $\Phi' \cap A = U \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cap A = \Phi \cap A = \Phi$