Class 10th NCERT

Chapter – 1 Notes

Real Numbers

Introduction

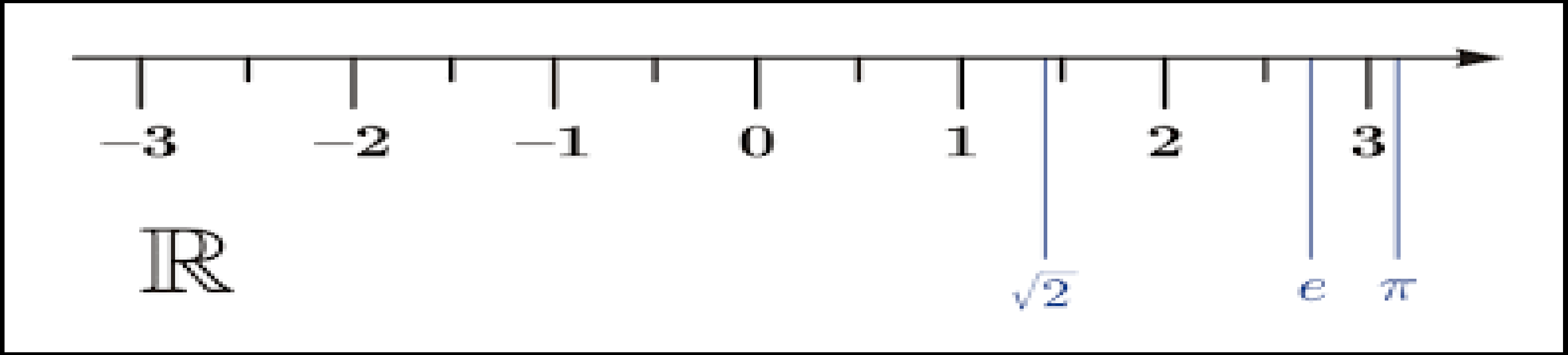
Exercise 1.1

Exercise 1.2

Real Number

Real numbers can be defined as the union of both rational and irrational numbers.

They can be both positive or negative and are denoted by the symbol “R”



Fundamental Theorem of Arithmetic

Factorization of every composite number can be expressed as a product of primes.

Eg. 66 =2x3x11

420 =2x2x3x5x7

HCF and LCM Using Prime Factorization

LCM(66,420)= 2x2x3x5x7x11 = 4620

HCF(66,420)= 2x3 = 6

EXERCISE 1.1

1.Express each number as a product of its prime factors:

(i)140

Solution:- Hence, **140** = 2×2x5×7

(ii) 156

Solution:- Hence 156 = 2 x 2 x 3 x 13

(iii) 3825

Solution:- Hence 3825= 3 x 3 x 5 x 5 x 17

(iv) 5005

Solution:- Hence 5005 = 5 x 7 x 11 x 13

(v) 7429

Solution:- Hence 7429 = 17 × 19 × 23

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

Prime factors of 26 = 2×13

Prime factors of 91 = 7×13

HCF of 26 and 91 = 13

LCM of 26 and 91 = $2 \times 7 \times 13 = 14 \times 13 = 182$

Product of these two numbers = $26 \times 91 = 2366$

LCM × HCF = $182 \times 13 = 2366$

(ii) 510 and 92

Solution:-

Prime factors of 510 = $2 \times 3 \times 5 \times 17$

Prime factors of 92 = $2 \times 2 \times 23$

HCF of the two numbers = 2

LCM of the two numbers = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of these two numbers = $510 \times 92 = 46920$

LCM x HCF = $2 \times 23460 = 46920$

(iii) 336 and 54

Solution:-

Prime factors of 336 = $2 \times 2 \times 2 \times 2 \times 3 \times 7$

Prime factors of 54 = $2 \times 3 \times 3 \times 3$

HCF of the two numbers = 6

LCM of the two numbers = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$

Product of these two numbers = $336 \times 54 = 18144$

LCM x HCF = $3024 \times 6 = 18144$

Thus, the product of two numbers = LCM × HCF

3. Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21

Solution:- Prime factors of 12 = 2 × 2 × 3

Prime factors of 15 = 3 × 5

Prime factors of 21 = 3 × 7

HCF of 12, 15 and 21 = 3

LCM of 12, 15 and 21 = 2 x 2× 3 × 5 × 7 = 420

(ii) 17, 23 and 29

Solution:- Prime factors of 17 = 17 × 1

Prime factors of 23 = 23 × 1

Prime factors of 29 = 29 × 1

HCF of 17, 23 and 29 = 1

LCM of 17, 23 and 29 = 17 × 23 × 29 = 11339

(iii) 8, 9 and 25

Solution:- Prime factors of 8 = $2 \times 2 \times 2 \times 1$

Prime factors of 9 = $3 \times 3 \times 1 = 3^2 \times 1$

Prime factors of 25 = $5 \times 5 \times 1 = 5^2 \times 1$

HCF of 8, 9 and 25 = 1

LCM of 8 , 9 and 25 = $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution:

We know that LCM × HCF = product of two given integers

Given, HCF (306, 657) = 9

We have to find, LCM (306, 657)

We have the given numbers 306 and 657.

Hence

LCM × HCF = product of two

LCM × 9 = 306 × 657

LCM = (306 × 657) / 9

LCM = 34 × 657

LCM = 22338

5. Check whether 6n can end with the digit 0 for any natural number n.

Solution:-

If any number ends with the digit 0 that means it should be divisible by 5. That is, if 6^n ends with the digit 0, Then the prime factorization of 6^n would contain the prime number 5.

Prime factors of $6^n = (2 \times 3)^n = (2)^n \times (3)^n$

We can clearly observe, 5 is not present in the prime factors of 6^n. That means 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution:-

Now $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$.

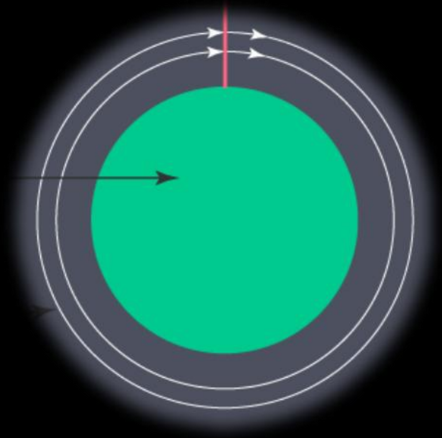
$7 \times 11 \times 13 + 13 = 13\,(7 \times 11 + 1)$
$= 13(77 + 1)$
$= 13 \times 78$
$= 13 \times 13 \times 6 \times 1$
$= 13 \times 13 \times 2 \times 3 \times 1$

The given number has 2, 3, 13, and 1 as its factors. Therefore, it is a composite number.

Now, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5 \times (1008 + 1)$
$= 5 \times 1009 \times 1$

1009 cannot be factorized further. Therefore, the given expression has 5,1009 and 1 as its factors. Hence, it is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?



Solution

Given: Sonia takes 18 minutes to drive one round of the field.
Ravi takes 12 minutes for the same.
They both start at the same point and at the same time and go in the same direction.

Let's find LCM of 18 and 12 by prime factorization method.

$18 = 2 \times 3 \times 3$

$12 = 2 \times 2 \times 3$

LCM of 12 and 18 = $2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

EXERCISE 1.2

1. Prove that √5 is irrational

Solution:

Let us prove that √5 is an irrational number

The contradiction method. Let's assume that √5 is a rational number. If √5 is rational, that means it can be written in the form of a/b, where a and b integers b ≠ 0. i.e., a and b are coprime numbers

$\sqrt{5}/1 = a/b$

$\sqrt{5}b = a$

Squaring both sides,

$5b^2 = a^2$ ... (1)

This means 5 divides a^2.

From this, 5 also divides a.

Then a = 5c, for some integer 'c'.

On squaring, we get

$a^2 = 25c^2$

Put the value of a^2 in equation (1).

$5b^2 = 25c^2$

$b^2 = 5c^2$

This means b^2 is divisible by 5 and so b is also divisible by 5. Therefore, a and b have 5 as common factor.

But this contradicts the fact that a and b are coprime.

This contradiction has arisen because of our incorrect assumption that √5 is a rational number.

So, we conclude that √5 is irrational.

2. Prove that $3 + 2\sqrt{5}$ is irrational

Solution

We have to prove that $3 + 2\sqrt{5}$ is irrational. We will be solving this question with the help of the contradiction method.
Let's assume that $3 + 2\sqrt{5}$ is rational. If $3 + 2\sqrt{5}$ is rational that means it can be written in the form of a/b where a and b are integers that have no common factor other than 1 and $b \neq 0$.
$3 + 2\sqrt{5} = a/b$
$b(3 + 2\sqrt{5}) = a$
$3b + 2\sqrt{5}b = a$
$2\sqrt{5}b = a - 3b$
$\sqrt{5} = (a - 3b)/2b$
Since $(a - 3b)/2b$ is a rational number, then $\sqrt{5}$ is also a rational number.
But, we know that $\sqrt{5}$ is irrational.
Therefore, our assumption was wrong that $3 + 2\sqrt{5}$ is rational. Hence, $3 + 2\sqrt{5}$ is irrational.

Prove that the following are irrationals:
(i) $1/\sqrt{2}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

(i) $1/\sqrt{2}$
Let us assume that $1/\sqrt{2}$ is a rational number.
Then, $1/\sqrt{2} = a/b$, where a and b have no common factors other than 1.
$\sqrt{2} \times a = b$
$\sqrt{2} = b/a$
Since b and a are integers, b/a is a rational number and so, $\sqrt{2}$ is rational.
But we know that $\sqrt{2}$ is irrational.
So, our assumption was wrong. Therefore, $1/\sqrt{2}$ is an irrational number.

(ii) $7\sqrt{5}$
Let us assume that $7\sqrt{5}$ is a rational number.
Then, $7\sqrt{5} = a/b$, where a and b have no common factors other than 1.
$(7\sqrt{5})\, b = a$
$\sqrt{5} = a/7b$
Since, a, 7, and b are integers, so, a/7b is a rational number. This means $\sqrt{5}$ is rational. But this contradicts the fact that $\sqrt{5}$ is irrational.
So, our assumption was wrong. Therefore, $7\sqrt{5}$ is an irrational number.

(iii) $6 + \sqrt{2}$

Let us assume that $6 + \sqrt{2}$ is rational.

Then, $6 + \sqrt{2} = a/b$, where a and b have no common factors other than 1.

$\sqrt{2} = (a/b) - 6$

Since, a, b, and 6 are integers, so, $a/b - 6$ is a rational number. This means $\sqrt{2}$ is also a rational number.

But this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption was wrong.

Therefore, $6 + \sqrt{2}$ is an irrational number.