10th NCERT Chapter – 2 *Polynomial*

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Introduction of **Polynomials**

A **polynomial's** degree is the highest power of its variable.

Degrees of **Polynomials**

Linear **polynomials** have degree 1

Quadratic degree 2

And cubic degree 3.

Types of Polynomials

- Monomials
- Binomials
- Trinomials

Zeroes of a Polynomial

The values of the variable that make the polynomial equal to zero.

Numbers of Zeroes in Polynomial equal to Degrees.

Eg= $2x - 4=0$

$2x=4$

$x=4/2$

$X=2$

EXERCISE 2.1

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

(i) In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.

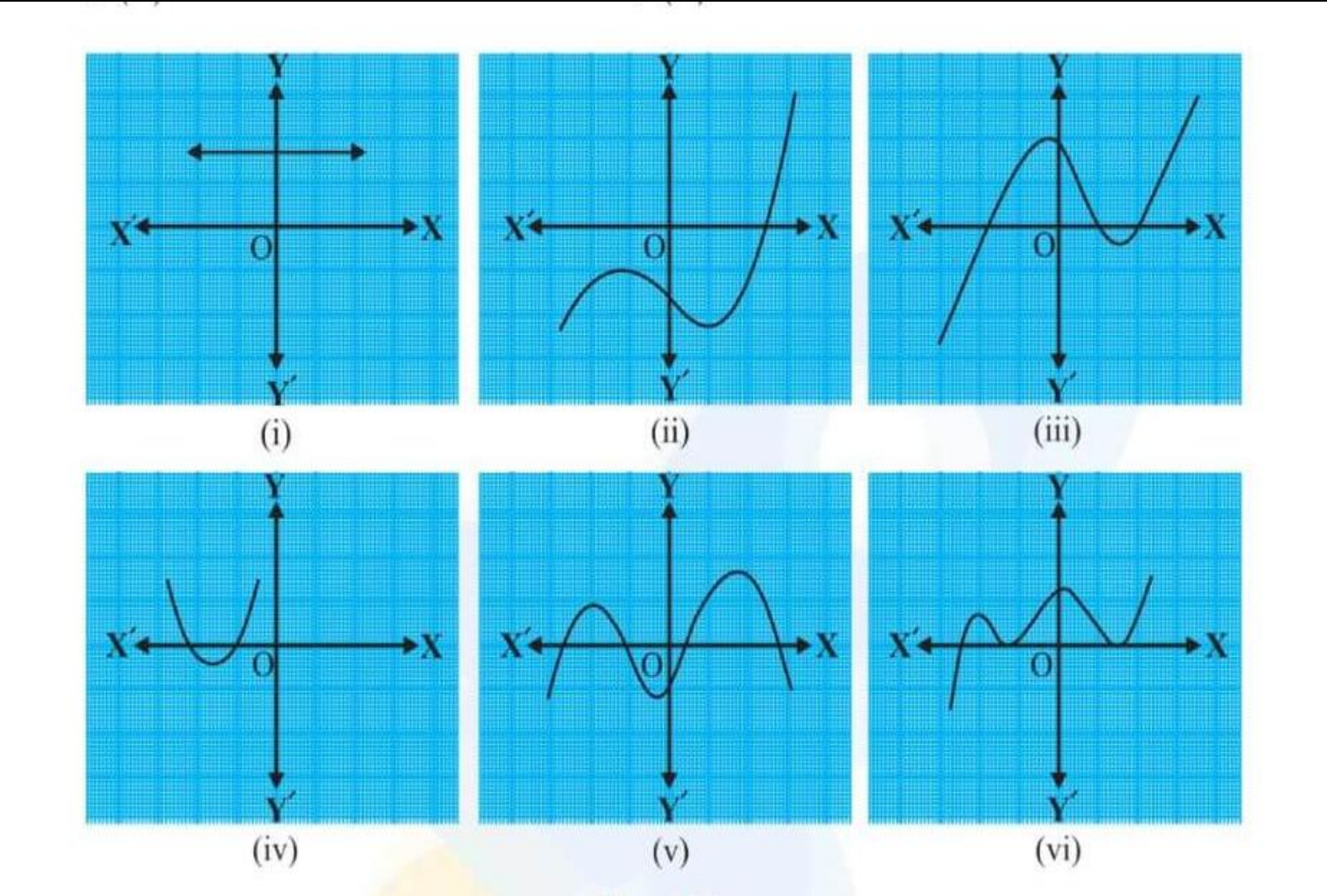
(ii) In the given graph, the number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.

(iii) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at any three points.

(iv) In the given graph, the number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.

(v) In the given graph, the number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.

(vi) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.



EXERCISE 2.2
Introduction

We know that the standard form of the quadratic equation is: $ax^2 + bx + c = 0$

Let α and β be the zeros of the polynomial.

Sum of zeroes = - coefficient of x / coefficient of x^2

$\alpha + \beta = - b / a$

Product of Zeroes = constant term / coefficient of x^2

$\alpha \times \beta = c / a$

Put the values in the above formula and find the relation between the zeroes and the coefficients.

EXERCISE 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

Solution :-(i) $x^2 - 2x - 8$

Let's find the zeros of the polynomial by factorization.

$x^2 - 4x + 2x - 8 = 0$

$x (x - 4) + 2 (x - 4) = 0$

$(x - 4) (x + 2) = 0$

$x = 4$, $x = -2$ are the zeroes of the polynomial.

Thus, $\alpha = 4$, $\beta = -2$

Now let's find the relationship between the zeroes and the coefficients.

Sum of zeroes = - coefficient of x / coefficient of x^2

For $x^2 - 2x - 8$,

$a = 1, b = -2, c = -8$

$\alpha + \beta = -b / a$

Here,

$\alpha + \beta = -2 + 4 = 2$

$-b / a = -(-2) / 1 = 2$

Hence, sum of the zeros $\alpha + \beta = -b/a$ is verified.

Now, Product of zeroes = constant term / coefficient of x^2

$\alpha \times \beta = c / a$

Here,

$\alpha \times \beta = -2 \times 4 = -8$

$c / a = -8 / 1 = -8$

Hence, product of zeros $\alpha \times \beta = c / a$ is verified.

Thus, $x = 4, -2$ are the zeroes of the polynomial.

(ii) $4s^2 - 4s + 1$

$4s^2 - 2s - 2s + 1 = 0$

$2s (2s - 1) - 1 (2s - 1) = 0$

$(2s - 1)(2s - 1) = 0$

$s = 1/2$, $s = 1/2$ are the zeroes of the polynomial.

Thus, $\alpha = 1/2$ and $\beta = 1/2$

Now, let's find the relationship between the zeroes and the coefficients.

For $4s^2 - 4s + 1$,

$a = 4$, $b = - 4$ and $c = 1$

Sum of zeroes = - coefficient of s / coefficient of s^2

$\alpha + \beta = - b/a$

Here,

$\alpha + \beta = 1/2 + 1/2 = 1$

$- b / a = - (- 4) / 4 = 1$

Hence, $\alpha + \beta = - b / a$, verified

Now, Product of zeroes = constant term / coefficient of s^2

$\alpha \times \beta = c / a$

Here,

$\alpha \times \beta = 1/2 \times 1/2 = 1/4$

$c / a = 1/4$

Hence, $\alpha \times \beta = c / a$, verified.

Thus, $s = 1/2, 1/2$ are the zeroes of the polynomial.

(iii) $6x^2 - 3 - 7x$

$6x^2 - 7x - 3 = 0$

$6x^2 - 9x + 2x - 3 = 0$

$3x(2x - 3) + 1(2x - 3) = 0$

$(2x - 3)(3x + 1) = 0$

$x = 3/2$, $x = -1/3$ are the zeroes of the polynomial.

Thus, $\alpha = 3/2$ and $\beta = -1/3$

Now, let's find the relationship between the zeroes and the coefficients:

For $6x^2 - 3 - 7x$,

$a = 6$, $b = -7$ and $c = -3$

Sum of zeroes = - coefficient of x / coefficient of x^2

$\alpha + \beta = - b / a$

Here,

$\alpha + \beta = 3/2 + (-1/3) = 7/6$

$- b / a = - (-7) / 6 = 7/6$

Hence, $\alpha + \beta = - b/a$, verified

Now, Product of zeroes = constant term/ coefficient of x^2

$\alpha \times \beta = c / a$

Here,

$\alpha \times \beta = 3/2 \times (- 1/3) = -1/2$

$c / a = (- 3) / 6 = -1/2$

Hence, $\alpha \times \beta = c/a$, verified.

Thus, $x = 3/2, - 1/3$ are the zeroes of the polynomial.

(iv) $4u^2 + 8u$

$4u(u + 2) = 0$

$u = 0$, $u = -2$ are the zeroes of the polynomial

Thus, $\alpha = 0$ and $\beta = -2$

Now, let's find the relationship between the zeroes and the coefficients

For $4u^2 + 8u$,

$a = 4$, $b = 8$, $c = 0$

Sum of zeroes = - coefficient of u / coefficient of u^2

$\alpha + \beta = -b/a$

Here,

$\alpha + \beta = 0 + (-2) = -2$

$-b / a = -(8) / 4 = -2$

Hence, $\alpha + \beta = -b / a$, verified

Now, Product of zeroes = constant term / coefficient of u^2

$\alpha \times \beta = c/a$

Here,

$\alpha \times \beta = 0 \times (-2) = 0$

$c / a = 0 / 4 = 0$

Hence, $\alpha \times \beta = c / a$, verified.

Thus, $u = 0, -2$ are the zeroes of the polynomial.

(v) $t^2 - 15$
$t^2 - 15 = 0$
$t^2 = 15$
$t = \pm\sqrt{15}$
$t = -\sqrt{15}$, $t = \sqrt{15}$ are the zeroes of the polynomial.
Thus, $\alpha = -\sqrt{15}$ and $\beta = \sqrt{15}$
Now, let's find the relationship between the zeroes and the coefficients
For $t^2 - 15$,
$a = 1, b = 0, c = -15$
Sum of zeroes = - coefficient of t / coefficient of t^2
$\alpha + \beta = - b / a$
Here,
$\alpha + \beta = -\sqrt{15} + \sqrt{15} = 0$
$- b / a = - 0 / 1 = 0$
Hence, $\alpha + \beta = - b / a$, verified
Now, Product of zeroes = constant term / coefficient of t^2
$\alpha \times \beta = c / a$
Here,
$\alpha \times \beta = -\sqrt{15} \times \sqrt{15} = -15$
$c / a = -15 / 1 = -15$
Hence, $\alpha \times \beta = c / a$, verified.
Thus, $t = -\sqrt{15}, \sqrt{15}$ are the zeroes of the polynomial.

(vi) $3x^2 - x - 4$

Solution:- $3x^2 - x - 4 = 0$

$3x^2 - 4x + 3x - 4 = 0$

$x (3x - 4) + 1(3x - 4) = 0$

$(x + 1)(3x - 4) = 0$

$x = - 1$, $x = 4/3$ are the zeroes of the polynomial.

Thus, $\alpha = - 1$ and $\beta = 4/3$

Now, let's find the relationship between the zeroes and the coefficients

For $3x^2 - x - 4$,

$a = 3$, $b = - 1$, $c = - 4$

Sum of zeroes = - coefficient of x / coefficient of x^2

$\alpha + \beta = - b / a$

Here,

$\alpha + \beta = - 1 + 4/3 = 1/3$

$- b / a = - (-1) / 3 = 1/3$

Hence, $\alpha + \beta = - b / a$, verified

Now, Product of zeroes = constant term / coefficient of x^2

$\alpha \times \beta = c/a$

Here,

$\alpha \times \beta = - 1 \times (4/3) = - 4/3$

$c / a = - 4/3$

Hence, $\alpha \times \beta = c / a$, verified.

Thus, $x = - 1, 4/3$ are the zeroes of the polynomial.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) 1/4, - 1

Solution:- We know that the general equation of a quadratic polynomial is:

x^2 - (sum of roots) x + (product of roots)

$x^2 - (1/4)x + (-1)$

$x^2 - (1/4)x - 1$

(ii) √2, 1/3

Solution:- We know that the general equation of a quadratic polynomial is:

x^2 - (sum of roots) x + (product of roots)

$x^2 - \sqrt{2}\,x + 1/3$

(iii) 0, √5

Solution:- We know that the general equation of a quadratic polynomial is:

x^2 - (sum of roots) x + (product of roots)

$x^2 - 0\,x + \sqrt{5}$

$x^2 + \sqrt{5}$

(iv) 1, 1

Solution:-

We know that the general equation of a quadratic polynomial is:

x^2 - (sum of roots) x + (product of roots)

$x^2 - 1x + 1$

$x^2 - x + 1$

(v) - 1/4, 1/4

Solution:-

We know that the general equation of a quadratic polynomial is:

x^2 - (sum of roots) x + (product of roots)

$x^2 - (- 1/4)x + 1/4$

$x^2 + (1/4)x + 1/4$

(vi) 4, 1

Solution:-

We know that the general equation of a quadratic polynomial is:

x^2 - (sum of roots) x + (product of roots)

$x^2 - 4x + 1$