

Vector Algebra

Question 1.

A unit vector perpendicular to the plane of

$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is

(a) $\frac{4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}}$

(b) $\frac{2\hat{i} - 6\hat{j} - \hat{k}}{7}$

(c) $\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

(d) $\frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$

Answer:

(c) $\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

Question 2.

The area of parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$ is

(a) $10\sqrt{6}$

(b) $5\sqrt{6}$

(c) $10\sqrt{3}$

(d) $5\sqrt{3}$

Answer:

(b) $5\sqrt{6}$

Question 3.

If $\vec{AB} \times \vec{AC} = 2\hat{i} - 4\hat{j} + 4\hat{k}$, then the area of $\triangle ABC$ is

(a) 3 sq. units

(b) 4 sq. units

(c) 16 sq. units

(d) 9 sq. units

Answer:

(a) 3 sq. units

Question 4.

A vector of magnitude 5 and perpendicular to $(\hat{i} - 2\hat{j} + \hat{k})$ and $(2\hat{i} + \hat{j} - 3\hat{k})$ is

- (a) $\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} - \hat{k})$
(c) $\frac{5\sqrt{3}}{3}(\hat{i} - \hat{j} + \hat{k})$ (d) $\frac{5\sqrt{3}}{3}(-\hat{i} + \hat{j} + \hat{k})$

Answer:

(a) $\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} + \hat{k})$

Question 5.

$|a \times b|^2 + |a \cdot b|^2 = 144$ and $|a| = 4$, then $|b|$ is equal to

- (a) 12
(b) 3
(c) 8
(d) 4

Answer:

(b) 3

Question 6.

If $|a \times b| = 4$ and $|a \cdot b| = 2$, then $|a|^2 |b|^2$ is equal to

- (a) 2
(b) 6
(c) 8
(d) 20

Answer:

(d) 20

Question 7.

If $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is

- (a) $\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{j} - \hat{k}$ (c) $\hat{i}$ (d) $2\hat{i}$

Answer:

(c) $\hat{i}$

Question 8.

The two vectors $a = 2\hat{i} + \hat{j} + 3\hat{k}$ and $b = 4\hat{i} - \lambda\hat{j} + 6\hat{k}$ are parallel, if λ is equal to

- (a) 2
- (b) -3
- (c) 3
- (d) 2

Answer:

- (d) 2

Question 9.

If $|a|=5$, $|b|=13$ and $|a \times b|=25$, find $a \cdot b$

- (a) ± 10
- (b) ± 40
- (c) ± 60
- (d) ± 25

Answer:

- (c) ± 60

Question 10.

Find the value of λ so that the vectors $2\hat{i} - 4\hat{j} + \hat{k}$ and $4\hat{i} - 8\hat{j} + \lambda\hat{k}$ are parallel.

- (a) -1
- (b) 3
- (c) -4
- (d) 2

Answer:

- (d) 2

Question 11.

If O is origin and C is the mid point of A(2, -1) and B(-4, 3), then the value of OC is

- (a) $\hat{i} + \hat{j}$
- (b) $\hat{i} - \hat{j}$
- (c) $-\hat{i} + \hat{j}$
- (d) $-\hat{i} - \hat{j}$

Answer:

- (c) $-\hat{i} + \hat{j}$

Question 12.

The vectors $\vec{AB} = 3\hat{i} + 4\hat{k}$ and $\vec{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the side of a ΔABC . The length of the median through A is

- (a) $\sqrt{18}$
- (b) $\sqrt{72}$
- (c) $\sqrt{33}$
- (d) $\sqrt{288}$

Answer:

(c) $\sqrt{33}$

Question 13.

The summation of two unit vectors is a third unit vector, then the modulus of the difference of the unit vector is

(a) $\sqrt{3}$

(b) $1 - \sqrt{3}$

(c) $1 + \sqrt{3}$

(d) $-\sqrt{3}$

Answer:

(a) $\sqrt{3}$

Question 14.

Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\hat{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then $\hat{c}$ is

(a) $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$

(b) $\frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$

(c) $\frac{1}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$

(d) $\frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} + \hat{k})$

Answer:

(d) $\frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} + \hat{k})$

Question 15.

If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a} + \vec{b}| < 1$, then

(a) $\theta = \frac{\pi}{2}$

(b) $\theta < \frac{\pi}{3}$ (c)

$\pi \geq \theta > \frac{2\pi}{3}$

(d) $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$

Answer:

(c) $\pi \geq \theta > \frac{2\pi}{3}$

Question 16.

The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

- (a) $\frac{2}{3}$
- (b) $\frac{3}{2}$
- (c) $\frac{5}{2}$
- (d) $\frac{2}{5}$

Answer:

- (a) $\frac{2}{3}$

Question 17.

The vectors from origin to the points A and B are $a = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $b = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively then the area of triangle OAB is

- (a) 340
- (b) $\sqrt{25}$
- (c) $\sqrt{229}$
- (d) $\frac{1}{2} \sqrt{229}$

Answer:

- (d) $\frac{1}{2} \sqrt{229}$

Question 18.

The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if

- (a) $\lambda = -2$
- (b) $\lambda = 0$
- (c) $\lambda = 1$
- (d) $\lambda = -1$

Answer:

- (a) $\lambda = -2$

Question 19.

If a, b, c are unit vectors such that $a + b + c = 0$, then the value of $a \cdot b + b \cdot c + c \cdot a$ is

- (a) 1
- (b) 3
- (c) $-\frac{3}{2}$
- (d) None of these

Answer:

- (c) $-\frac{3}{2}$

Question 20.

If $|a| = 4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda a|$ is

- (a) $[0, 8]$

(b) [-12, 8]

(c) [0, 12]

(d) [8, 12]

Answer:

(c) [0, 12]

Question 21.

The number of vectors of unit length perpendicular to the vectors $a = 2\hat{i} + \hat{j} + 2\hat{k}$ and $b = \hat{j} + \hat{k}$ is

(a) one

(b) two

(c) three

(d) infinite

Answer:

(b) two

Question 22.

Let a, b and c be vectors with magnitudes 3, 4 and 5 respectively and $a + b + c = 0$, then the values of $a \cdot b + b \cdot c + c \cdot a$ is

(a) 47

(b) 25

(c) 50

(d) -25

Answer:

(d) -25

Question 23.

If $|a| = |b| = 1$ and $|a + b| = \sqrt{3}$, then the value of $(3a - 4b) \cdot (2a + 5b)$ is

(a) -21

(b) $-\frac{21}{2}$

(c) 21

(d) $\frac{21}{2}$

Answer:

(b) $-\frac{21}{2}$

Question 24.

The unit vector perpendicular to $\hat{i} - \hat{j}$ and coplanar with $\hat{i} + 2\hat{j}$ and $2\hat{i} + 3\hat{j}$ is

- (a) $\frac{2\hat{i} - 5\hat{j}}{\sqrt{29}}$ (b) $2\hat{i} + 5\hat{j}$
(c) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ (d) $\hat{i} + \hat{j}$

Answer:

(c) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

Question 25.

If $|a - b| = |a| = |b| = 1$, then the angle between a and b is

- (a) $\frac{\pi}{3}$
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{2}$
(d) 0

Answer:

(a) $\frac{\pi}{3}$

Question 26.

$(\vec{a} \cdot \hat{i})^2 + (\vec{a} \cdot \hat{j})^2 + (\vec{a} \cdot \hat{k})^2$ is equal to

- (a) 1 (b) $\vec{a}$ (c) $-\vec{a}$ (d) $|\vec{a}|^2$

Answer:

(d) $|\vec{a}|^2$

Question 27.

a, b, c are three vectors, such that $a + b + c = 0$, $|a| = 1$, $|b| = 2$, $|c| = 3$, then $a \cdot b + b \cdot c + c \cdot a$ is equal to

- (a) 0
(b) -7
(c) 7
(d) 1

Answer:

(b) -7

Question 28.

If $|a + b| = |a - b|$, then angle between a and b is ($a \neq 0$, $b \neq 0$)

- (a) $\frac{\pi}{3}$
- (b) $\frac{\pi}{6}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{2}$

Answer:

- (d) $\frac{\pi}{2}$

Question 29.

If a and b are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then $|a + b|$ equals

- (a) $\sqrt{\frac{2}{3}}$
- (b) $\sqrt{2}$
- (c) $\sqrt{3}$
- (d) 2

Answer:

- (d) 2

Question 30.

If the angle between $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j} + a\hat{k}$ is $\frac{\pi}{3}$, then the value of a is

- (a) 0 or 2
- (b) -4 or 0
- (c) 0 or -3
- (d) 2 or -2

Answer:

- (b) -4 or 0

Question 31.

The length of longer diagonal of the parallelogram constructed on $5a + 2b$ and $a - 3b$. If it is given that

$|a| = 2\sqrt{2}$, $|b| = 3$ and angle between a and b is $\frac{\pi}{4}$, is

- (a) 15
- (b) $\sqrt{113}$
- (c) $\sqrt{593}$
- (d) $\sqrt{369}$

Answer:

- (c) $\sqrt{593}$

Question 32.

If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then the value of n is

- (a) $\frac{\sqrt{23}}{6}$

(b) $\frac{23}{6}$

(c) $\frac{2}{3}$

(d) $\frac{3}{2}$

Answer:

(a) $\frac{\sqrt{23}}{6}$

Question 33.

Find the magnitude of vector $3\hat{i} + 2\hat{j} + 12\hat{k}$.

(a) $\sqrt{157}$

(b) $4\sqrt{11}$

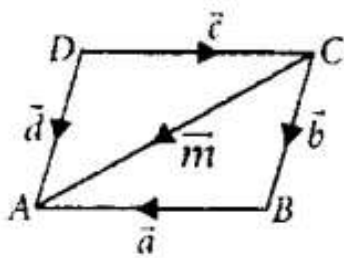
(c) $\sqrt{213}$

(d) $9\sqrt{3}$

Answer:

(a) $\sqrt{157}$

Direction (34 – 36): Study the given parallelogram and answer the following questions.



Question 34.

Which of the following represents equal vectors?

(a) a, c

(b) b, d

(c) b, c

(d) m, d

Answer:

(b) b, d

Question 35.

Which of the following represents collinear but not equal vectors?

(a) a, c

(b) b, d

(c) b, m

(d) Both (a) and (b)

Answer:

(a) a, c

Question 36.

Which of the following represents coinitial vector?

- (a) c, d
- (b) m, b
- (c) b, d
- (d) Both (a) and (b)

Answer:

- (d) Both (a) and (b)

Question 37.

The unit vector in the direction of the sum of vectors

$\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + 3\hat{j} + 4\hat{k}$ is

- (a) $\frac{1}{5\sqrt{2}}(3\hat{i} + 4\hat{j} + 5\hat{k})$
- (b) $\frac{1}{5\sqrt{2}}(3\hat{i} - 4\hat{j} - 5\hat{k})$
- (c) $\frac{1}{2\sqrt{2}}(4\hat{i} + 3\hat{j} + 5\hat{k})$
- (d) $\frac{1}{3\sqrt{2}}(-3\hat{k} + 4\hat{i} + 5\hat{j})$

Answer:

- (a) $\frac{1}{5\sqrt{2}}(3\hat{i} + 4\hat{j} + 5\hat{k})$

Question 38.

The vectors $3\hat{i} + 5\hat{j} + 2\hat{k}$, $2\hat{i} - 3\hat{j} - 5\hat{k}$ and $5\hat{i} + 2\hat{j} - 3\hat{k}$ form the sides of

- (a) Isosceles triangle
- (b) Right triangle
- (c) Scalene triangle
- (d) Equilateral triangle

Answer:

- (d) Equilateral triangle

Question 39.

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are

linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then

- (a) $\alpha = 1, \beta = -1$
- (b) $\alpha = 1, \beta = \pm 1$
- (c) $\alpha = -1, \beta = \pm 1$
- (d) $\alpha = \pm 1, \beta = 1$

Answer:

- (d) $\alpha = \pm 1, \beta = 1$

Question 40.

The vectors $a = x\hat{i} - 2\hat{j} + 5\hat{k}$ and $b = \hat{i} + y\hat{j} - z\hat{k}$ are collinear, if

- (a) $x=1, y=-2, z=-5$
- (b) $x=1.2, y=-4, z=-10$
- (c) $x=-1/2, y=4, z=10$
- (d) All of these

Answer:

- (d) All of these

Question 41.

The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{i} + 2\hat{k}$. The value of x is

- (a) $\left\{-\frac{2}{3}, 2\right\}$
- (b) $\left\{\frac{1}{3}, 2\right\}$
- (c) $\left\{\frac{2}{3}, 0\right\}$
- (d) $\{2, 7\}$

Answer:

- (a) $\left\{-\frac{2}{3}, 2\right\}$

Question 42.

If $a + b + c = 0$, then $a \times b =$

- (a) $c \times a$
- (b) $b \times c$
- (c) 0
- (d) Both (a) and (b)

Answer:

- (d) Both (a) and (b)

Question 43.

If a is perpendicular to b and c , $|a| = 2$, $|b| = 3$, $|c| = 4$ and the angle between b and c is $\frac{2\pi}{3}$, $|abc|$ is equal to

- (a) $4\sqrt{3}$
- (b) $6\sqrt{3}$
- (c) $12\sqrt{3}$
- (d) $18\sqrt{3}$

Answer:

- (c) $12\sqrt{3}$

Question 44.

If $\vec{b}$ and $\vec{c}$ are any two non-collinear unit vectors and $\vec{a}$ is any vector, then

$(\vec{a} \cdot \vec{b})\vec{b} + (\vec{a} \cdot \vec{c})\vec{c} + \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} (\vec{b} \times \vec{c})$ is equal to

- (a) $\vec{0}$ (b) $\vec{a}$ (c) $\vec{b}$ (d) $\vec{c}$

Answer:

(b) a

Question 45.

Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and

$\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then, $[\vec{a} \vec{b} \vec{c}]$ depends on

- (a) neither x nor y (b) both x and y
(c) only x (d) only y

Answer:

(a) neither x nor y

Question 46.

If a, b, c are three non-coplanar vectors, then $(a + b + c) \cdot [(a + b) \times (a + c)]$ is

- (a) 0
(b) $2[abc]$
(c) $-[abc]$
(d) $[abc]$

Answer:

(c) $-[abc]$

Question 47.

If u, v and w are three non-coplanar vectors, then $(u + v - w) \cdot [(u - v) \times (v - w)]$ equals

- (a) 0
(b) $u \cdot v \times w$
(c) $u \cdot w \times v$
(d) $3u \cdot v \times w$

Answer:

(b) $u \cdot v \times w$

Question 48.

If unit vector c makes an angle $\frac{\pi}{3}$ with $\hat{i} \times \hat{j}$, then minimum and maximum values of $(\hat{i} \times \hat{j}) \cdot c$

respectively are

- (a) $0, \frac{\sqrt{3}}{2}$
- (b) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$
- (c) $-1, \frac{\sqrt{3}}{2}$
- (d) None of these

Answer:

- (b) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$

Question 49.

The volume of the tetrahedron whose conterminous edges are $\hat{j} + \hat{k}, \hat{i} + \hat{k}, \hat{i} + \hat{j}$ is

- (a) $\frac{1}{6}$ cu. unit
- (b) $\frac{1}{3}$ cu. unit
- (c) $\frac{1}{2}$ cu. unit
- (d) $\frac{2}{3}$ cu. unit

Answer:

- (b) $\frac{1}{3}$ cu. unit

Question 50.

If the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} - \hat{k}$ form three concurrent edges of a parallelopiped, then the volume of the parallelopiped is

- (a) 8
- (b) 10
- (c) 4
- (d) 14

Answer:

- (c) 4

Question 51.

The volume of the parallelopiped whose edges are represented by $-12\hat{i} + \alpha\hat{k}, 3\hat{j} - \hat{k}$ and $2\hat{i} + \hat{j} - 15\hat{k}$ is 546 cu. units. Then $\alpha =$

- (a) 3
- (b) 2
- (c) -3
- (d) -2

Answer:

- (c) -3

Question 52.

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{k}$, $\vec{c} = \hat{i} + \lambda\hat{j} + 6\hat{k}$ are coplanar, then the value of λ is

- (a) $\frac{5}{2}$ (b) $\frac{3}{5}$
(c) $\frac{7}{3}$ (d) None of these

Answer:

(d) None of these

Question 53.

The value of λ , for which the four points $2\hat{i} + 3\hat{j} - \hat{k}$,

$\hat{i} + 2\hat{j} + 3\hat{k}$, $3\hat{i} + 4\hat{j} - 2\hat{k}$, $\hat{i} - \lambda\hat{j} + 6\hat{k}$ are coplanar, is

- (a) -2 (b) 8 (c) 6 (d) 0

Answer:

(a) -2

Question 54.

The vectors $x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}$,

$(x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k}$ and

$(x+6)\hat{i} + (x+7)\hat{j} + (x+8)\hat{k}$ are coplanar for

- (a) all values of x (b) $x < 0$
(c) $x \leq 0$ (d) None of these

Answer:

(a) all values of x

Question 55.

If the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$, $\lambda\hat{i} - \hat{j} + 2\hat{k}$ are coplanar, then the value of λ is equal to

- (a) 0
(b) 1
(c) 2
(d) 3

Answer:

(a) 0

Question 56.

Find the value of λ if the vectors, $a = 2\hat{i} - \hat{j} + \hat{k}$, $b = \hat{i} + 2\hat{j} - 3\hat{k}$ and $c = 3\hat{i} - \lambda\hat{j} + 5\hat{k}$ are coplanar.

- (a) 4
- (b) -2
- (c) -6
- (d) 5

Answer:

- (a) 4

Question 57.

If a, b, c are unit vectors, then $|a - b| + |b - c| + |c - a|$ does not exceed

- (a) 4
- (b) 9
- (c) 8
- (d) 6

Answer:

- (b) 9

Question 58.

Find the value of λ so that the vectors $2\hat{i} - 4\hat{j} + \hat{k}$ and $4\hat{i} - 8\hat{j} + \lambda\hat{k}$ are perpendicular.

- (a) -15
- (b) 10
- (c) -40
- (d) 20

Answer:

- (c) -40

Question 59.

The dot product of a vector with the vectors $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

- (a) $\hat{i} + 2\hat{j} + \hat{k}$
- (b) $-\hat{i} + 3\hat{j} - 2\hat{k}$
- (c) $\hat{i} + 2\hat{j} + 3\hat{k}$
- (d) $\hat{i} - 3\hat{j} - 3\hat{k}$

Answer:

- (a) $\hat{i} + 2\hat{j} + \hat{k}$

Question 60.

If a, b, c are three mutually perpendicular vectors of equal magnitude, find the angle between a and $a + b + c$.

- (a) $\cos^{-1}(1/\sqrt{3})$

(b) $\cos^{-1}(1/2\sqrt{2})$

(c) $\cos^{-1}(1/3\sqrt{3})$

(d) $\cos^{-1}(1/2\sqrt{3})$

Answer:

(a) $\cos^{-1}(1/\sqrt{3})$