Unit 11 (Area Related To Circles)

Exercise 11.1 Multiple Choice Questions (MCQs)

Question 1:

If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(a) $R_1 + R_2 = R$
(b) $R_1^2 + R_2^2 = R^2$
(c) $R_1 + R_2 < R$
(d) $R_1^2 + R_2^2 < R^2$

Solution:

(b) According to the given condition,

Area of circle =Area of first circle + Area of second circle

$\therefore \quad \pi R^2 = \pi R_1^2 + \pi R_2^2$

$\Rightarrow \quad R^2 = R_1^2 + R_2^2$

Question 2:

If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) Nothing definite can be said about the relation among R_1, R_2 and R

Solution:

(a) According to the given condition,

Circumference of circle = Circumference of first circle + Circumference of second circle

$\therefore \quad 2\pi R = 2\pi R_1 + 2\pi R_2$

$\Rightarrow \quad R = R_1 + R_2$

Question 3:

If the circumference of a circle and the perimeter of a square are equal, then

(a) Area of the circle = Area of the square

(b) Area of the circle > Area of the square

(c) Area of the circle < Area of the square

(d) Nothing definite can be said about the relation between the areas of the circle and square

Solution:

(b) According to the given condition,

Circumference of a circle = Perimeter of square

$$2\pi r = 4a$$

[where, r and a are radius of circle and side of square respectively]

$\Rightarrow \quad \frac{22}{7} r = 2a \Rightarrow 11r = 7a$

$\Rightarrow \quad a = \frac{11}{7} r \Rightarrow r = \frac{7a}{11}$ ...(i)

Now, area of circle, $A_1 = \pi r^2$

$= \pi \left(\frac{7a}{11}\right)^2 = \frac{22}{7} \times \frac{49a^2}{121}$ [from Eq. (i)]

$= \frac{14a^2}{11}$ ...(ii)

and area of square, $A_2 = (a)^2$ ...(iii)

From Eqs. (ii) and (iii), $A_1 = \frac{14}{11} A_2$

$\therefore \quad A_1 > A_2$

Hence, Area of the circle > Area of the square.

Question 4:

Area of the largest triangle that can be inscribed in a semi-circle of radius r units is

(a) r^2 squnits (b) $\frac{1}{2}$ r^2 sq units (c) $2r^2$ sq units (d) $\sqrt{2}\, r^2$ sq units

Solution:

(a) Take a point C on the circumference of the semi-circle and join it by the end points of diameter A and B.

$\therefore \quad \angle C = 90°$ [by property of circle]

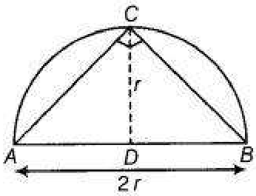
[angle in a semi-circle are right angle]

So, ΔABC is right angled triangle.

$\therefore$ Area of largest $\Delta ABC = \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times 2r \times r$

$= r^2$ sq units



Question 5:

If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(a) 22 :7 (b) 14:11 (c) 7:22 (d) 11:14

Solution:

(b) Let radius of circle be r and side of a square be a.

According to the given condition,

Perimeter of a circle = Perimeter of a square

$\therefore \quad 2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$ ...(i)

Now, $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$ [from Eq. (i)]

$= \frac{\pi r^2}{\pi^2 r^2 / 4} = \frac{4}{\pi} = \frac{4}{22/7} = \frac{28}{22} = \frac{14}{11}$

Question 6:

It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(a) 10 m (b)15m (c) 20 m (d) 24 m

Solution:

(a) Area of first circular park, whose diameter is 16 m

$$= \pi r^2 = \pi \left(\frac{16}{2}\right)^2 = 64\,\pi\,m^2 \qquad \left[\because r = \frac{d}{2} = \frac{16}{2} = 8\,m\right]$$

Area of second circular park, whose diameter is 12 m

$$= \pi \left(\frac{12}{2}\right)^2 = \pi\,(6)^2 = 36\pi m^2 \qquad \left[\because r = \frac{d}{2} = \frac{12}{2} = 6\,m\right]$$

According to the given condition,

Area of single circular park = Area of first circular park + Area of second circular park

$$\pi R^2 = 64\,\pi + 36\,\pi \qquad [\because R \text{ be the radius of single circular park}]$$

$$\Rightarrow \quad \pi R^2 = 100\pi \Rightarrow R^2 = 100$$

$$\therefore \quad R = 10\,m$$

Question 7:

The area of the circle that can be inscribed in a square of side 6 cm is

(a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2

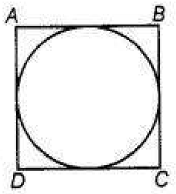
Solution:

(d) Given, side of square = 6 cm

$\therefore$ Diameter of a circle, (d) = Side of square = 6 cm

$$\therefore \quad \text{Radius of a circle } (r) = \frac{d}{2} = \frac{6}{2} = 3\,\text{cm}$$

$$\therefore \quad \text{Area of circle} = \pi\,(r)^2 = \pi\,(3)^2 = 9\,\pi\,\text{cm}^2$$



Question 8:

The area of the square that can be inscribed in a circle of radius 8 cm is

(a) 256 cm^2 (b) 128 cm^2 (c)$64\sqrt{2}$ cm^2 (d)64 cm^2

Solution:

(b) Given, radius of circle, r = OC = 8cm.

$\therefore$ Diameter of the circle = AC = 2 x OC = 2 x 8= 16 cm

which is equal to the diagonal of a square.

Let side of square be x.

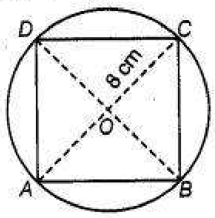
In right angled ΔABC, $AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$$\Rightarrow \quad (16)^2 = x^2 + x^2$$

$$\Rightarrow \quad 256 = 2x^2$$

$$\Rightarrow \quad x^2 = 128$$

$$\therefore \quad \text{Area of square} = x^2 = 128\,\text{cm}^2$$



Alternate Method

Radius of circle (r) = 8 cm

Diameter of circle $(d) = 2r = 2 \times 8 = 16$ cm

Since, square inscribed in circle.

$\therefore$ Diagonal of the squre = Diameter of circle

$$\text{Now,} \quad \text{Area of square} = \frac{(\text{Diagonal})^2}{2} = \frac{(16)^2}{2} = \frac{256}{2} = 128\,\text{cm}^2$$

Question 9:

The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

Solution:

(c) $\because$ Circumference of first circle = $2\,\pi r = \pi d_1 = 36\,\pi$ cm [given, $d_1 = 36$ cm]

and circumference of second circle = $\pi d_2 = 20\,\pi$ cm [given, $d_2 = 20$ cm]

According to the given condition,

Circumference of circle = Circumference of first circle + Circumference of second circle

$\Rightarrow \quad \pi D = 36\pi + 20\pi$ [where, D is diameter of a circle]

$\Rightarrow \quad D = 56$ cm

So, diameter of a circle is 56 cm.

$\therefore \quad$ Required radius of circle $= \frac{56}{2} = 28$cm

Question 10:

The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(a) 31 cm (b) 25 cm (c) 62 cm (d) 50 cm

Solution:

(d) Let r_1 = 24 cm and r_2 = 7 cm

$\therefore \quad$ Area of first circle $= \pi r_1^2 = \pi (24)^2 = 576\pi$ cm^2

and area of second circle $= \pi r_2^2 = \pi (7)^2 = 49\pi$ cm^2

According to the given condition,

Area of circle = Area of first circle + Area of second circle

$\therefore \quad \pi R^2 = 576\pi + 49\pi$ [where, R be radius of circle]

$\Rightarrow \quad R^2 = 625 \Rightarrow R = 25$ cm

$\therefore \quad$ Diameter of a circle $= 2R = 2 \times 25 = 50$ cm

Exercise 11.2 Very Short Answer Type Questions

Write whether True or False and justify your answer

Question 1:

Is the area of the circle inscribed in a square of side a cm, πa^2 cm^2 ? Give reasons for your answer

Solution:

False

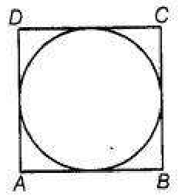
Let $ABCD$ be a square of side a.

$\therefore$ Diameter of circle = Side of square = a

$\therefore \quad$ Radius of circle $= \frac{a}{2}$

$\therefore \quad$ Area of circle $= \pi$ (Radius)$^2 = \pi \left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

Hence, area of the circle is $\frac{\pi a^2}{4}$ cm^2.



Question 2:

Will it be true to say that the perimeter of a square circumscribing a circle of radius a cm is 80 cm? Give reason for your answer.

Solution:

True

Given, radius of circle, r = a cm

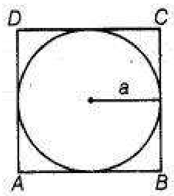
$\therefore$ Diameter of circle, $d = 2 \times$ Radius $= 2a$ cm

$\therefore \quad$ Side of a square = Diameter of circle

$= 2a$ cm

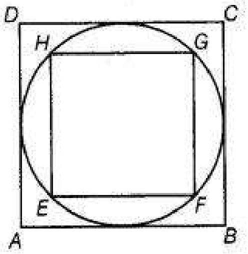
$\therefore$ Perimeter of a square $= 4 \times$ (Side) $= 4 \times 2a$

$= 8a$ cm



Question 3:

In figure, a square is inscribed in a circle of diameter d and another square is circumscribing the circle. Is the area of the outer square four times the area of the inner square? Give reason for your answer.



Solution:

False

Given diameter of circle is d.

∴ Diagonal of inner square = Diameter of circle = d

Let side of inner square EFGH be x.

∴ In right angled ΔEFG,

$$EG^2 = EF^2 + FG^2 \quad \text{[by Pythagoras theorem]}$$

$$\Rightarrow \quad d^2 = x^2 + x^2$$

$$\Rightarrow \quad d^2 = 2x^2 \Rightarrow x^2 = \frac{d^2}{2}$$

$$\therefore \quad \text{Area of inner square } EFGH = (\text{Side})^2 = x^2 = \frac{d^2}{2}$$

But side of the outer square ABCS = Diameter of circle = d

∴ Area of outer square = d^2

Hence, area of outer square is not equal to four times the area of the inner square.

Question 4:

Is it true to say that area of segment of a circle is less than the area of its corresponding sector? Why?

Solution:

False

It is true only in the case of minor segment. But in case of major segment area is always greater than the area of sector.

Question 5:

Is it true that the distance travelled by a circular wheel of diameter d cm in one revolution is 2πd cm? Why?

Solution:

False

Because the distance travelled by the wheel in one revolution is equal to its circumference i.e., πd.

i.e., $\pi(2r) = 2\pi r$ = Circumference of wheel [∵d = 2r]

Question 6:

In covering a distance s m, a circular wheel of radius r m makes $\frac{s}{2\pi r}$ revolution. Is this statement true? Why?

Solution:

True

The distance covered in one revolution is 2πr i.e., its circumference.

Question 7:

The numerical value of the area of a circle is greater than the numerical value of its circumference. Is this statement true? Why?

Solution:

False

If 0< r< 2, then numerical value of circumference is greater than numerical value of area of circle and if r > 2, area is greater than circumference.

Question 8:

If the length of an arc of a circle of radius r is equal to that of an arc of a circle of radius 2r, then the angle of the corresponding sector of the first circle is double the angle of the corresponding sector of the other circle. Is this statement false? Why?

Solution:

False

Let two circles C_1 and C_2 of radius r and 2r with centres O and O', respectively.

It is given that, the arc length $\widehat{AB}$ of C_1 is equal to arc length $\widehat{CD}$ of C_2 *i.e.*, $\widehat{AB} = \widehat{CD} = l$ (say)

Now, let θ_1 be the angle subtended by arc $\widehat{AB}$ of θ_2 be the angle subtended by arc $\widehat{CD}$ at the centre.

$$\therefore \quad \widehat{AB} = l = \frac{Q_1}{360} \times 2\pi r \quad \ldots(i)$$

$$\text{and} \quad \widehat{CD} = l = \frac{\theta_2}{360} \times 2\pi(2r) = \frac{\theta_2}{360} \times 4\pi r \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$\frac{\theta_1}{360} \times 2\pi r = \frac{\theta_2}{360} \times 4\pi r$$

$$\Rightarrow \quad \theta_1 = 2\theta_2$$

i.e., angle of the corresponding sector of C_1 is double the angle of the corresponding sector of C_2.

It is true statement

Question 9:

The area of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true? Why?

Solution:

False

It is true for arcs of the same circle. But in different circle, it is not possible.

Question 10:

The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal? Why?

Solution:

False

It is true for arcs of the same circle. But in different circle, it is not possible

Question 11:

Is the area of the largest circle that can be drawn inside a rectangle of length a cm and breadth b cm (a > b) is πb^2 cm? Why?

Solution:

False

The area of the largest circle that can be drawn inside a rectangle is $\pi \left(\frac{b}{2}\right)^2$ cm, where $\pi \frac{b}{2}$ is the radius of the circle and it is possible when rectangle becomes a square.

Question 12:

Circumference of two circles are equal. Is it necessary that their areas be equal? Why?

Solution:

True

If circumference of two circles are equal, then their corresponding radii are equal. So, their areas will be equal.

Question 13:

Areas of two circles are equal. Is it necessary that their circumferences are equal? Why?

Solution:

True

If areas of two circles are equal, then their corresponding radii are equal. So, their circumference will be equal.

Question 14:

Is it true to say that area of a square inscribed in a circle of diameter p cm is p^2 cm^2 ? Why?

Solution:

True

When the square is inscribed in the circle, the diameter of a circle is equal to the diagonal of a square but not the side of the square.

Exercise 11.3 Short Answer Type Questions

Question 1:

Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of radii 15 cm and 18 cm.

Solution:

Let the radius of a circle be r.

$\therefore$ Circumference of a circle = $2\pi r$

Let the radii of two circles are r_1 and r_2 whose values are 15 cm and 18 cm respectively.

i.e. $r_1 = 15$cmand $r_2 = 18$cm

Now, by given condition,

Circumference of circle = Circumference of first circle + Circumference of second circle

$\Rightarrow \quad 2\pi r = 2\pi r_1 + 2\pi r_2$

$\Rightarrow \quad r = r_1 + r_2$

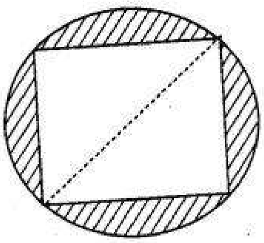
$\Rightarrow \quad r = 15 + 18$

$\therefore \quad r = 33$ cm

Hence, required radius of a circle is 33 cm.

Question 2:

In figure, a square of diagonal 8 cm is inscribed in a circle. Find the area of the shaded region.



Solution:

Let the side of a square be a and the radius of circle be r.

Given that, length of diagonal of square = 8 cm

$\Rightarrow \quad a\sqrt{2} = 8$

$\Rightarrow \quad a = 4\sqrt{2}$ cm

Now, Diagonal of a square = Diameter of a circle

$\Rightarrow$ Diameter of circle = 8

$\Rightarrow$ Radius of circle = $r = \dfrac{\text{Diameter}}{2}$

$\Rightarrow \quad r = \dfrac{8}{2} = 4$ cm

$\therefore$ Area of circle = $\pi r^2 = \pi(4)^2$

$= 16\pi \times$ cm^2

and Area of square = $a^2 = (4\sqrt{2})^2$

$= 32$ cm^2

So, the area of the shaded region = Area of circle – Area of square

$= (16\pi - 32)\ cm^2$

Hence, the required area of the shaded region is $(16\pi - 32)\ cm^2$.

Question 3:

Find the area of a sector of a circle of radius 28 cm and central angle 45°.

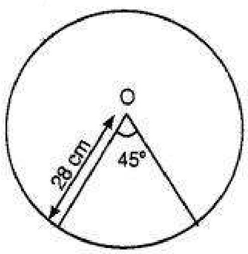
Solution:

Given that, Radius of a circle, r = 28 cm

and measure of central angle θ= 45°

Hence, the required area of a sector of a circle is 308 cm

$$\therefore \quad \text{Area of a sector of a circle} = \frac{\pi r^2}{360^\circ} \times \theta$$
$$= \frac{22}{7} \times \frac{(28)^2}{360} \times 45^\circ$$
$$= \frac{22 \times 28 \times 28}{7} \times \frac{45^\circ}{360^\circ}$$
$$= 22 \times 4 \times 28 \times \frac{1}{8}$$
$$= 22 \times 14$$
$$= 308\ cm^2$$



Question 4:

The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make, so as to keep a speed of 66 km/h?

Solution:

Given, radius of wheel, r = 35 cm

$$\text{Circumference of the wheel} = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 35$$
$$= 220\ cm$$
$$\text{But speed of the wheel} = 66\ kmh^{-1} = \frac{66 \times 1000}{60}\ m/min$$
$$= 1100 \times 100\ cm\,min^{-1}$$
$$= 110000\ cm\,min^{-1}$$
$$\therefore \text{Number of revolutions in 1 min} = \frac{110000}{220} = 500 \text{ revolution}$$

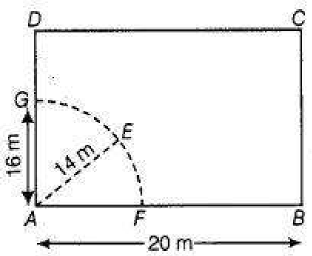
Hence, required number of revolutions per minute is 500.

Question 5:

A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20 m x 16 m. Find the area of the field in which the cow can graze.

Solution:

Let ABCD be a rectangular field of dimensions 20 m x 16 m . Suppose, a cow is tied at a point A Let length of rope be AE = 14 m = r (say).



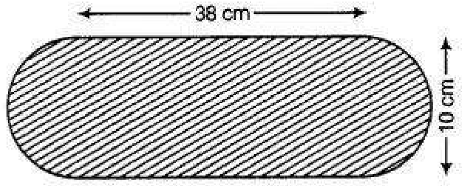
$$\therefore \text{Area of the field in which the cow graze} = \text{Area of sector } AFEG = \frac{\theta}{360^\circ} \times \pi r^2$$
$$= \frac{90}{360} \times \pi (14)^2$$

[so, the angle between two adjacent sides of a rectangle is 90°]

$$= \frac{1}{4} \times \frac{22}{7} \times 196$$
$$= 154\ m^2$$

Question 6:

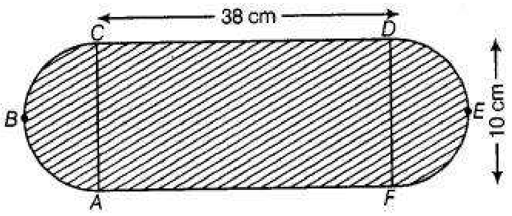
Find the area of the flower bed (with semi-circular ends) shown in figure



Solution:

Length and breadth of a circular bed are 38 cm and 10 cm.

$\therefore$ Area of rectangle ACDF = Length x Breadth = 38 x 10 = 380 cm^2



Both ends of flower bed are semi-circles.

$\therefore$ Radius of semi-circle $= \frac{DF}{2} = \frac{10}{2} = 5$ cm

$\therefore$ Area of one semi-circles $= \frac{\pi r^2}{2} = \frac{\pi}{2}(5)^2 = \frac{25\pi}{2}$ cm^2

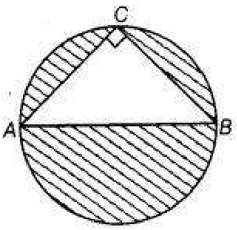
$\therefore$ Area of two semi-circles $= 2 \times \frac{25}{2}\pi = 25\pi$ cm^2

$\therefore$ Total area of flower bed = Area of rectangle $ACDF$ + Area of two semi-circles

$= (380 + 25\pi)$ cm^2

Question 7:

In figure, AB is a diameter of the circle, AC = 6 cm and BC = 8 cm. Find the area of the shaded region, (use π = 3.14)



Solution:

Given, AC = 6 cm and BC = 8 cm

We know that, triangle in a semi-circle with hypotenuse as diameter is right angled triangle.

$\therefore$ $\angle C = 90°$

In right angled ΔACB, use Pythagoras theorem,

$\therefore$ $AB^2 = AC^2 + CB^2$

$\Rightarrow$ $AB^2 = 6^2 + 8^2 = 36 + 64$

$\Rightarrow$ $AB^2 = 100$

$\Rightarrow$ $AB = 10$ cm [since, side cannot be negative]

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} \times 8 \times 6 = 24$ cm^2

Here, diameter of circle, $AB = 10$ cm

$\therefore$ Radius of circle, $r = \frac{10}{2} = 5$ cm

Area of circle $= \pi r^2 = 3.14 \times (5)^2$

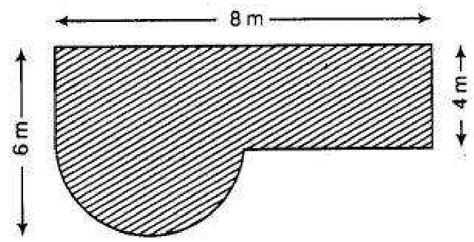
$= 3.14 \times 25 = 78.5$ cm^2

$\therefore$ Area of the shaded region = Area of circle − Area of ΔABC

$= 78.5 - 24 = 54.5$ cm^2

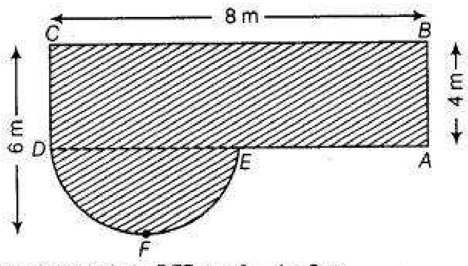
Question 8:

Find the area of the shaded field shown in figure.



Solution:

In a figure, join ED



From figure, radius of semi-circle *DFE*, $r = 6 - 4 = 2$ m

Now, area of rectangle $ABCD = BC \times AB = 8 \times 4 = 32\ m^2$

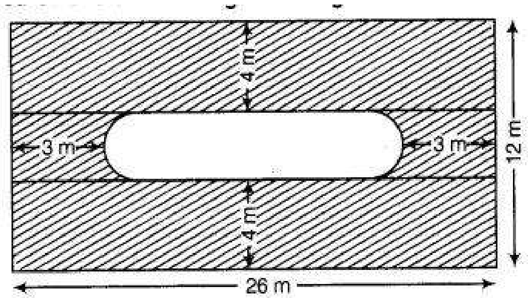
and area of semi-circle $DFE = \dfrac{\pi r^2}{2} = \dfrac{\pi}{2}(2)^2 = 2\pi\ m^2$

$\therefore$ Area of shaded region = Area of rectangle *ABCD* + Area of semi-circle *DFE*

$= (32 + 2\pi)\ m^2$

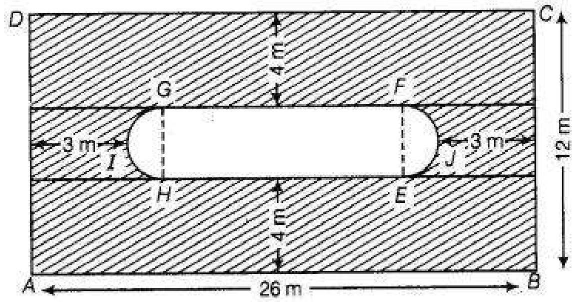
Question 9:

Find the area of the shaded region in figure.



Solution:

join GH and FE



Here, breadth of the rectangle $BC = 12$ m

$\therefore$ Breadth of the inner rectangle $EFGH = 12 - (4 + 4) = 4$ cm

which is equal to the diameter of the semi-circle *EJF*, $d = 4$ m

$\therefore$ Radius of semi-circle *EJF*, $r = 2$ m

$\therefore$ Length of inner rectangle $EFGH = 26 - (5 + 5) = 16$ m

$\therefore$ Area of two semi-circles *EJF* and $HIG = 2\left(\dfrac{\pi r^2}{2}\right) = 2 \times \pi \dfrac{(2)^2}{2} = 4\pi$ m

Now, area of inner rectangle $EFGH = EH \times FG = 16 \times 4 = 64\ m^2$

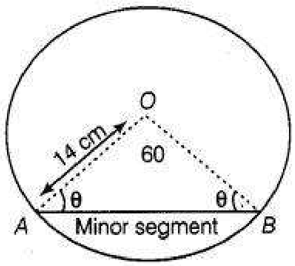
and area of outer rectangle $ABCD = 26 \times 12 = 312\ m^2$

$\therefore$ Area of shaded region = Area of outer rectangle − (Area of two semi-circles + Area of inner rectangle)

$= 312 - (64 + 4\pi) = (248 - 4\pi)\ m^2$

Question 10:

Find the area of the minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is 60°.



Solution:

Given that, radius of circle (r) = 14 cm

and angle of the corresponding sector i.e., central angle (θ) = 60°

Since, in ΔAOB, OA = OB = Radius of circle i.e., ΔAOB is isosceles.

$\Rightarrow \quad \angle OAB = \angle OBA = \theta$

Now, in $\Delta OAB \quad \angle AOB + \angle OAB + \angle OBA = 180°$

[since,sum of interior angles of any triangle is 180°]

$\Rightarrow \quad 60° + \theta + \theta = 180°$ [given, $\angle AOB = 60°$]

$\Rightarrow \quad 2\theta = 120°$

$\Rightarrow \quad \theta = 60°$

i.e. $\quad \angle OAB = \angle OBA = 60° = \angle AOB$

Since, all angles of ΔAOB are equal to 60° *i.e.*, ΔAOB is an equilateral triangle.

Also, $\quad OA = OB = AB = 14 \text{ cm}$

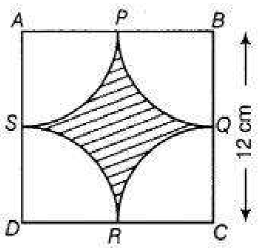
So, $\quad$ Area of $\Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (14)^2$ [$\because$ area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$]

$= \frac{\sqrt{3}}{4} \times 196 = 49\sqrt{3} \text{ cm}^2$

Question 11:

Find the area of the shaded region in figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-point P, Q, R and 5 of the sides AB, BC, CD and DA, respectively of a square ABCD. (use π = 3.14)



Solution:

Given, side of a square BC = 12 cm

Since, Q is a mid-point of BC.

$\therefore \quad \text{Radius} = BQ = \frac{12}{2} = 6 \text{ cm}$

Now, $\quad$ area of quadrant $BPQ = \frac{\pi r^2}{4} = \frac{3.14 \times (6)^2}{4} = \frac{113.04}{4} \text{cm}^2$

Area of four quadrants $= \frac{4 \times 113.04}{4} = 1123.04 \text{ cm}^2$

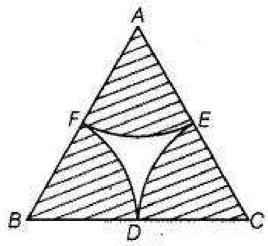
Now, $\quad$ area of square $ABCD = (12)^2 = 144 \text{ cm}^2$

$\therefore \quad$ Area of the shaded region = Area of square − Area of four quadrants

$= 144 - 113.04 = 30.96 \text{ cm}^2$

Question 12:

In figure arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, To intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region, (use π = 3.14)



Solution:

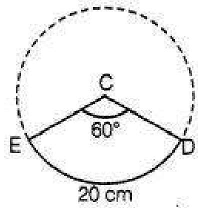
Since, ABC is an equilateral triangle.

$\therefore \quad \angle A = \angle B = \angle C = 60^\circ$

and $\quad AB = BC = AC = 10$ cm

So, E, F and D are mid-points of the sides.

$\therefore \quad AE = EC = CD = BD = BF = FA = 5$ cm



Now, area of sector $CDE = \dfrac{\theta \pi r^2}{360} = \dfrac{60 \times 3.14}{360}(5)^2$

$= \dfrac{3.14 \times 25}{6} = \dfrac{78.5}{6} = 13.0833 \text{ cm}^2$

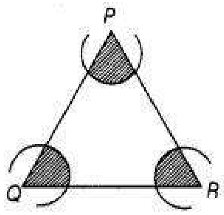
$\therefore$ Area of shaded region $= 3$ (Area of sector CDE)

$= 3 \times 13.0833$

$= 39.25 \text{ cm}^2$

Question 13:

In figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



Solution:

Given that, radii of each arc (r) = 14 cm

Now, area of the sector with central $\angle P = \dfrac{\angle P}{360^\circ} \times \pi r^2$

$= \dfrac{\angle P}{360^\circ} \times \pi \times (14)^2 \text{ cm}^2$

[$\because$ area of any sector with central angle θ and radius $r = \dfrac{\pi r^2}{360^\circ} \times \theta$]

Area of the sector with central angle $= \dfrac{\angle Q}{360^\circ} \times \pi r^2 = \dfrac{\angle Q}{360^\circ} \times \pi \times (14)^2 \text{ cm}^2$

and area of the sector with central angle $R = \dfrac{\angle R}{360^\circ} \times \pi r^2 = \dfrac{\angle R}{360^\circ} \times \pi \times (14)^2 \text{ cm}^2$

Therefore, sum of the areas (in cm^2) of three sectors

$= \dfrac{\angle P}{360^\circ} \times \pi \times (14)^2 + \dfrac{\angle \theta}{360^\circ} \times \pi \times (14)^2 + \dfrac{\angle R}{360^\circ} \times \pi \times (14)^2$

$= \dfrac{\angle P + \angle Q + \angle R}{360} \times 196 \times \pi = \dfrac{180^\circ}{360^\circ} \times 196\pi \text{ cm}^2$

[since, sum of all interior angles in any triangle is 180°]

$= 98\,\pi \text{ cm}^2 = 98 \times \dfrac{22}{7}$

$= 14 \times 22 = 308 \text{ cm}^2$

Hence, the required area of the shaded region is 308 cm^2.

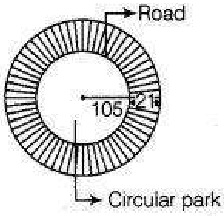
Question 14:

A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, then find the area of the road.

Solution:

Given that, a circular park is surrounded by a road.
Width of the road = 21 m
Radius of the park (r_i) = 105 m
∴ Radius of whole circular portion (park + road),

$$r_e = 105 + 21 = 126 \text{ m}$$



Now, area of road = Area of whole circular portion − Area of circular park

$$= \pi r_e^2 - \pi r_i^2 \quad [\because \text{area of circle} = \pi r^2]$$

$$= \pi(r_e^2 - r_i^2)$$

$$= \pi\{(126^2 - (105)^2\}$$

$$= \frac{22}{7} \times (126 + 105)(126 - 105)$$

$$= \frac{22}{7} \times 231 \times 21 \quad [\because (a^2 - b^2) = (a - b)(a + b)]$$

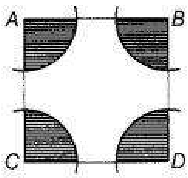
$$= 66 \times 231$$

$$= 15246 \text{cm}^2$$

Hence, the required area of the road is 15246 cm^2.

Question 15:

In figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.



Solution:

Given that, radius of each arc (r) = 21 cm

$$\text{Area of sector with } \angle A = \frac{\angle A}{360^\circ} \times \pi r^2 = \frac{\angle A}{360^\circ} \times \pi \times (21)^2 \text{ cm}^2$$

$$\left[\because \text{area of any sector with central angle } \theta \text{ and radius } r = \frac{\pi r^2}{360^\circ} \times \theta\right]$$

$$\text{Area of sector with } \angle B = \frac{\angle B}{360^\circ} \times \pi r^2 = \frac{\angle B}{360^\circ} \times \pi \times (21)^2 \text{ cm}^2$$

$$\text{Area of sector with } \angle C = \frac{\angle C}{360^\circ} \times \pi r^2 = \frac{\angle C}{360^\circ} \times \pi \times (21)^2 \text{ cm}^2$$

$$\text{and area of sector with } \angle D = \frac{\angle D}{360^\circ} \times \pi r^2 = \frac{\angle D}{360^\circ} \times \pi \times (21)^2 \text{cm}^2$$

Therefore, sum of the areas (in cm^2) of the four sectors

$$= \frac{\angle A}{360^\circ} \times \pi \times (21)^2 + \frac{\angle B}{360^\circ} \times \pi \times (21)^2 + \frac{\angle C}{360^\circ} \times \pi \times (21)^2 + \frac{\angle D}{360^\circ} \times \pi \times (21)^2$$

$$= \frac{(\angle A + \angle B + \angle C + \angle D)}{360^\circ} \times \pi \times (21)^2$$

$$[\because \text{sum of all interior angles in any quadrilateral} = 360^\circ]$$

$$= 22 \times 3 \times 21 = 1386 \text{ cm}^2$$

Hence, required area of the shade region is 1386 cm^2

Question 16:

A piece of wire 20 cm long is bent into the from of an arc of a circle, subtending an angle of 60° at its centre. Find the radius of the circle.

Solution:

Length of arc of circle = 20 cm

Here, central angle $\theta = 60^\circ$

∴ $\text{Length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$

⇒ $20 = \frac{60^\circ}{360^\circ} \times 2\pi r \Rightarrow \frac{20 \times 6}{2\pi} = r$

∴ $r = \frac{60}{\pi}$ cm

Hence, the radius of circle is $\frac{60}{\pi}$ cm.

Exercise 11.4 Long Answer Type Questions

Question 1:

The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹ 50 per m.

Solution:

Given, area of a circular playground = 22176 m^2

$$\therefore \quad \pi r^2 = 22176 \quad [\because \text{area of circle} = \pi r^2]$$

$$\Rightarrow \quad \frac{22}{7} r^2 = 22176 \Rightarrow r^2 = 1008 \times 7$$

$$\Rightarrow \quad r^2 = 7056 \Rightarrow r = 84 \text{ m}$$

$$\therefore \quad \text{Circumference of a circle} = 2\pi r = 2 \times \frac{22}{7} \times 84$$

$$= 44 \times 12 = 528 \text{ m}$$

$$\therefore \quad \text{Cost of fencing this ground} = 528 \times 50 = ₹\, 26400$$

Question 2:

The diameters of front and rear wheels of a tractor are 80 cm and 2m, respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

Solution:

Given, diameter of front wheels, d_1 = 80 cm

and diameter of rear wheels, d_2 = 2 m = 200 cm

$$\therefore \quad \text{Radius of front wheel } (r_1) = \frac{80}{2} = 40 \text{ cm}$$

$$\text{and} \quad \text{radius of rear wheel } (r_2) = \frac{200}{2} = 100 \text{ cm}$$

$$\therefore \quad \text{Circumference of the front wheel} = 2\pi r_1 = \frac{2 \times 22}{7} \times 40 = \frac{1760}{7}$$

$$\therefore \quad \text{Total distance covered by front wheel} = 1400 \times \frac{1760}{7} = 200 \times 1760$$

$$= 352000 \text{ cm}$$

$$\text{Number of revolutions by rear wheel} = \frac{\text{Distance coverd}}{\text{Circumference}}$$

$$= \frac{352000}{2 \times \frac{22}{7} \times 100} = \frac{7 \times 3520}{2 \times 22} = \frac{24640}{44} = 560$$

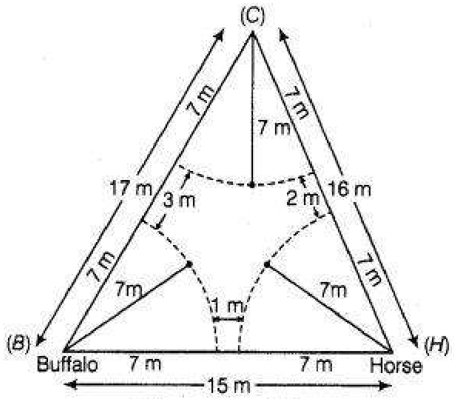
Question 3:

Sides of a triangular field are 15 m, 16m and 17m. with the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.

Solution:

Given that, a triangular field with the three corners of the field a cow, a buffalo and a horse are tied separately with ropes. So, each animal grazed the field in each corner of triangular field as a sectorial form.

Given, radius of each sector (r) = 7m



Now, area of sector with $\angle C = \frac{\angle C}{360°} \times \pi r^2 = \frac{\angle C}{360°} \times \pi \times (7)^2 m^2$

Area of the sector with $\angle B = \frac{\angle B}{360°} \times \pi r^2 = \frac{\angle B}{360°} \times \pi \times (7)^2 m^2$

and area of the sector with $\angle H = \frac{\angle H}{360°} \times \pi r^2 = \frac{\angle H}{360°} \times \pi \times (7)^2 m^2$

Therefore, sum of the areas (in cm^2) of the three sectors

$$= \frac{\angle C}{360°} \times \pi \times (7)^2 + \frac{\angle B}{360°} \times \pi \times (7)^2 + \frac{\angle H}{360°} \times \pi \times (7)^2$$

$$= \frac{(\angle C + \angle B + \angle H)}{360°} \times \pi \times 49$$

$$= \frac{180°}{360°} \times \frac{22}{7} \times 49 = 11 \times 7 = 77 \text{ cm}^2$$

Given that, sides of triangle are $a = 15$, $b = 16$ and $c = 17$

Now, semi-perimeter of triangle, $s = \frac{a + b + c}{2}$

$\Rightarrow \quad = \frac{15 + 16 + 17}{2} = \frac{48}{2} = 24$

$\therefore$ Area of triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$ [by Heron's formula]

$$= \sqrt{24 \cdot 9 \cdot 8 \cdot 7}$$

$$= \sqrt{64 \cdot 9 \cdot 21}$$

$$= 8 \times 3\sqrt{21} = 24\sqrt{21} \text{ m}^2$$

So, area of the field which cannot be grazed by the three animals

= Area of triangular field – Area of each sectorial field

$= 24\sqrt{21} - 77 \text{ m}^2$

Hence, the required area of the field which can not be grazed by the three animals is $(24\sqrt{21} - 77) \text{m}^2$.

Question 4:

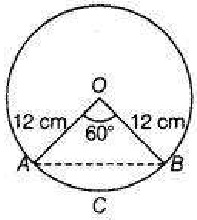
Find the area of the segment of a circle of radius 12 cm whose corresponding sector has a centrel angle of 60°. (use π = 3.14)

Solution:

Given that, radius of a circle (r) = 12 cm

and central angle of sector OBCA (θ) = 60°

$\therefore$ Area of sector $OBCA = \frac{\pi r^2}{360} \times \theta$ [here, $OBCA$ = sector and $ABCA$ = segment]



$$= \frac{3.14 \times 12 \times 12}{360°} \times 60°$$
$$= 3.14 \times 2 \times 12$$
$$= 3.14 \times 24 = 75.36 \text{ cm}^2$$

Since, ΔOAB is an isosceles triangle.

Let $\angle OAB = \angle OBA = \theta_1$

and $OA = OB = 12$ cm

$\angle AOB = \theta = 60°$

$\angle OAB + \angle OBA + \angle AOB = 180°$ [$\because$ sum of all interior angles of a triangle is 180°]

$\Rightarrow \theta_1 + \theta_1 + 60° = 180°$

$\Rightarrow 2\theta_1 = 120°$

$\Rightarrow \theta_1 = 60°$

$\therefore \theta_1 = \theta = 60°$

So, the required ΔAOB is an equilateral triangle.

Now, area of $\Delta AOB = \frac{\sqrt{3}}{4}(\text{side})^2$ [$\because$ area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$]

$$= \frac{\sqrt{3}}{4}(12)^2$$
$$= \frac{\sqrt{3}}{4} \times 12 \times 12 = 36\sqrt{3} \text{ cm}^2$$

Now, area of the segment of a circle *i.e.*,

$ABCA$ = Area of sector $OBCA$ – Area of ΔAOB

$$= (75.36 - 36\sqrt{3}) \text{ cm}^2$$

Hence, the required area of segment of a circle is $(75.36 - 36\sqrt{3}) \text{ cm}^2$.

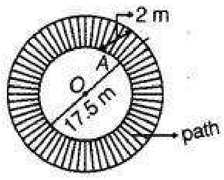
Question 5:

A circular pond is 17.5 m is of diameter. It is surrounded by a 2m wide path. Find the cost of constructing the path at the rate of ₹ 25 Per m^2?

Solution:

Given that, a circular pond is surrounded by a wide path.

The diameter of circular pond = 17.5 m



$\therefore$ Radius of circular pond $(r_i) = \frac{\text{Diameter}}{2}$

i.e., $OA = r_i = \frac{17.5}{2} = 8.75$ m

and the width of the path = 2 m

i.e., $AB = 2$ m

Now, length of $OB = OA + AB = r_i + AB$

Let $(r_e) = 8.75 + 2 = 10.75$ m

So, area of circular path = Area of outer circle *i.e.*, (circular pond + path) – Area of circular pond

$$= \pi r_e^2 - \pi r_i^2 \quad [\because \text{area of circle} = \pi r^2]$$
$$= \pi(r_e^2 - r_i^2)$$
$$= \pi\{(10.75)^2 - (8.75)^2\}$$
$$= \pi\{(10.75 + 8.75)(10.75 - 8.75)\}$$
$$= 3.14 \times 19.5 \times 2$$
$$= 122.46 \text{ m}^2$$

Now, cost of constructing the path per square metre = ₹ 25

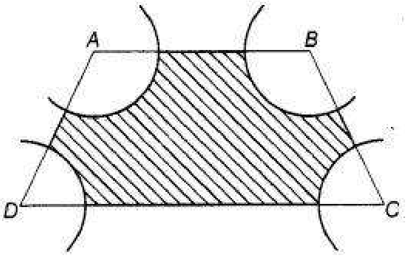
$\therefore$ Cost constructing the path ₹ 122.46 m^2 = 122.46 × 25

= ₹ 3061.50

Hence, required cost of constructing the path at the rate of ₹ 25 per m^2 is ₹ 3061.50.

Question 6:

In figure, ABCD is a trapezium with AB || DC. AB = 18 cm, DC = 32 cm and distance between AB and DC = 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure.



Solution:

Given, AB = 18 cm, DC = 32 cm, height, (h) = 14cm

and arc of radii = 7 cm

Since, $AB \parallel DC$

$\therefore$ $\angle A + \angle D = 180°$

and $\angle B + \angle C = 180°$

$\therefore$ Area of sector with angle A and $D = \frac{\theta \times \pi r^2}{360}$

$$= \frac{180°}{360} \times \frac{22}{7} \times (7)^2$$

$$= 11 \times 7 = 77 \text{ cm}^2$$

Similarly, area of sector with angle B and C = 77 cm

Now, area of trapezium $= \frac{1}{2}(AB + DC) \times h$

$$= \frac{1}{2}(18 + 32) \times 14 = \frac{50}{2} \times 14 = 350 \text{ cm}^2$$

$\therefore$ Area of shaded region = Area of trapezium − (Area of sector points A and D + Area of sector points B and C)

$$= 350 - (77 + 77) = 196 \text{ cm}^2$$

Hence, the required area of shaded region is 1996 cm^2

Question 7:

Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles.

Solution:

Given that, three circles are in such a way that each of them touches the other two.

Now, we join centre of all three circles to each other by a line segment. Since, radius of each circle is 3.5 cm.

So; AB = 2 x Radius of circle

= 2 x 3.5 = 7 cm.

⇒ AC = BC = AB = 7cm

which shows that, ΔABC is an equilateral triangle with side 7 cm.

We know that, each angle between two adjacent sides of an equilateral triangle is 60°

∴ Area of sector with angle ∠A = 60°.

$$= \frac{\angle A}{360°} \times \pi r^2 = \frac{60°}{360°} \times \pi \times (3.5)^2$$

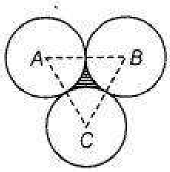
So, area of each sector = 3 × Area of sector with angle A.

$$= 3 \times \frac{60°}{360°} \times \pi \times (3.5)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 11 \times \frac{5}{10} \times \frac{35}{10} = \frac{11}{2} \times \frac{7}{2}$$

$$= \frac{77}{4} = 19.25 \text{ cm}^2$$



and Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (7)^2$ $\left[\because \text{area of an equilateral triangle} = \frac{\sqrt{3}}{4}(\text{side})^2\right]$

$$= 49\frac{\sqrt{3}}{4} \text{cm}^2$$

$\therefore$ Area of shaded region enclosed between these circles = Area of ΔABC − Area of each sector

$$= 49\frac{\sqrt{3}}{4} - 1925 = 12.25 \times \sqrt{3} - 1925$$

$$= 21.2176 - 1925 = 1.9676 \text{ cm}^2$$

Hence, the required area enclosed between these circles is 1.967 cm^2 (approx).

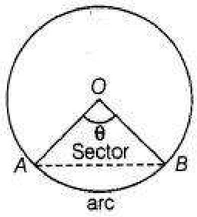
Question 8:

Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Solution:

Let the central angle of the sector be θ.

Given that, radius of the sector of a circle (r) = 5 cm·



and arc length (l) = 3.5 cm.

$\therefore$ Central angle of the sector, $\theta = \dfrac{\text{arc length } (l)}{\text{radius}}$

$$\Rightarrow \quad \theta = \frac{3.5}{5} = 0.7R \qquad \left[\because \theta = \frac{l}{r}\right]$$

$$\Rightarrow \quad \theta = \left(0.7 \times \frac{180}{\pi}\right)^{\circ} \qquad \left[\because 1R = \frac{180^{\circ}}{\pi} D^{\circ}\right]$$

Now, area of sector with angle θ = 0.7

$$= \frac{\pi r^2}{360^{\circ}} \times (0.7) \times \frac{180^{\circ}}{\pi}$$

$$= \frac{(5)^2}{2} \times 0.7 = \frac{25 \times 7}{2 \times 10} = \frac{175}{20} = 8.75 \text{ cm}^2$$

Hence, required area of the sector of a circle is 8.75 cm^2

Question 9:

Four circular cardboard pieces of radii 7 cm are placed on a paperin such a way that each piece touches other two pieces. Find the area of the portion enclosed between these pieces.

Solution:

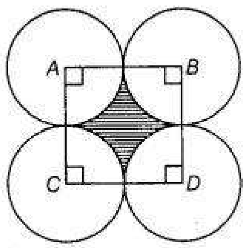
Given that, four circular cardboard pieces arc placed on a paper in such a way that each piece touches other two pieces.

Now, we join centre of all four circles to each other by a line segment. Since, radius of each circle is 7 cm.

So, AB = 2 x Radius of circle

= 2×7 = 14cm

⇒ AB = BC = CD = AD = 14cm



which shows that, quadrilateral $ABCD$ is a square with each of its side is 14 cm.

We know that, each angle between two adjacent sides of a square is 90°.

$\therefore$ Area of sector with $\angle A = 90^{\circ}$

$$= \frac{\angle A}{360^{\circ}} \times \pi r^2 = \frac{90^{\circ}}{360^{\circ}} \times \pi \times (7)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 49 = \frac{154}{4} = \frac{77}{2}$$

$$= 38.5 \text{ cm}^2$$

$\therefore$ Area of each sector = 4 × Area of sector with $\angle A$

$$= 4 \times 38.5$$

$$= 154 \text{ cm}^2$$

and area of square $ABCD$ = (side of square)^2

$$= (14)^2 = 196 \text{ cm}^2 \qquad [\because \text{area of square} = (\text{side})^2]$$

So, area of shaded region enclosed between these pieces = Area of square $ABCD$ − Area of each sector

$$= 196 - 154$$

$$= 42 \text{ cm}^2$$

Hence, required area of the portion enclosed between these pieces is 42 cm^2.

Question 10:

On a square cardboard sheet of area 784 cm^2, four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.

Solution:

$\because$ Area of square = 784

$\therefore$ $(\text{Side})^2 = (28)^2$

$\Rightarrow$ Side = 28 cm

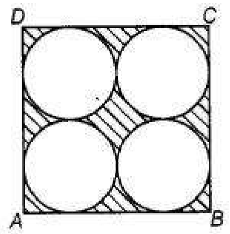
Since, all four are congruent circular plates.

$\therefore$ Diameter of each circular plate = 14 cm

$\therefore$ Radius of each circular plate = 7 cm

Now, area of one circular plate $= \pi r^2 = \frac{22}{7}(7)^2$

$= 154 \text{ cm}^2$



$\therefore$ Area of four circular plates $= 4 \times 154 = 616 \text{ cm}^2$

$\therefore$ Area of the square sheet not covered by the circular plates $= 784 - 616 = 168 \text{ cm}^2$

Question 11:

Floor of a room is of dimensions 5m x 4m and it is covered with circular tiles of diameters 50 cm each as shown infigure. Find area of floor that remains uncovered with tiles, (use π = 3.14)

Solution:

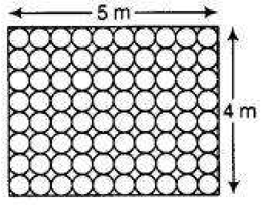
Given, floor of a room is covered with circular tiles.

Length of a floor of a room (l) = 5 m

and breadth of floor of a room (b) = 4 m

$\therefore$ Area of floor of a room $= l \times b$

$= 5 \times 4 = 20 \text{ m}^2$



Diameter of each circular tile = 50 cm

$\Rightarrow$ Radius of each circular tile $= \frac{50}{2} = 25$ cm

$= \frac{25}{100} \text{ m} = \frac{1}{4} \text{ m}$ [$\because$ diameter = 2 × radius]

Now, area of a circular tile $= \pi (\text{radius})^2$

$= 3.14 \times \left(\frac{1}{4}\right)^2 = \frac{3.14}{16} \text{ m}^2$

$\therefore$ Area of 80 circular tiles $= 80 \times \frac{3.14}{16} = 5 \times 3.14 = 15.7 \text{ m}^2$

[$\because$ 80 congruent circular tiles covering the floor of a room]

So, area of floor that remains uncovered with tiles = Area of floor of a room − Area of 80 circular tiles

$= 20 - 15.7 = 4.3 \text{ m}^2$

Hence, the required area of floor that remains uncovered with tiles is 4.3 m^2.

Question 12:

All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 1256 cm^2, (use π = 3.14)

Solution:

Let the radius of the circle be r.

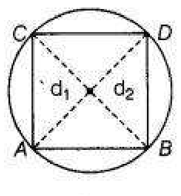
Given that, Area of the circle $= 1256 \text{ cm}^2$

$\pi r^2 = 1256$

$\Rightarrow$ $r^2 = \frac{1256}{\pi} = \frac{1256}{3.14} = 400$

$\Rightarrow$ $r^2 = (20)^2$

$\Rightarrow$ $r = 20$ cm



$\therefore$ So, the radius of circle is 20 cm.

$\Rightarrow$ Diameter of circle = 2 × Radius

$= 2 \times 20$

$= 40$ cm

Since, all the vertices of a rhombus lie on a circle that means each diagonal of a rhombus must pass through the centre of a circle that is why both diagonals are equal and same as the diameter of the given circle.

Let d_1 and d_2 be the diagonals of the rhombus.

$\therefore$ $d_1 = d_2$ = Diameter of circle = 40 cm

So, Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

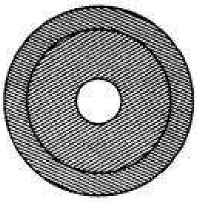
$= \frac{1}{2} \times 40 \times 40$

$= 20 \times 40 = 800 \text{ cm}^2$

Hence, the required area of rhombus is 800 cm^2

Question 13:

An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1:2:3, then find the ratio of the areas of three regions.



Solution:

Let the diameters of concentric circles be k, 2k and 3k.

$\therefore$ Radius of concentric circles are $\frac{k}{2}$, k and $\frac{3k}{2}$.

$\therefore$ Area of inner circle, $A_1 = \pi\left(\frac{k}{2}\right)^2 = \frac{k^2\pi}{4}$

$\therefore$ Area of middle region, $A_2 = \pi(k)^2 - \frac{k^2\pi}{4} = \frac{3k^2\pi}{4}$

[$\because$ area of ring $= \pi(R^2 - r^2)$, where R is radius of outer ring and r is radius of inner ring]

and area of outer region, $A_3 = \pi\left(\frac{3k}{2}\right)^2 - \pi k^2$

$$= \frac{9\pi k^2}{4} - \pi k^2 = \frac{5\pi k^2}{4}$$

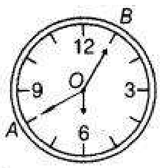
$\therefore$ Required ratio $= A_1 : A_2 : A_3$

$$= \frac{k^2\pi}{4} : \frac{3k^2\pi}{4} : \frac{5\pi k^2}{4} = 1:3:5$$

Question 14:

The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6 : 05 am and 6 : 40 am

Solution:



We know that, in 60 min, minute hand revolving = 360°

In 1 min, minute hand revolving $= \frac{360°}{60°}$

$\therefore$ In (6:05 am to 6:40 am) = 35 min,

minute hand revolving $= \frac{360°}{60°} \times 35 = 6 \times 35$

Given that, length of minute hand (r) = 5 cm.

$\therefore$ Area of sector $AOBA$ with angle $\angle O = \frac{\pi r^2}{360} \times \angle O$

$$= \frac{22}{7}\frac{(5)^2}{360°} \times 6 \times 35$$

$$= \frac{22}{7} \times \frac{5 \times 5}{360°} \times 6 \times 35$$

$$= \frac{22 \times 5 \times 5 \times 5}{60°} = \frac{22 \times 5 \times 5}{12}$$

$$= \frac{11 \times 5 \times 5}{6} = \frac{275}{6} = 45\frac{5}{6}\ \text{cm}^2$$

Hence, the required area swept by the minute land is $45\frac{5}{6}$ cm^2.

Question 15:

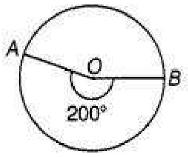
Area of a sector of central angle 200° of a circle is 770 cm^2. Find the length of the corresponding arc of this sector.

Solution:

Let the radius of the sector AOBA be r.

Given that, Central angle of sector $AOBA = \theta = 200°$

and area of the sector $AOBA = 770\ cm^2$



We know that, area of the sector $= \frac{\pi r^2}{360°} \times \theta°$

$\therefore$ Area of the sector, $770 = \frac{\pi r^2}{360°} \times 200$

$\Rightarrow \frac{77 \times 18}{\pi} = r^2$

$\Rightarrow r^2 = \frac{77 \times 18}{22} \times 7 \Rightarrow r^2 = 9 \times 49$

$\Rightarrow r = 3 \times 7$

$\therefore r = 21\ cm$

So, radius of the sector $AOBA = 21$ cm.

Now, the length of the correspoding arc of this sector = Central angle × Radius $\left[\because \theta = \frac{l}{r}\right]$

$$= 200 \times 21 \times \frac{\pi}{180°} \qquad \left[\because 1° = \frac{\pi}{180}R\right]$$

$$= \frac{20}{18} \times 21 \times \frac{22}{7}$$

$$= \frac{220}{3}\ cm = 73\frac{1}{3}\ cm$$

Hence, the required length of the corresponding arc is $73\frac{1}{3}$ cm.

Question 16:

The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the lengths of the corresponding arcs. What do you observe?

Solution:

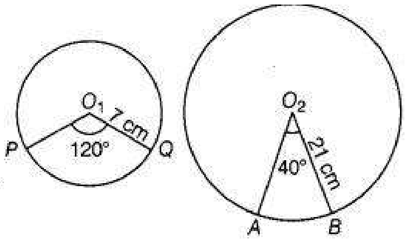
Let the lengths of the corresponding arc be l_1 and l_2

Given that, radius of sector $PO_1QP = 7$ cm

and radius of sector $AO_2BA = 21$ cm

Central angle of the sector $PO_1QP = 120°$

and central angle of the sector $AO_2BA = 40°$



$\therefore$ Area of the sector with central angle O_1

$$= \frac{\pi r^2}{360°} \times \theta = \frac{\pi (7)^2}{360°} \times 120°$$

$$= \frac{22}{7} \times \frac{7 \times 7}{360°} \times 120$$

$$= \frac{22 \times 7}{3} = \frac{154}{3} cm^2$$

and area of the sector with central angle O_2

$$= \frac{\pi r^2}{360°} \times \theta = \frac{\pi (21)^2}{360°} \times 40°$$

$$= \frac{22}{7} \times \frac{21 \times 21}{360°} \times 40°$$

$$= \frac{22 \times 3 \times 21}{9} = 22 \times 7 = 154\ cm^2$$

Now, corresponding arc length of the sector PO_1QP

$$= \text{Central angle} \times \text{Radius of the sector}$$

$$= 120° \times 7 \times \frac{\pi}{180°} \qquad \left[\because \theta = \frac{l}{r} \text{ and } 1° = \frac{\pi}{180°}R\right]$$

$$= \frac{2}{3} \times 7 \times \frac{22}{7}$$

$$= \frac{44}{3} cm$$

and corresponding arc length of the sector AO_2BA

$$= \text{Central angle} \times \text{Radius of the sector}$$

$$= 40° \times 21 \times \frac{\pi}{180°} \qquad \left[\therefore \theta = \frac{l}{r} \text{ and } 1° = \frac{\pi}{180°}R\right]$$

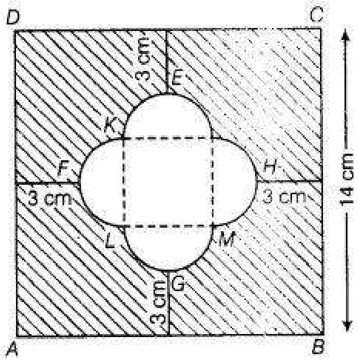
$$= \frac{2}{9} \times 21 \times \frac{22}{7}$$

$$= \frac{2}{3} \times 22 = \frac{44}{3}\ cm$$

Hence, we observe that arc lengths of two sectors of two different circles may be equal but their area need not be equal.

Question 17:

Find the area of the shaded region given in figure



Solution:

Join JK, KL, LM and MJ,

Their are four equally semi-circles and LMJK formed a square.

Their are four equally semi-circles and *LMJK* formed a square.

$\therefore \quad FH = 14 - (3 + 3) = 8\,\text{cm}$

So, the side of square should be 4 cm and radius of semi-circle of both ends are 2 cm each.

$\therefore \quad \text{Area of square } JKLM = (4)^2 = 16\,\text{cm}^2$

$$\text{Area of semi-circle } HJM = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times (2)^2}{2} = 2\pi\,\text{cm}^2$$

$\therefore \quad \text{Area of four semi-circle} = 4 \times 6.28 = 25.12\,\text{cm}^2$

Now, $\quad \text{area of square } ABCD = (14)^2 = 196\,\text{cm}^2$

$\therefore \quad \text{Area of shaded region} = \text{Area of square } ABCD$

$- [\text{Area of four semi-circle} + \text{Area of square } JKLM]$

$= 196 - [8\pi + 16] = 196 - 16 - 8\pi$

$= (180 - 8\pi)\,\text{cm}^2$

Hence, the required of the shaded region is $(180 - 8\pi)\text{cm}^2$.

Question 18:

Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

Solution:

Let the number of revolutions made by a circular wheel be n and the radius of circular wheel be r.

Given that, $\quad \text{area of circular wheel} = 1.54\,\text{m}^2$

$\Rightarrow \quad \pi r^2 = 1.54 \qquad [\because \text{area of circular} = \pi r^2]$

$\Rightarrow \quad r^2 = \frac{1.54}{22} \times 7 \Rightarrow r^2 = 0.49$

$\therefore \quad r = 0.7\,\text{m}$

So, the radius of the wheel is 0.7 m.

Distance travelled by a circular wheel in one revolution = Circumference of circular wheel

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 0.7 = \frac{22}{5} = 4.4\,\text{m} \qquad [\because \text{circumference of a circle} = 2\pi r]$

Since, distance travelled by a circular wheel = 176 m

$\therefore \quad \text{Number of revolutions} = \frac{\text{Total distance}}{\text{Distance in one revolution}} = \frac{176}{4.4} = 40$

Hence, the required number of revolutions made by a circular wheel is 40.

Question 19:

Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre.

Solution:

Let the radius of the circle be r

$\therefore$ $OA = OB = r$ cm

Given that, length of chord of a circle, $AB = 5$ cm

and central angle of the sector $AOBA$ $(\theta) = 90°$

Now, in ΔAOB $(AB)^2 = (OA)^2 + (OB)^2$ [by Pythagoras theorem]

$$(5)^2 = r^2 + r^2$$

$\Rightarrow$ $2r^2 = 25$

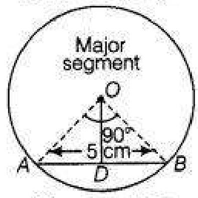
$\therefore$ $r = \frac{5}{\sqrt{2}}$ cm

Now, in ΔAOB we drawn a perpendicular line OD, which meets at D on AB and divides chord AB into two equal parts.

So, $AD = DB = \frac{AB}{2} = \frac{5}{2}$ cm

[since, the perpendicular drawn from the centre to the chord of a circle divides the chord into two equal parts]

By Pythagoras theorem, in ΔADO,



Minor segment

$$(OA)^2 = OD^2 + AD^2$$

$\Rightarrow$ $OD^2 = OA^2 - AD^2$

$$= \left(\frac{5}{\sqrt{2}}\right)^2 - \left(\frac{5}{2}\right)^2 = \frac{25}{2} - \frac{25}{4}$$

$$= \frac{50 - 25}{4} = \frac{25}{4}$$

$\Rightarrow$ $OD = \frac{5}{2}$cm

$\therefore$ Area of an isosceles $\Delta AOB = \frac{1}{2} \times$ Base $(= AB) \times$ Height $(= OD)$

$$= \frac{1}{2} \times 5 \times \frac{5}{2} = \frac{25}{4}\text{cm}^2$$

Now, area of sector $AOBA = \frac{\pi r^2}{360°} \times \theta = \frac{\pi \times \left(\frac{5}{\sqrt{2}}\right)^2}{360°} \times 90°$

$$= \frac{\pi \times 25}{2 \times 4} = \frac{25\pi}{8}\text{cm}^2$$

$\therefore$ Area of minor segment = Area of sector $AOBA$ − Area of an isosceles ΔAOB

$$= \left(\frac{25\pi}{8} - \frac{25}{4}\right)\text{cm}^2 \quad \text{...(i)}$$

Now, area of the circle $= \pi r^2 = \pi\left(\frac{5}{\sqrt{2}}\right) = \frac{25\pi}{2}$ cm^2

$\therefore$ Area of major segment = Area of circle − Area of minor segment

$$= \frac{25\pi}{2} - \left(\frac{25\pi}{8} - \frac{25}{4}\right)$$

$$= \frac{25\pi}{8}(4 - 1) + \frac{25}{4}$$

$$= \left(\frac{75\pi}{8} + \frac{25}{4}\right)\text{cm}^2 \quad \text{...(i)}$$

$\therefore$ Difference of the areas of two segments of a circle = | Area of major segment − Area of minor segment |

$$= \left|\left(\frac{75\pi}{8} + \frac{25}{4}\right) - \left(\frac{25\pi}{4} - \frac{25}{4}\right)\right|$$

$$= \left|\left(\frac{75\pi}{8} - \frac{25\pi}{8}\right) - \left(\frac{25\pi}{8} + \frac{25}{4}\right)\right|$$

$$= \left|\frac{75\pi - 25\pi}{8} + \frac{50}{4}\right| = \left|\frac{50\pi}{8} + \frac{50}{4}\right|$$

$$= \left(\frac{25\pi}{4} + \frac{25}{2}\right)\text{cm}^2$$

Hence, the required difference of the areas of two segments is $\left(\frac{25\pi}{4} + \frac{25}{2}\right)\text{cm}^2$

Question 20:

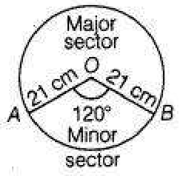
Find the difference of the areas of a sector of angle 120° and its corresponding major sector of a circle of radius 21 cm.

Solution:

Given that, radius of the circle (r) = 21 cm and central angle of the sector AOBA (θ) = 120°

So, area of the circle $= \pi r^2 = \frac{22}{7} \times (21)^2 = \frac{22}{7} \times 21 \times 21$

$= 22 \times 3 \times 21 = 1386 \text{ cm}^2$



Now, area of the minor sector $AOBA$ with central angle 120°

$$= \frac{\pi r^2}{360°} \times \theta = \frac{22}{7} \times \frac{21 \times 21}{360°} \times 120$$

$$= \frac{22 \times 3 \times 21}{3} = 22 \times 21 = 462 \text{ cm}^2$$

$\therefore$ Area of the major sector $ABOA$

= Area of the circle − Area of the sector $AOBA$

$= 1386 - 462 = 924 \text{ cm}^2$

$\therefore$ Difference of the areas of a sector $AOBA$ and its corresponding major sector $ABOA$

$= |\text{Area of major sector } ABOA - \text{ Area of minor sector } AOBA|$

$= |924 - 462| = 462 \text{ cm}^2$

Hence, the required difference of two sectors is 462 cm^2