IN
FU

BASIC CONCEPTS

Principal value branches

one onto) and thus for existe

to make it bijective. This restr

function which are as follows

Functions

5. (*i*) $3\sin^{-1}x = \sin^{-1}(3x - 4$

(*ii*) $3\cos^{-1}x = \cos^{-1}(4x^3 -$

(*iii*) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x -}{1 - 3}\right.$

6. (*i*) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1 + x}\right.$

(*ii*) $2\tan^{-1}x = \cos^{-1}\left(\frac{1 - x}{1 + x}\right.$

7. (*i*) $\sin^{-1}x = \cos^{-1}\left(\sqrt{1 - x^2}\right.$

$= \cot^{-1}\left(\frac{\sqrt{1 - x^2}}{x}\right) =$ se

3. **The value of $\sin^{-1}\Big(\cos$**

(a) $\frac{\pi}{9}$

4. **Let $\theta = \sin^{-1}(\sin(-600°)$**

(a) $\frac{\pi}{3}$

5. **The value of the expres**

(a) $\frac{\pi}{6}$

6. **The value of $\tan^2(\sec^{-1}$**

(a) 5

7. **The domain of the fun**

(a) $[1, 2]$

8. **The value of $\cot\Big[\cos^{-1}$**

17. The domain of $y = \cos$

(*a*) $[-1, 1]$

(*c*) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{}$

18. The domain of the fun

(*a*) ϕ

19. The value of sin (2 sin

(*a*) $\sin 1.6$

20. $\sin^{-1}(\sin 5) > x^2 - 4x$ **if**

(*a*) $x = 2 - \sqrt{9 - 2\pi}$

(*c*) $x > 2 + \sqrt{9 - 2\pi}$

21. If $\sin^{-1}x + \sin^{-1}y + \sin$

$x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{}{x^{f(1)}}$

(*a*) 2

32. If $\sin(\pi \cos x) = \cos(\pi$

(a) $\frac{1}{2}\sin^{-1}\frac{3}{4}$

33. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha,$

(a) 4

34. $3\sin x + 4\cos x = y^2 - 2$

(a) $y = 1$

(c) $xy = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

35. If $(\cot^{-1}x)^2 - 7(\cot^{-1}x)$

(a) $(\cot 5, \cot 2)$

(c) $(-\infty, \cot 5)$

36. The domain of $f(x) =$

(a) $[-1, 1]$

45. **The value of** $\sin^{-1}\left[\cos\right.$

(a) $\frac{3\pi}{5}$

46. **The domain of the fun**

(a) $[0, 1]$

47. **The value of** $\cos^{-1}\left(\cos\right.$

(a) $\frac{\pi}{2}$

48. **The value of** $2\sec^{-1}2 +$

(a) $\frac{\pi}{6}$

49. **The value of** $\cot\left[\cos^{-1}\right.$

58. The value of $\cot\left[\frac{1}{2}\sin\right.$

(a) 1

59. The value of $\sin^{-1}\left[-\left(\frac{1}{2}\right.\right.$

$+ \sec^{-1}(\sqrt{2})$ equals

(a) $\frac{9\pi}{4}$

60. The value of $\cos^{-1}(2x^2$

(a) $2\cos^{-1}x$

Answers

1. (c)	2. (d)
7. (a)	8. (d)
13. (b)	14. (b)
19. (c)	20. (d)

(ii) $\angle CAB = \alpha =$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$

(iii) $\angle BCA = \beta =$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$

(iv) $\angle ABC =$

(a) $\frac{\pi}{4}$

(v) Domain and Range

(a) $(-1, 1), (0, \pi)$

Sol. We have,

(v) Let $\cos^{-1} x = y \quad \Rightarrow$

$-1 \leq \cos y \leq 1 \Rightarrow$

$0 \leq y \leq \pi \qquad \Rightarrow$

$\therefore$ Option (c) is c

2. Read the following an

$\Rightarrow \quad \alpha = \tan^{-}$

$\Rightarrow \quad \angle CAB = \tan^{-}$

$\therefore$ Option (b) is co

(ii) We have, $\tan \alpha = \frac{1}{2}$

$\therefore \quad \tan 2\alpha =$

$\tan 2\alpha =$

$\Rightarrow \quad 2\alpha =$ t

$\Rightarrow \quad \angle DAB =$ t

$\therefore$ Option (c) is co

$\therefore$ Option (a) is co

(v) Domain and Rang

$\therefore$ Option (c) is co

ASSERTION-REASON

In the following questions, a

Choose the correct answer ou

(*a*) *Both A and R are true a*

(*b*) *Both A and R are true b*

(*c*) *A is true but R is false.*

(*d*) *A is false and R is also*

1. **Assertion (A) :** Domain

Reason (R) : Domain

$$\Rightarrow \qquad 1 + \cot^2\theta = \frac{1}{x^2}$$

Option (*d*) is correct.

3. $\sin^{-1}\left(\cos\frac{\pi}{9}\right) = \sin^{-1}\left(\text{si}\right.$

Option (*d*) is correct.

4.
$$\begin{aligned}\theta &= \sin^{-1}(\sin(-600)\\ &= -\sin^{-1}(\sin 600)\\ &= -\sin^{-1}(-\sin 120)\\ &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}\end{aligned}$$

Option (*a*) is correct.

5. We have,

$$2\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right.$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{1+x^2}}$$

$$\therefore \sin\alpha = \sqrt{1-\cos$$

$$= \frac{x}{\sqrt{1+x^2}}$$

$$\therefore \sin(\tan^{-1}x) = \sin(\alpha)$$

Option (d) is correct.

10. We have, $\cos^{-1}\alpha$

We know that, $0 \le \cos$

$\cos^{-1}\alpha$

If and only if, $\cos^{-1}\alpha$

$\Rightarrow$ $\cos\pi =$

$\Rightarrow$ $\alpha = \beta =$

$\therefore$ $\alpha(\beta+\gamma)+\beta($

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$$

Option (b) is correct.

14. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}($

$$= \frac{5\pi}{6}\left[\because \frac{5\pi}{6} \in [0, \pi]\right]$$

Option (b) is correct.

15. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \mathrm{s}$

$$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\frac{\pi}{2} =$$

Option (d) is correct.

16. $\because \tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}$

$$= \tan^{-1}(\sqrt{3}) - (\pi - \cot^{-1}($$

(i) and (ii) $x \in \{(-\infty, -\sqrt{}$

Option (c) is correct.

18. The domain of cos func

$\Rightarrow$ The domain of $\sin^{-1}$

Option (c) is correct.

19. Let $\sin^{-1}(0.8) = \theta \Rightarrow \sin$

$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$ =

$\Rightarrow \cos\theta = \sqrt{1-0.64}$ =

$\Rightarrow \cos\theta = 0.6$

$\therefore \sin(2\sin^{-1}(0.8)) = \sin$

Option (c) is correct.

20. $\because$ 1 rad = 57.75° $\Rightarrow$ 5 ra

$\Rightarrow \frac{3\pi}{} < 5 < \frac{5\pi}{} \Rightarrow \sin$

$$\Rightarrow \; u = \frac{\pi}{2} - 2\tan^{-1}x \Rightarrow$$

$$\Rightarrow \; \tan^{-1}x = \frac{\pi}{4} - \frac{u}{2}$$

$$\Rightarrow \; x = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) \Rightarrow$$

Option (a) is correct.

23. $\cos^{-1}x > \sin^{-1}x \Rightarrow \cos^{-1}$

$$\Rightarrow \; 2\cos^{-1}x > \frac{\pi}{2} \Rightarrow \cos$$

$$\Rightarrow \; x < \cos\left(\frac{\pi}{4}\right) \left[\because \cos x\right.$$

$$\Rightarrow \; x < \frac{1}{\sqrt{2}}$$

Hence $x \in \left[-1, \frac{1}{\sqrt{2}}\right]$ [A

Option (c) is correct.

$$\therefore \frac{x+y}{1-xy} = \frac{2ac}{a^2-c^2}$$

Option (c) is correct.

27. $\alpha = \tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right), \beta =$

$$\Rightarrow \tan\alpha = \tan\frac{5\pi}{4} = \tan$$

$$\beta = \tan^{-1}\left(-\tan\left(\frac{2\pi}{3}\right.\right.$$

$$\Rightarrow \tan\beta = \tan\left(-\frac{2\pi}{3}\right) =$$

$$\Rightarrow \tan\beta = \tan\frac{\pi}{3} \Rightarrow \beta =$$

$\therefore 4\alpha = 3\beta$

So option (a) is correct o

28. If $x = \frac{\sqrt{3}}{2}$, then

$$\text{But } [x] = \begin{cases} -1 & \forall \quad -1 \le \\ 0 & \forall \quad 0 \le \\ 1 & \forall \quad 1 \le \end{cases}$$

$\therefore$ Domain of $\sin^{-1}[x]$ is

Option (b) is correct.

32. We have $\sin(\pi \cos x) =$

$\Rightarrow \sin(\pi \cos x) = \sin(\pi$

$$\Rightarrow \pi \cos x = \frac{\pi}{2} \pm \pi \sin x$$

$$\Rightarrow \cos x \mp \sin x = \frac{1}{2}$$

Squaring both sides, we

$$1 \pm \sin 2x = \frac{1}{4} \Rightarrow \pm \sin$$

$$\Rightarrow \pm \sin 2x = -\frac{3}{4} \Rightarrow \sin$$

$$\Rightarrow x = \frac{1}{2}\sin^{-1}\left(\pm\frac{3}{4}\right) \Rightarrow$$

$\Rightarrow r^2 = 25 \Rightarrow r = 5$. Also

$\Rightarrow r \cos\theta \sin x + r \sin\theta$

$\Rightarrow r \sin(x + \theta) = r \Rightarrow$ si

$\Rightarrow x + \theta = \sin^{-1}(1) = \dfrac{\pi}{2}$

$\Rightarrow x = \dfrac{\pi}{2} - \theta = \dfrac{\pi}{2} - \tan^{-}$

$\therefore xy = \dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{4}{3}\right)$

So all option are true.

Option (d) is correct.

35. $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 1$

$\Rightarrow \cos^2\theta - \sin^2\theta = (\sin$

$\Rightarrow \cos^2\theta = 2\sin^2\theta$ whe

$\therefore \tan^2\theta = \frac{1}{2} \Rightarrow \tan\theta =$

Also $\cos^2\theta - \sin^2\theta = \sin$

$\Rightarrow 1 - 2\sin^2\theta = \sin^2\theta,$ s

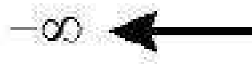
$\Rightarrow 1 = 3\sin^2\theta$ where si

$\Rightarrow \sin^2\theta = \frac{1}{3}, \sin\theta \in (0,$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}}, \sin\theta \in$

But $\sin\theta = -\frac{1}{\sqrt{3}}$ is not

$\therefore \sin\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin$



$\Rightarrow -1 \le \alpha \le 3 \Rightarrow -1 \le [\mathrm{t}$

$\Rightarrow -1 \le \tan^{-1} x < 4$

$\Rightarrow \tan(-1) \le x < \tan 4 <$

$\Rightarrow x \in [-\tan 1, \infty)$

Option (a) is correct.

42. $\cos^{-1}\left[\cos\left(\left(-\frac{17}{15}\right)\pi\right)\right] =$

$= \cos^{-1}\left[\cos\left(2\pi - \frac{13}{15}\right.\right.$

Option (b) is correct.

43. Principal value branch o

Option (c) is correct.

$$= \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$$

Option (b) is correct.

49. $\cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right) = \cot\left(\text{co}\right.$

$$= \cot\left(\cot^{-1}\left(\frac{7/25}{24/25}\right)\right) = \text{c}$$

$$= \frac{7}{24}$$

Option (d) is correct.

50. $\sqrt{1 + \cos 2x} = \sqrt{2}\cos^{-1}($

$$= \frac{\pi}{3} + \pi - \cot^{-1}(1) + \pi -$$

$$= \frac{\pi}{3} + 2\pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{7}{}$$

$$= \frac{28\pi - 5\pi}{12} = \frac{23\pi}{12}$$

Option (d) is correct.

54. $2\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(-\right.$

$$= 2\left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right) - 2\sin$$

$$= 2\left(\pi - \frac{\pi}{3}\right) - 2\left(\frac{\pi}{6}\right) - (\pi -$$

$$= 2\left(\frac{2\pi}{3}\right) - \frac{\pi}{3} - \pi$$

$$= -\frac{\pi}{6} + \pi - \frac{\pi}{3} + \pi - \frac{\pi}{6} + \frac{\pi}{4}$$

$$= -\frac{2\pi}{3} + \frac{9\pi}{4} = \frac{19\pi}{12}$$

Option (b) is correct.

60. $\cos^{-1}(2x^2 - 1)$

Put $x = \cos\alpha \Rightarrow \alpha = \cos$

$\therefore \cos^{-1}(2x^2-1) = \cos^{-1}$

$$\left[\begin{array}{l} \because 0 \leq x \leq 1 \\ \Rightarrow \cos\left(-\frac{\pi}{2}\right) \leq \cos\alpha \\ \Rightarrow -\frac{\pi}{2} \leq \alpha \leq 0 \\ \Rightarrow -\pi \leq 2\alpha \leq 0 \end{array}\right.$$

$= 2\alpha = 2\cos^{-1}x$

Option (a) is correct.