

Unit 2 (Polynomials)

Exercise 2.1 Multiple Choice Questions (MCQs)

Question 1:

If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3 , then the value of k is

(a) $\frac{4}{3}$

(b) $\frac{-4}{3}$

(c) $\frac{2}{3}$

(d) $\frac{-2}{3}$

Solution:

(a) Given that, one of the zeroes of the quadratic polynomial say $p(x) = (k - 1)x^2 + kx + 1$

is -3 , then $p(-3) = 0$

$$\Rightarrow (k - 1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow 9(k - 1) - 3k + 1 = 0$$

$$\Rightarrow 9k - 9 - 3k + 1 = 0$$

$$\Rightarrow 6k - 8 = 0$$

$$\therefore k = 4/3$$

Question 2:

A quadratic polynomial, whose zeroes are -3 and 4 , is

(a) $x^2 - x + 12$

(b) $x^2 + x + 12$

(c) $\frac{x^2}{2} - \frac{x}{2} - 6$

(d) $2x^2 + 2x - 24$

Solution:

(c) Let $ax^2 + bx + c$ be a required polynomial whose zeroes are -3 and 4 .

Then, sum of zeroes $= -3 + 4 = 1$

$$\left[\because \text{sum of zeroes} = \frac{-b}{a} \right]$$

$$\Rightarrow \frac{-b}{a} = \frac{1}{1} \Rightarrow \frac{-b}{a} = -\frac{(-1)}{1} \quad \dots(i)$$

and product of zeroes $= -3 \times 4 = -12$

$$\left[\because \text{product of zeroes} = \frac{c}{a} \right]$$

$$\Rightarrow \frac{c}{a} = \frac{-12}{1} \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$\begin{aligned} a &= 1, b = -1 \text{ and } c = -12 \\ &= ax^2 + bx + c \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ Required polynomial} &= 1 \cdot x^2 - 1 \cdot x - 12 \\
 &= x^2 - x - 12 \\
 &= \frac{x^2}{2} - \frac{x}{2} - 6
 \end{aligned}$$

We know that, if we multiply/divide any polynomial by any constant, then the zeroes of polynomial do not change.

Alternate Method

Let the zeroes of a quadratic polynomial are $\alpha = -3$ and $\beta = 4$.

Then, sum of zeroes $= \alpha + \beta = -3 + 4 = 1$ and product of zeroes $= \alpha\beta = (-3)(4) = -12$

Question 3:

If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

- (a) $a = -7, b = -1$ (b) $a = 5, b = -1$
 (c) $a = 2, b = -6$ (d) $a = 0, b = -6$

Solution:

(d) Let $p(x) = x^2 + (a+1)x + b$

Given that, 2 and -3 are the zeroes of the quadratic polynomial $p(x)$.

$$\begin{aligned}
 \therefore p(2) &= 0 \text{ and } p(-3) = 0 \\
 \Rightarrow 2^2 + (a+1)(2) + b &= 0 \\
 \Rightarrow 4 + 2a + 2 + b &= 0 \\
 \Rightarrow 2a + b &= -6 \quad \dots(i) \\
 \text{and } (-3)^2 + (a+1)(-3) + b &= 0 \\
 \Rightarrow 9 - 3a - 3 + b &= 0 \\
 \Rightarrow 3a - b &= 6 \quad \dots(ii)
 \end{aligned}$$

On adding Eqs. (i) and (ii), we get

$$5a = 0 \Rightarrow a = 0$$

Put the value of a in Eq. (i), we get

$$2 \times 0 + b = -6 \Rightarrow b = -6$$

required values are $a = 0$ and $b = -6$.

Question 4:

The number of polynomials having zeroes as -2 and 5 is

- (a) 1 (b) 2 (c) 3 (d) more than 3

Solution:

(d) Let $p(x) = ax^2 + bx + c$ be the required polynomial whose zeroes are -2 and 5.

$$\begin{aligned}
 \therefore \text{ Sum of zeroes} &= \frac{-b}{a} \\
 \Rightarrow \frac{-b}{a} &= -2 + 5 = \frac{3}{1} = \frac{-(-3)}{1} \quad \dots(i) \\
 \text{and product of zeroes} &= \frac{c}{a} \\
 \Rightarrow \frac{c}{a} &= -2 \times 5 = \frac{-10}{1} \quad (ii)
 \end{aligned}$$

From Eqs. (i) and (ii),

$$a = 1, b = -3 \text{ and } c = -10$$

$$\therefore p(x) = ax^2 + bx + c = 1 \cdot x^2 - 3x - 10$$

$$= x^2 - 3x - 10$$

But we know that, if we multiply/divide any polynomial by any arbitrary constant. Then, the zeroes of polynomial never change.

$$\therefore p(x) = kx^2 - 3kx - 10k \quad [\text{where, } k \text{ is a real number}]$$

$$\Rightarrow p(x) = \frac{x^2}{k} - \frac{3}{k}x - \frac{10}{k}, \quad [\text{where, } k \text{ is a non-zero real number}]$$

Hence, the required number of polynomials are infinite i.e., more than 3.

Question 5:

If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of then other two zeroes is

- (a) $-\frac{c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

Solution:

(b) Let $p(x) = ax^3 + bx^2 + cx + d$

Given that, one of the zeroes of the cubic polynomial $p(x)$ is zero.

Let α, β and γ are the zeroes of cubic polynomial $p(x)$, where $a = 0$.

We know that,

$$\text{Sum of product of two zeroes at a time} = \frac{c}{a}$$

$$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a} \quad [\because \alpha = 0, \text{ given}]$$

$$\Rightarrow 0 + \beta\gamma + 0 = \frac{c}{a}$$

$$\Rightarrow \beta\gamma = \frac{c}{a}$$

$$\text{Hence, product of other two zeroes} = \frac{c}{a}$$

Question 6:

If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

- (a) $b - a + 1$ (b) $b - a - 1$ (c) $a - b + 1$ (d) $a - b - 1$

Solution:

(a) Let $p(x) = x^3 + ax^2 + bx + c$

Let α, β and γ be the zeroes of the given cubic polynomial $p(x)$.

$$\therefore \alpha = -1 \quad [\text{given}]$$

and $p(-1) = 0$

$$\Rightarrow (-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$\Rightarrow -1 + a - b + c = 0$$

$$\Rightarrow c = 1 - a + b \quad \dots(i)$$

We know that,

$$\text{Product of all zeroes} = (-1)^3 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{c}{1}$$

$$\alpha\beta\gamma = -c$$

$$\Rightarrow (-1)\beta\gamma = -c \quad [\because \alpha = -1]$$

$$\Rightarrow \beta\gamma = c$$

$$\Rightarrow \beta\gamma = 1 - a + b \quad [\text{from Eq. (i)}]$$

Hence, product of the other two roots is $1 - a + b$.

Alternate Method

Since, -1 is one of the zeroes of the cubic polynomial $f(x) = x^3 + ax^2 + bx + c$ i.e., $(x + 1)$ is a factor of $f(x)$.

Now, using division algorithm,

$$\begin{array}{r}
 x^2 + (a-1)x + (b-a+1) \\
 x+1 \overline{) x^3 + ax^2 + bx + c} \\
 \underline{x^3 + x^2} \\
 (a-1)x^2 + bx \\
 \underline{(a-1)x^2 + (a-1)x} \\
 (b-a+1)x + c \\
 \underline{(b-a+1)x + (b-a+1)} \\
 (c-b+a-1)
 \end{array}$$

$$\Rightarrow x^3 + ax^2 + bx + c = (x+1) \times \{x^2 + (a-1)x + (b-a+1)\} + (c-b+a-1)$$

$$\Rightarrow x^3 + ax^2 + bx + (b-a+1) = (x+1) \{x^2 + (a-1)x + (b-a+1)\}$$

Let α and β be the other two zeroes of the given polynomial, then

$$\text{Product of zeroes} = (-1) \alpha \cdot \beta = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow -\alpha \cdot \beta = \frac{-(b-a+1)}{1}$$

$$\Rightarrow \alpha\beta = -a + b + 1$$

Hence, the required product of other two roots is $(-a + b + 1)$.

Question 7:

The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

- (a) both positive (b) both negative
(c) one positive and one negative (d) both equal

Solution:

(b) Let given quadratic polynomial be $p(x) = x^2 + 99x + 127$.

On comparing $p(x)$ with $ax^2 + bx + c$, we get

$a = 1$, $b = 99$ and $c = 127$

We know that,

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{[by quadratic formula]} \\
 &= \frac{-99 \pm \sqrt{(99)^2 - 4 \times 1 \times 127}}{2 \times 1} \\
 &= \frac{-99 \pm \sqrt{9801 - 508}}{2} \\
 &= \frac{-99 \pm \sqrt{9293}}{2} = \frac{-99 \pm 96.4}{2} \\
 &= \frac{-99 + 96.4}{2}, \frac{-99 - 96.4}{2} \\
 &= \frac{-2.6}{2}, \frac{-195.4}{2} \\
 &= -1.3, -97.7
 \end{aligned}$$

Hence, both zeroes of the given quadratic polynomial $p(x)$ are negative.

Alternate Method

In quadratic polynomial, if $\left. \begin{array}{l} a > 0 \\ a < 0 \end{array} \right\} \text{ or } \left. \begin{array}{l} b > 0, c > 0 \\ b < 0, c < 0 \end{array} \right\}$, then both zeroes are negative.

In given polynomial, we see that

$$a = 1 > 0, b = 99 > 0 \text{ and } c = 127 > 0$$

the above condition.

So, both zeroes of the given quadratic polynomial are negative.

Question 8:

The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$,

- (a) cannot both be positive (b) cannot both be negative
(c) are always unequal (d) are always equal

Solution:

(a) Let $p(x) = x^2 + kx + k, k \neq 0$

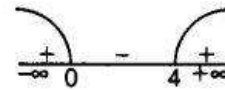
On comparing $p(x)$ with $ax^2 + bx + c$, we get

$$a = 1, b = k \text{ and } c = k$$

Now,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-k \pm \sqrt{k^2 - 4k}}{2 \times 1} \\ &= \frac{-k \pm \sqrt{k(k-4)}}{2}, k \neq 0 \end{aligned}$$

[by quadratic formula]



Here, we see that

$$k(k-4) > 0$$

$$\Rightarrow k \in (-\infty, 0) \cup (4, \infty)$$

Now, we know that

In quadratic polynomial $ax^2 + bx + c$

If $a > 0, b > 0, c > 0$ or $a < 0, b < 0, c < 0$,

then the polynomial has always all negative zeroes.

and if $a > 0, c < 0$ or $a < 0, c > 0$, then the polynomial has always zeroes of opposite sign

Case I If $k \in (-\infty, 0)$ i.e., $k < 0$

$$\Rightarrow a = 1 > 0, b, c = k < 0$$

So, both zeroes are of opposite sign.

Case II If $k \in (4, \infty)$ i.e., $k > 4$

$$\Rightarrow a = 1 > 0, b, c > 4$$

So, both zeroes are negative.

Hence, in any case zeroes of the given quadratic polynomial cannot both be positive.

Question 9:

If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal, then

- (a) c and a have opposite signs (b) c and b have opposite signs
(c) c and a have same signs (d) c and b have the same signs

Solution:

(c) The zeroes of the given quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal. If coefficient of x^2 and constant term have the same sign i.e., c and a have the same sign. While b i.e., coefficient of x can be positive/negative but not zero.

e.g., (i) $x^2 + 4x + 4 = 0$

(ii) $x^2 - 4x + 4 = 0$

$$\Rightarrow (x+2)^2 = 0$$

$$\Rightarrow (x-2)^2 = 0$$

$$\Rightarrow x = -2, -2$$

$$\Rightarrow x = 2, 2$$

Alternate Method

Given that, the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal i.e., discriminant $(D) = 0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow b^2 = 4ac$$

$$\Rightarrow ac = \frac{b^2}{4}$$

$$\Rightarrow ac > 0$$

which is only possible when a and c have the same signs.

Question 10:

If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

- (a) has no linear term and the constant term is negative
- (b) has no linear term and the constant term is positive
- (c) can have a linear term but the constant term is negative
- (d) can have a linear term but the constant term is positive

Solution:

(a) Let $p(x) = x^2 + ax + b$.

Put $a = 0$, then,

$$p(x) = x^2 + b = 0$$

$\Rightarrow$

$$x^2 = -b$$

$\Rightarrow$

$$x = \pm \sqrt{-b}$$

$[\because b < 0]$

Hence, if one of the zeroes of quadratic polynomial $p(x)$ is the negative of the other, then it has no linear term i.e., $a = 0$ and the constant term is negative i.e., $b < 0$.

Alternate Method

Let $f(x) = x^2 + ax + b$

and by given condition the zeroes are α and $-\alpha$.

Sum of the zeroes = $\alpha - \alpha = a$

$\Rightarrow a = 0$

$f(x) = x^2 + b$, which cannot be linear,

and product of zeroes = $\alpha \cdot (-\alpha) = b$

$\Rightarrow$

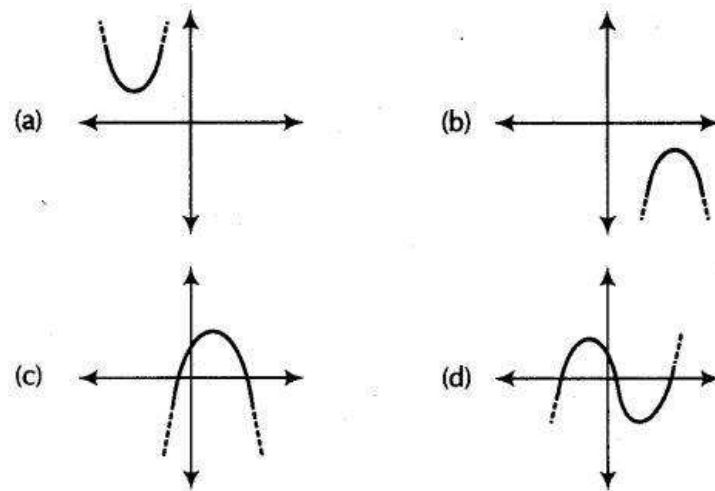
$$-\alpha^2 = b$$

which is possible when, $b < 0$.

Hence, it has no linear term and the constant term is negative.

Question 11:

Which of the following is not the graph of a quadratic polynomial?

**Solution:**

(d) For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like u or open downwards like n depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (d) cannot be possible.

Also, the curve of a quadratic polynomial crosses the X-axis on at most two points but in option (d) the curve crosses the X-axis on the three points, so it does not represent the quadratic polynomial.

Exercise 2.2 Very Short Answer Type Questions

Question 1:

Answer the following and justify.

- (i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?
- (ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s$, $p \neq 0$?
- (iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degree of $p(x)$ and $g(x)$?
- (vi) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
- (v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Solution:

(i) No. because whenever we divide a polynomial $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5, then we get quotient always as in linear form i.e., polynomial in x of degree 1. Let

divisor = a polynomial in x of degree 5

$$= ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

$$\text{quotient} = x^2 - 1$$

$$\text{and dividend} = x^6 + 2x^3 + x - 1$$

By division algorithm for polynomials,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$= (ax^5 + bx^4 + cx^3 + dx^2 + ex + f)(x^2 - 1) + \text{Remainder}$$

$$= (\text{a polynomial of degree 7}) + \text{Remainder}$$

[in division algorithm, degree of divisor > degree of remainder]

$$= (\text{a polynomial of degree 7})$$

But dividend = a polynomial of degree 6

So, division algorithm is not satisfied.

Hence, $x^2 - 1$ is not a required quotient.

(ii) Given that, Divisor $px^3 + qx^2 + rx + s$, $p \neq 0$

$$\text{and dividend} = ax^2 + bx + c$$

We see that,

$$\text{Degree of divisor} > \text{Degree of dividend}$$

So, by division algorithm,

$$\text{quotient} = 0 \text{ and remainder} = ax^2 + bx + c$$

If degree of dividend < degree of divisor, then quotient will be zero and remainder as same as dividend.

(iii) If division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, then relation between the degrees of $p(x)$ and $g(x)$ is degree of $p(x) < \text{degree of } g(x)$.

(iv) If division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, then $g(x)$ is a factor of $p(x)$ and has degree less than or equal to the degree of $p(x)$. e., degree of $g(x) \leq \text{degree of } p(x)$.

(v) No, let $p(x) = x^2 + kx + k$

If $p(x)$ has equal zeroes, then its discriminant should be zero.

$$D = B^2 - 4AC = 0 \quad \dots(i)$$

On comparing $p(x)$ with $Ax^2 + Bx + C$, we get

$$A = 1, B = k \text{ and } C = k$$

$$\therefore (k)^2 - 4(1)(k) = 0 \quad [\text{from Eq. (i)}]$$

$$\Rightarrow k(k - 4) = 0$$

$$\Rightarrow k = 0, 4$$

So, the quadratic polynomial $p(x)$ have equal zeroes only at $k = 0, 4$.

Question 2:

Are the following statements True or False? Justify your answer.

(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a , b and c all

have the same sign.

(ii) If the graph of a polynomial intersects the X-axis at only one point, it cannot be a quadratic polynomial.

(iii) If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then atleast one of a, b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $\frac{1}{2}$

Solution:

(i) **False**, if the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha \cdot \beta = \frac{c}{a}$$

where α and β are the zeroes of quadratic polynomial.

$$\therefore \quad c < 0, a < 0 \quad \text{and} \quad b > 0$$

$$\text{or} \quad c > 0, a > 0 \quad \text{and} \quad b < 0$$

(ii) True, if the graph of a polynomial intersects the X-axis at only one point, then it cannot be a quadratic polynomial because a quadratic polynomial may touch the X-axis at exactly one point or intersects X-axis at exactly two points or do not touch the X-axis.

(iii) True, if the graph of a polynomial intersects the X-axis at exactly two points, then it may or may not be a quadratic polynomial. As, a polynomial of degree more than 2 is possible which intersects the X-axis at exactly two points when it has two real roots and other imaginary roots.

(iv) True, let a, p and y be the zeroes of the cubic polynomial and given that two of the zeroes have value 0.

$$\text{Let} \quad \alpha = \beta = 0$$

$$\text{and} \quad f(x) = (x - \alpha)(x - \beta)(x - \gamma)$$

$$= (x - a)(x - 0)(x - 0)$$

$$= x^3 - ax^2$$

which does not have linear and constant terms.

(v) True, if $f(x) = ax^3 + bx^2 + cx + d$. Then, for all negative roots, a, b, c and d must have same sign.

(vi) False, let α, β and γ be the three zeroes of cubic polynomial $x^3 + ax^2 - bx + c$.

Then, product of zeroes = $(-1)^3 \frac{\text{Constant term}}{\text{Coefficient of } x^3}$

$$\Rightarrow \alpha\beta\gamma = -\frac{(+c)}{1}$$

$$\Rightarrow \alpha\beta\gamma = -c \quad \dots(i)$$

Given that, all three zeroes are positive. So, the product of all three zeroes is also positive

$$\text{i.e., } \alpha\beta\gamma > 0$$

$$\Rightarrow -c > 0 \quad [\text{from Eq. (i)}]$$

$$\Rightarrow c < 0$$

Now, sum of the zeroes = $\alpha + \beta + \gamma = (-1) \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$\Rightarrow \alpha + \beta + \gamma = -\frac{a}{1} = -a$$

But α, β and γ are all positive.

Thus, its sum is also positive.

$$\text{So, } \alpha + \beta + \gamma > 0$$

$$\Rightarrow -a > 0$$

$$\Rightarrow a < 0$$

and sum of the product of two zeroes at a time = $(-1)^2 \cdot \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{-b}{1}$

$$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = -b$$

$$\therefore \alpha\beta + \beta\gamma + \gamma\alpha > 0 \Rightarrow -b > 0$$

$$\Rightarrow b < 0$$

So, the cubic polynomial $x^3 + ax^2 - bx + c$ has all three zeroes which are positive only when all constants a, b and c are negative.

(vii) False, let $f(x) = kx^2 + x + k$

For equal roots. Its discriminant should be zero i.e., $D = b^2 - 4ac = 0$

$$\Rightarrow 1 - 4k.k = 0$$

$$\Rightarrow k = \pm \frac{1}{2}$$

So, for two values of k , given quadratic polynomial has equal zeroes

Exercise 2.3 Short Answer Type Questions

Question 1:

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials

(i) $4x^2 - 3x - 1$.

Solution:

$$\begin{aligned} \text{Let } f(x) &= 4x^2 - 3x - 1 \\ &= 4x^2 - 4x + x - 1 \quad [\text{by splitting the middle term}] \\ &= 4x(x - 1) + 1(x - 1) \\ &= (x - 1)(4x + 1) \end{aligned}$$

So, the value of $4x^2 - 3x - 1$ is zero when $x - 1 = 0$ or $4x + 1 = 0$ i.e., when $x = 1$ or $x = -\frac{1}{4}$.

So, the zeroes of $4x^2 - 3x - 1$ are 1 and $-\frac{1}{4}$.

$$\therefore \text{Sum of zeroes} = 1 - \frac{1}{4} = \frac{3}{4} = \frac{-(-3)}{4}$$

$$= (-1) \left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \right)$$

$$\text{and product of zeroes} = (1) \left(-\frac{1}{4} \right) = -\frac{1}{4}$$

$$= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } x^2} \right)$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(ii) $3x^2 + 4x - 4$.

Solution:

Let
$$\begin{aligned} f(x) &= 3x^2 + 4x - 4 \\ &= 3x^2 + 6x - 2x - 4 && \text{[by splitting the middle term]} \\ &= 3x(x + 2) - 2(x + 2) \\ &= (x + 2)(3x - 2) \end{aligned}$$

So, the value of $3x^2 + 4x - 4$ is zero when $x + 2 = 0$ or $3x - 2 = 0$, i.e., when $x = -2$ or $x = \frac{2}{3}$. So, the zeroes of $3x^2 + 4x - 4$ are -2 and $\frac{2}{3}$.

$\therefore$ Sum of zeroes $= -2 + \frac{2}{3} = -\frac{4}{3}$
 $= (-1) \cdot \left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \right)$

and product of zeroes $= (-2) \left(\frac{2}{3} \right) = -\frac{4}{3}$
 $= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } x^2} \right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(iii) $5t^2 + 12t + 7$.

Solution:

Let
$$\begin{aligned} f(t) &= 5t^2 + 12t + 7 \\ &= 5t^2 + 7t + 5t + 7 && \text{[by splitting the middle term]} \\ &= t(5t + 7) + 1(5t + 7) \\ &= (5t + 7)(t + 1) \end{aligned}$$

So, the value of $5t^2 + 12t + 7$ is zero when $5t + 7 = 0$ or $t + 1 = 0$,

i.e., when $t = -\frac{7}{5}$ or $t = -1$.

So, the zeroes of $5t^2 + 12t + 7$ are $-7/5$ and -1 .

$\therefore$ Sum of zeroes $= -\frac{7}{5} - 1 = -\frac{12}{5}$
 $= (-1) \cdot \left(\frac{\text{Coefficient of } t}{\text{Coefficient of } t^2} \right)$

and product of zeroes $= \left(-\frac{7}{5} \right) (-1) = \frac{7}{5}$
 $= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } t^2} \right)$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(iv) $t^3 - 2t^2 - 15t$.

Solution:

Let $f(t) = t^3 - 2t^2 - 15t$
 $= t(t^2 - 2t - 15)$
 $= t(t^2 - 5t + 3t - 15)$ [by splitting the middle term]
 $= t[t(t - 5) + 3(t - 5)]$
 $= t(t - 5)(t + 3)$

So, the value of $t^3 - 2t^2 - 15t$ is zero when $t = 0$ or $t - 5 = 0$ or $t + 3 = 0$

i.e., when $t = 0$ or $t = 5$ or $t = -3$.

So, the zeroes of $t^3 - 2t^2 - 15t$ are $-3, 0$ and 5 .

$\therefore$ Sum of zeroes $= -3 + 0 + 5 = 2 = \frac{-(-2)}{1}$
 $= (-1) \cdot \left(\frac{\text{Coefficient of } t^2}{\text{Coefficient of } t^3} \right)$

Sum of product of two zeroes at a time

$$= (-3)(0) + (0)(5) + (5)(-3)$$

$$= 0 + 0 - 15 = -15$$

$$= (-1)^2 \cdot \left(\frac{\text{Coefficient of } t}{\text{Coefficient of } t^3} \right)$$

and product of zeroes $= (-3)(0)(5) = 0$

$$= (-1)^3 \left(\frac{\text{Constant term}}{\text{Coefficient of } t^3} \right)$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(v) $2x^2 + \frac{7}{2}x + \frac{3}{4}$

Solution:

Let $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4} = 8x^2 + 14x + 3$
 $= 8x^2 + 12x + 2x + 3$ [by splitting the middle term]
 $= 4x(2x + 3) + 1(2x + 3)$
 $= (2x + 3)(4x + 1)$

So, the value of $8x^2 + 14x + 3$ is zero when $2x + 3 = 0$ or $4x + 1 = 0$,

i.e., when $x = -\frac{3}{2}$ or $x = -\frac{1}{4}$.

So, the zeroes of $8x^2 + 14x + 3$ are $-\frac{3}{2}$ and $-\frac{1}{4}$.

$\therefore$ Sum of zeroes $= -\frac{3}{2} - \frac{1}{4} = -\frac{7}{4} = \frac{-7}{2 \times 2}$
 $= - \frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

And product of zeroes $= \left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right) = \frac{3}{8} = \frac{3}{2 \times 4}$
 $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

(vi) $4x^2 + 5\sqrt{2}x - 3$.

Solution:

$$\begin{aligned}
 \text{Let } f(x) &= 4x^2 + 5\sqrt{2}x - 3 \\
 &= 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 \quad [\text{by splitting the middle term}] \\
 &= 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3) \\
 &= (\sqrt{2}x + 3)(2\sqrt{2}x - 1)
 \end{aligned}$$

So, the value of $4x^2 + 5\sqrt{2}x - 3$ is zero when $\sqrt{2}x + 3 = 0$ or $2\sqrt{2}x - 1 = 0$,

$$\text{i.e.,} \quad \text{when } x = -\frac{3}{\sqrt{2}} \text{ or } x = \frac{1}{2\sqrt{2}}.$$

So, the zeroes of $4x^2 + 5\sqrt{2}x - 3$ are $-\frac{3}{\sqrt{2}}$ and $\frac{1}{2\sqrt{2}}$.

$$\begin{aligned}
 \therefore \text{Sum of zeroes} &= -\frac{3}{\sqrt{2}} + \frac{1}{2\sqrt{2}} \\
 &= -\frac{5}{2\sqrt{2}} = \frac{-5\sqrt{2}}{4} \\
 &= -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{and product of zeroes} &= -\frac{3}{\sqrt{2}} \cdot \frac{1}{2\sqrt{2}} = -\frac{3}{4} \\
 &= \frac{\text{Constant term}}{\text{Coefficient of } x^2}
 \end{aligned}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

$$(vii) 2s^2 - (1+2\sqrt{2})s + \sqrt{2}$$

Solution:

$$\begin{aligned}
 \text{Let } f(s) &= 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2} \\
 &= 2s^2 - s - 2\sqrt{2}s + \sqrt{2} \\
 &= s(2s - 1) - \sqrt{2}(2s - 1) \\
 &= (2s - 1)(s - \sqrt{2})
 \end{aligned}$$

So, the value of $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$ is zero when $2s - 1 = 0$ or $s - \sqrt{2} = 0$,

$$\text{i.e.,} \quad \text{when } s = \frac{1}{2} \text{ or } s = \sqrt{2}.$$

So, the zeroes of $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$ are $\frac{1}{2}$ and $\sqrt{2}$.

$$\therefore \text{Sum of zeroes} = \frac{1}{2} + \sqrt{2} = \frac{1 + 2\sqrt{2}}{2} = \frac{-[-(1 + 2\sqrt{2})]}{2} = \frac{(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

$$\text{and product of zeroes} = \frac{1}{2} \cdot \sqrt{2} = \frac{1}{\sqrt{2}} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

$$(viii) v^2 + 4\sqrt{3}v - 15.$$

Solution:

$$\begin{aligned}
 \text{Let } f(v) &= v^2 + 4\sqrt{3}v - 15 \\
 &= v^2 + (5\sqrt{3} - \sqrt{3})v - 15 \quad [\text{by splitting the middle term}] \\
 &= v^2 + 5\sqrt{3}v - \sqrt{3}v - 15 \\
 &= v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) \\
 &= (v + 5\sqrt{3})(v - \sqrt{3})
 \end{aligned}$$

So, the value of $v^2 + 4\sqrt{3}v - 15$ is zero when $v + 5\sqrt{3} = 0$ or $v - \sqrt{3} = 0$,

i.e., when $v = -5\sqrt{3}$ or $v = \sqrt{3}$.

So, the zeroes of $v^2 + 4\sqrt{3}v - 15$ are $-5\sqrt{3}$ and $\sqrt{3}$.

$$\begin{aligned}
 \therefore \text{Sum of zeroes} &= -5\sqrt{3} + \sqrt{3} = -4\sqrt{3} \\
 &= (-1) \cdot \left(\frac{\text{Coefficient of } v}{\text{Coefficient of } v^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{and product of zeroes} &= (-5\sqrt{3})(\sqrt{3}) \\
 &= -5 \times 3 = -15 \\
 &= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } v^2} \right)
 \end{aligned}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

$$(ix) \ y^2 + \frac{3}{2}\sqrt{5}y - 5.$$

Solution:

$$\begin{aligned}
 \text{Let } f(y) &= y^2 + \frac{3}{2}\sqrt{5}y - 5 = 2y^2 + 3\sqrt{5}y - 10 \\
 &= 2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10 \quad [\text{by splitting the middle term}] \\
 &= 2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5}) \\
 &= (y + 2\sqrt{5})(2y - \sqrt{5})
 \end{aligned}$$

So, the value of $y^2 + \frac{3}{2}\sqrt{5}y - 5$ is zero when $(y + 2\sqrt{5}) = 0$ or $(2y - \sqrt{5}) = 0$,

i.e., when $y = -2\sqrt{5}$ or $y = \frac{\sqrt{5}}{2}$.

So, the zeroes of $2y^2 + 3\sqrt{5}y - 10$ are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{2}$.

$$\therefore \text{Sum of zeroes} = -2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{-3\sqrt{5}}{2} = -\frac{(\text{Coefficient of } y)}{(\text{Coefficient of } y^2)}$$

$$\text{And product of zeroes} = -2\sqrt{5} \times \frac{\sqrt{5}}{2} = -5 = \frac{\text{Constant term}}{\text{Coefficient of } y^2}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

$$(x) \ 7y^2 - \frac{11}{3}y - \frac{2}{3}.$$

Solution:

Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$$= 21y^2 - 11y - 2$$

$$= 21y^2 - 14y + 3y - 2 \quad [\text{by splitting the middle term}]$$

$$= 7y(3y - 2) + 1(3y - 2)$$

$$= (3y - 2)(7y + 1)$$

So, the value of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ is zero when $3y - 2 = 0$ or $7y + 1 = 0$,

i.e., when $y = \frac{2}{3}$ or $y = -\frac{1}{7}$.

So, the zeroes of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ are $\frac{2}{3}$ and $-\frac{1}{7}$.

$$\therefore \text{Sum of zeroes} = \frac{2}{3} - \frac{1}{7} = \frac{14 - 3}{21} = \frac{11}{21} = -\left(\frac{-11}{3 \times 7}\right)$$

$$= (-1) \cdot \left(\frac{\text{Coefficient of } y}{\text{Coefficient of } y^2}\right)$$

$$\text{and product of zeroes} = \left(\frac{2}{3}\right)\left(-\frac{1}{7}\right) = \frac{-2}{21} = \frac{-2}{3 \times 7}$$

$$= (-1)^2 \cdot \left(\frac{\text{Constant term}}{\text{Coefficient of } y^2}\right)$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

Exercise 2.4 Long Answer Type Questions

Question 1:

For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorisation.

(i) $-\frac{8}{3}, \frac{4}{3}$ (ii) $\frac{21}{8}, \frac{5}{16}$ (iii) $-2\sqrt{3}, -9$ (iv) $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

Solution:

(i) Given that, sum of zeroes (S) = $-\frac{8}{3}$

and product of zeroes (P) = $\frac{4}{3}$

$$\therefore \text{Required quadratic expression, } f(x) = x^2 - Sx + P$$

$$= x^2 + \frac{8}{3}x + \frac{4}{3} = 3x^2 + 8x + 4$$

$$\text{Using factorisation method, } = 3x^2 + 6x + 2x + 4$$

$$= 3x(x + 2) + 2(x + 2) = (x + 2)(3x + 2)$$

Hence, the zeroes of $f(x)$ are -2 and $-\frac{2}{3}$.

(ii) Given that, S = $\frac{21}{8}$ and P = $\frac{5}{16}$

$$\therefore \text{Required quadratic expression, } f(x) = x^2 - Sx + P$$

$$= x^2 - \frac{21}{8}x + \frac{5}{16} = 16x^2 - 42x + 5$$

$$\text{Using factorisation method} = 16x^2 - 40x - 2x + 5$$

$$= 8x(2x - 5) - 1(2x - 5) = (2x - 5)(8x - 1)$$

Hence, the zeroes of $f(x)$ are $\frac{5}{2}$ and $\frac{1}{8}$.

(iii) Given that, $S = -2\sqrt{3}$ and $P = -9$

∴ Required quadratic expression,

$$f(x) = x^2 - Sx + P = x^2 + 2\sqrt{3}x - 9$$

$$= x^2 + 3\sqrt{3}x - \sqrt{3}x - 9$$

[using factorisation method]

$$= x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3})$$

$$= (x + 3\sqrt{3})(x - \sqrt{3})$$

Hence, the zeroes of $f(x)$ are $-3\sqrt{3}$ and $\sqrt{3}$.

(iv) Given that, $S = -\frac{3}{2\sqrt{5}}$ and $P = -\frac{1}{2}$

∴ Required quadratic expression,

$$f(x) = x^2 - Sx + P = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$$

$$= 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

Using factorisation method, $= 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$

$$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5})$$

$$= (2x + \sqrt{5})(\sqrt{5}x - 1)$$

Hence, the zeroes of $f(x)$ are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}}$

Question 2:

If the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, a + b$ and $a + 2b$ for some real numbers a and b , find the values of a and b as well as the zeroes of the given polynomial.

Solution:

Let $f(x) = x^3 - 6x^2 + 3x + 10$

Given that, $a, (a + b)$ and $(a + 2b)$ are the zeroes of $f(x)$. Then,

$$\text{Sum of the zeroes} = -\frac{(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$$

$$\Rightarrow a + (a + b) + (a + 2b) = -\frac{(-6)}{1}$$

$$\Rightarrow 3a + 3b = 6$$

$$\Rightarrow a + b = 2$$

...(i)

$$\text{Sum of product of two zeroes at a time} = \left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} \right)$$

$$\Rightarrow a(a + b) + (a + b)(a + 2b) + a(a + 2b) = \frac{3}{1}$$

$$\Rightarrow a(a + b) + (a + b)\{(a + b) + b\} + a\{(a + b) + b\} = 3$$

$$\Rightarrow 2a + 2(2 + b) + a(2 + b) = 3$$

[using Eq. (i)]

$$\Rightarrow 2a + 2(2 + 2 - a) + a(2 + 2 - a) = 3$$

[using Eq. (i)]

$$\Rightarrow 2a + 8 - 2a + 4a - a^2 = 3$$

$$\Rightarrow -a^2 + 8 = 3 - 4a$$

$$\Rightarrow a^2 - 4a - 5 = 0$$

Using factorisation method,

$$a^2 - 5a + a - 5 = 0 \Rightarrow 3$$

$$\Rightarrow a(a - 5) + 1(a - 5) = 0$$

$$\Rightarrow (a - 5)(a + 1) = 0$$

$$\Rightarrow a = -1, 5$$

when $a = -1$, then $b = 3$

When $a = 5$, then $b = -3$

[using Eq.

(i)]

∴ Required zeroes of $f(x)$ are

When $a = -1$ and $b = 3$

then, $a, (a + b), (a + 2b) = -1, (-1 + 3), (-1 + 6)$ or $-1, 2, 5$

When $a = 5$ and $b = -3$ then

$a, (a + b), (a + 2b) = 5, (5 - 3), (5 - 6)$ or $5, 2, -1$.

Hence, the required values of a and b are $a = -1$ and $d = 3$ or $a = 5, b = -3$ and the zeroes

are -1, 2 and 5.

Question 3:

If $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, then find its other two zeroes.

Solution:

Let $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ and given that $\sqrt{2}$ is one of the zeroes of $f(x)$ i.e., $(x - \sqrt{2})$ is one of the factors of given cubic polynomial.

Now, using division algorithm,

$$\begin{array}{r}
 6x^2 + 7\sqrt{2}x + 4 \\
 (x - \sqrt{2}) \overline{) 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}} \\
 \underline{6x^3 - 6\sqrt{2}x^2} \phantom{- 10x - 4\sqrt{2}} \\
 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 \underline{7\sqrt{2}x^2 - 14x} \phantom{- 4\sqrt{2}} \\
 4x - 4\sqrt{2} \\
 \underline{4x - 4\sqrt{2}} \\
 0
 \end{array}$$

$$\therefore 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} = (6x^2 + 7\sqrt{2}x + 4)(x - \sqrt{2}) + 0$$

[$\because$ dividend = divisor $\times$ quotient + remainder]

$$\begin{aligned}
 &= (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4) \\
 &= (x - \sqrt{2})\{\sqrt{2}x(3\sqrt{2}x + 4) + 1(3\sqrt{2}x + 4)\} \\
 &= (x - \sqrt{2})\{(3\sqrt{2}x + 4)(\sqrt{2}x + 1)\} \\
 &= (x - \sqrt{2})(\sqrt{2}x + 1)(3\sqrt{2}x + 4)
 \end{aligned}$$

So, its other zeroes are $-\frac{1}{\sqrt{2}}$ and $-\frac{4}{3\sqrt{2}}$.

Question 4:

Find k , so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also, find all the zeroes of the two polynomials.

Solution:

Given that, $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$, then we apply division algorithm,

$$\begin{array}{r}
 2x^2 - 3x + (-8 - 2k) \\
 x^2 + 2x + k \overline{) 2x^4 + x^3 - 14x^2 + 5x + 6} \\
 \underline{2x^4 + 4x^3 + 2kx^2} \\
 -3x^3 - (2k + 14)x^2 + 5x + 6 \\
 \underline{-3x^3 - 6x^2 - 3kx} \\
 (6 - 2k - 14)x^2 + (3k + 5)x + 6 \\
 \underline{(-8 - 2k)x^2 + 2(-8 - 2k)x + k(-8 - 2k)} \\
 (3k + 5 + 16 + 4k)x + (6 + 8k + 2k^2)
 \end{array}$$

Since, $(x^2 + 2x + k)$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$.

So, when we apply division algorithm remainder should be zero.

$$\therefore (7k + 21)x + (2k^2 + 8k + 6) = 0 \cdot x + 0$$

$$\Rightarrow 7k + 21 = 0 \text{ and } 2k^2 + 8k + 6 = 0$$

$$\Rightarrow k = -3 \text{ or } k^2 + 4k + 3 = 0$$

$$\Rightarrow k^2 + 3k + k + 3 = 0 \quad [\text{by splitting middle term}]$$

$$\Rightarrow k(k + 3) + 1(k + 3) = 0$$

$$\Rightarrow (k + 1)(k + 3) = 0$$

$$\Rightarrow k = -1 \text{ or } -3$$

Here, if we take $k = -3$, then remainder will be zero.

Thus, the required value of k is -3 .

Now, Dividend = Divisor $\times$ Quotient + Remainder

$$\Rightarrow 2x^4 + x^3 - 14x^2 + 5x + 16 = (x^2 + 2x - 3)(2x^2 - 3x - 2)$$

Using factorisation method,

$$= (x^2 + 3x - x - 3)(2x^2 - 4x + x - 2) \quad [\text{by splitting middle term}]$$

$$= \{x(x + 3) - 1(x + 3)\} \{2x(x - 2) + 1(x - 2)\}$$

$$= (x - 1)(x + 3)(x - 2)(2x + 1)$$

Hence, the zeroes of $x^2 + 2x - 3$ are $1, -3$ and the zeroes of $2x^2 - 3x - 2$ are $1, -3, 2, \frac{-1}{2}$.

Question 5:

If $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeroes of the polynomial.

Solution:

Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$ and given that, $(x - \sqrt{5})$ is a one of the factor of $f(x)$.

Now, using division algorithm,

$$\begin{array}{r} x^2 - 2\sqrt{5}x + 3 \\ x - \sqrt{5} \overline{) x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}} \\ \underline{x^3 - \sqrt{5}x^2} \phantom{+ 13x - 3\sqrt{5}} \\ -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ \underline{-2\sqrt{5}x^2 + 10x} \phantom{- 3\sqrt{5}} \\ 3x - 3\sqrt{5} \\ \underline{3x - 3\sqrt{5}} \\ 0 \end{array}$$

$$\therefore x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} = (x^2 - 2\sqrt{5}x + 3)(x - \sqrt{5})$$

$[\because \text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}]$

$$= (x - \sqrt{5})[x^2 - \{(\sqrt{5} + \sqrt{2}) + (\sqrt{5} - \sqrt{2})\}x + 3] \quad [\text{by splitting the middle term}]$$

$$= (x - \sqrt{5})[x^2 - (\sqrt{5} + \sqrt{2})x - (\sqrt{5} - \sqrt{2})x + (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})]$$

$$[\because 3 = (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})]$$

$$= (x - \sqrt{5})\{x\{x - (\sqrt{5} + \sqrt{2})\} - (\sqrt{5} - \sqrt{2})\{x - (\sqrt{5} + \sqrt{2})\}\}$$

$$= (x - \sqrt{5})\{x - (\sqrt{5} + \sqrt{2})\}\{x - (\sqrt{5} - \sqrt{2})\}$$

Hence, all the zeroes of polynomial are $\sqrt{5}, (\sqrt{5} + \sqrt{2})$ and $(\sqrt{5} - \sqrt{2})$.

Question 6:

For which values of a and b , the zeroes of $q(x) = x^3 + 2x^2 + a$ are also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

Solution:

Given that the zeroes of $q(x) = x^3 + 2x^2 + a$ are also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$ i.e., $q(x)$ is a factor of $p(x)$. Then, we use a division algorithm.

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x^3 + 2x^2 + a \overline{) x^5 - x^4 - 4x^3 + 3x^2 + 3x + b} \\
 \underline{x^5 + 2x^4 + ax^2} \\
 -3x^4 - 4x^3 + (3-a)x^2 + 3x + b \\
 \underline{-3x^4 - 6x^3 - 3ax} \\
 2x^3 + (3-a)x^2 + (3+3a)x + b \\
 \underline{2x^3 + 4x^2 + 2a} \\
 -(1+a)x^2 + (3+3a)x + (b-2a)
 \end{array}$$

If $(x^3 + 2x^2 + a)$ is a factor of $(x^5 - x^4 - 4x^3 + 3x^2 + 3x + b)$, then remainder should be zero.

$$\begin{aligned}
 \text{i.e., } & -(1+a)x^2 + (3+3a)x + (b-2a) = 0 \\
 & = 0x^2 + 0x + 0
 \end{aligned}$$

On comparing the coefficient of x , we get

$$a + 1 = 0$$

$$\Rightarrow a = -1$$

$$\text{and } b - 2a = 0$$

$$\Rightarrow b = 2a$$

$$b = 2(-1) = -2$$

$$[a = -1]$$

For $a = -1$ and $b = -2$, the zeroes of $q(x)$ are also the zeroes of the polynomial $p(x)$.

$$\therefore q(x) = x^3 + 2x^2 - 1$$

$$\text{and } p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$$

Now, Dividend = divisor $\times$ quotient + remainder

$$p(x) = (x^3 + 2x^2 - 1)(x^2 - 3x + 2) + 0$$

$$= (x^3 + 2x^2 - 1)\{x^2 - 2x - x + 2\}$$

$$= (x^3 + 2x^2 - 1)(x - 2)(x - 1)$$

Hence, the zeroes of $p(x)$ are 1 and 2 which are not the zeroes of $q(x)$.