

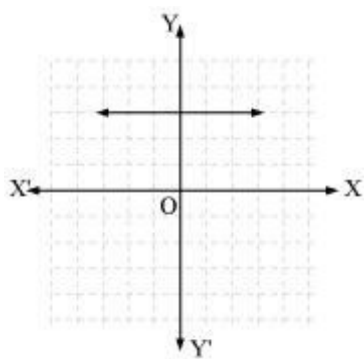
Polynomials

Exercise 2.1

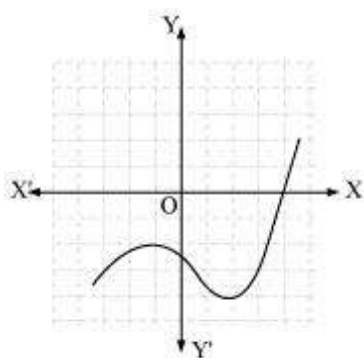
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

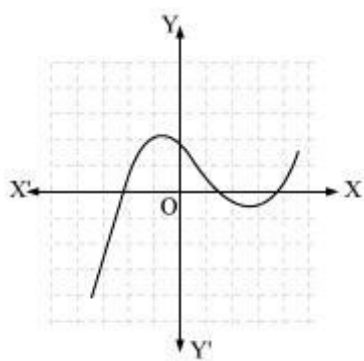
(i)



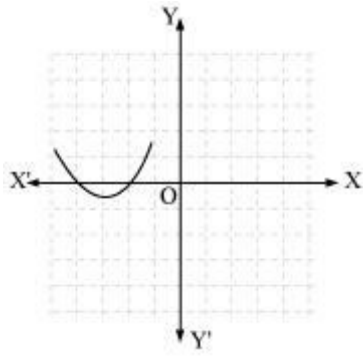
(ii)



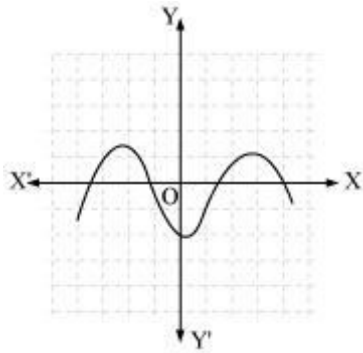
(iii)



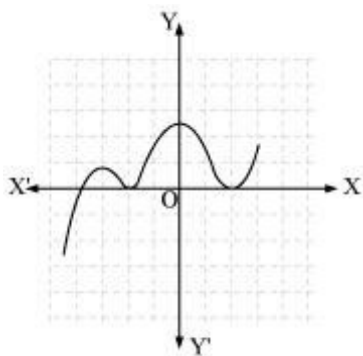
(iv)



(v)



(v)



Answer:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$$(i) x^2 - 2x - 8 \quad (ii) 4s^2 - 4s + 1 \quad (iii) 6x^2 - 3 - 7x$$

$$(iv) 4u^2 + 8u \quad (v) t^2 - 15 \quad (vi) 3x^2 - x - 4$$

Answer:

$$(i) \quad x^2 - 2x - 8 = (x - 4)(x + 2)$$

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

$$\text{Sum of zeroes} = 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$(ii) \quad 4s^2 - 4s + 1 = (2s - 1)^2$$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

$$(iii) \quad 6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x + 1)(2x - 3)$$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$(iv) \quad 4u^2 + 8u = 4u^2 + 8u + 0 \\ = 4u(u + 2)$$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

$$(v) \quad t^2 - 15 \\ = t^2 - 0t - 15 \\ = (t - \sqrt{15})(t + \sqrt{15})$$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$$

$$\text{Product of zeroes} = (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\begin{aligned} \text{(vi)} \quad & 3x^2 - x - 4 \\ &= (3x - 4)(x + 1) \end{aligned}$$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1 .

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

$$\text{(i)} \quad \frac{1}{4}, -1 \quad \text{(ii)} \quad \sqrt{2}, \frac{1}{3} \quad \text{(iii)} \quad 0, \sqrt{5}$$

$$\text{(iv)} \quad 1, 1 \quad \text{(v)} \quad -\frac{1}{4}, \frac{1}{4} \quad \text{(vi)} \quad 4, 1$$

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

$$\text{If } a = 1, \text{ then } b = 0, c = \sqrt{5}$$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

$$(iv) \quad 1, 1$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

$$\text{If } a = 1, \text{ then } b = -1, c = 1$$

Therefore, the quadratic polynomial is $x^2 - x + 1$.

$$(v) \quad -\frac{1}{4}, \frac{1}{4}$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$

$$\text{If } a = 4, \text{ then } b = 1, c = 1$$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

$$(vi) \quad 4, 1$$

Let the polynomial be $ax^2 + bx + c$.

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -4$, $c = 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Answer:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$
 $q(x) = x^2 - 2$

$$\begin{array}{r} x-3 \\ x^2-2 \overline{) x^3-3x^2+5x-3} \\ \underline{x^3 -2x} \\ -3x^2+7x-3 \\ \underline{-3x^2 +6} \\ 7x-9 \end{array}$$

Quotient = $x - 3$

Remainder = $7x - 9$

$$(ii) \quad p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0x^3 - 3x^2 + 4x + 5$$

$$q(x) = x^2 + 1 - x = x^2 - x + 1$$

$$\begin{array}{r}
 x^2 - x + 1 \overline{) x^4 + 0x^3 - 3x^2 + 4x + 5} \\
 \underline{x^4 - x^3 + x^2} \\
 x^3 - 4x^2 + 4x + 5 \\
 \underline{x^3 - x^2 + x} \\
 -3x^2 + 3x + 5 \\
 \underline{-3x^2 + 3x - 3} \\
 8
 \end{array}$$

$$\text{Quotient} = x^2 + x - 3$$

$$\text{Remainder} = 8$$

$$(iii) \quad p(x) = x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$$

$$q(x) = 2 - x^2 = -x^2 + 2$$

$$\begin{array}{r}
 -x^2 + 2 \overline{) x^4 + 0x^2 - 5x + 6} \\
 \underline{x^4 - 2x^2} \\
 2x^2 - 5x + 6 \\
 \underline{2x^2 - 4} \\
 -5x + 10
 \end{array}$$

$$\text{Quotient} = -x^2 - 2$$

$$\text{Remainder} = -5x + 10$$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

- (i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$
(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$
(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$t^2 - 3 = t^2 + 0t - 3$$

$$\begin{array}{r}
 \overline{2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 t^2 + 0t - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{2t^4 + 0t^3 - 6t^2} \\
 - + \\
 3t^3 + 4t^2 - 9t - 12 \\
 3t^3 + 0t^2 - 9t \\
 \underline{ - + } \\
 4t^2 + 0t - 12 \\
 4t^2 + 0t - 12 \\
 \underline{ - + } \\
 0
 \end{array}$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r}
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \\
 2x^2 + 6x + 2 \\
 \underline{2x^2 + 6x + 2} \\
 0
 \end{array}$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 - 3x^3 + x^2} \\
 -x^3 + 3x + 1 \\
 \underline{-x^3 + 3x - 1} \\
 2
 \end{array}$$

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Question 3:

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are

$$\sqrt{\frac{5}{3}} \text{ and } -\sqrt{\frac{5}{3}}.$$

Answer:

$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right) \text{ is a factor of } 3x^4 + 6x^3 - 2x^2 - 10x - 5.$$

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$\begin{array}{r} x^2 + 0x - \frac{5}{3} \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 + 0x^3 - 5x^2} \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{6x^3 + 0x^2 - 10x} \\ 3x^2 + 0x - 5 \\ \underline{3x^2 + 0x - 5} \\ 0 \end{array}$$

$$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$$

$$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$$

We factorize $x^2 + 2x + 1$

$$= (x + 1)^2$$

Therefore, its zero is given by $x + 1 = 0$

$$x = -1$$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1 .

Question 4:

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer:

$$p(x) = x^3 - 3x^2 + x + 2 \quad (\text{Dividend})$$

$$g(x) = ? \quad (\text{Divisor})$$

$$\text{Quotient} = (x - 2)$$

$$\text{Remainder} = (-2x + 4)$$

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

$$\begin{array}{r}
 x^2 - x + 1 \\
 x-2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \\
 -x^2 + 3x - 2 \\
 \underline{-x^2 + 2x} \\
 +x - 2 \\
 \underline{+x - 2} \\
 0
 \end{array}$$

$$\therefore g(x) = (x^2 - x + 1)$$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

(ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Answer:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x) \times q(x) + r(x),$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $\deg p(x) = \deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

$$\text{Here, } p(x) = 6x^2 + 2x + 2$$

$$g(x) = 2$$

$$q(x) = 3x^2 + x + 1 \text{ and } r(x) = 0$$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$$

$$= 6x^2 + 2x + 2$$

Thus, the division algorithm is satisfied.

$$(ii) \deg q(x) = \deg r(x)$$

Let us assume the division of $x^3 + x$ by x^2 ,

$$\text{Here, } p(x) = x^3 + x$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = x$$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + x = (x^2) \times x + x$$

$$x^3 + x = x^3 + x$$

Thus, the division algorithm is satisfied.

$$(iii) \deg r(x) = 0$$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$ by x^2 .

$$\text{Here, } p(x) = x^3 + 1$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = 1$$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + 1 = (x^2) \times x + 1$$

$$x^3 + 1 = x^3 + 1$$

Thus, the division algorithm is satisfied.

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

$$(i) \quad 2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$$

$$(ii) \quad x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$$

Answer:

(i) $p(x) = 2x^3 + x^2 - 5x + 2$.

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 \\ &= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(1) &= 2 \times 1^3 + 1^2 - 5 \times 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(-2) &= 2(-2)^3 + (-2)^2 - 5(-2) + 2 \\ &= -16 + 4 + 10 + 2 = 0 \end{aligned}$$

Therefore, $\frac{1}{2}$, 1, and -2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 2, b = 1, c = -5, d = 2$

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$\begin{aligned}
 p(2) &= 2^3 - 4(2^2) + 5(2) - 2 \\
 &= 8 - 16 + 10 - 2 = 0 \\
 p(1) &= 1^3 - 4(1)^2 + 5(1) - 2 \\
 &= 1 - 4 + 5 - 2 = 0
 \end{aligned}$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

$$\begin{aligned}
 \text{Multiplication of zeroes taking two at a time} &= (2)(1) + (1)(1) + (2)(1) = 2 + 1 + 2 = 5 \\
 &= \frac{(5)}{1} = \frac{c}{a}
 \end{aligned}$$

$$\text{Multiplication of zeroes} = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β , and γ .

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If $a = 1$, then $b = -2$, $c = -7$, $d = 14$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a-b, a, a+b$, find a and b .

Answer:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are $a-b, a, a+b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$$p = 1, q = -3, r = 1, t = 1$$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1-b, 1, 1+b$.

$$\text{Multiplication of zeroes} = 1(1-b)(1+b)$$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

$$\text{Therefore, } (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = x^2 + 4 - 4x - 3$$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r}
 x^2 - 2x - 35 \\
 x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 \underline{x^4 - 4x^3 + x^2} \\
 -2x^3 - 27x^2 + 138x - 35 \\
 \underline{-2x^3 + 8x^2 - 2x} \\
 +35x^2 + 140x - 35 \\
 \underline{-35x^2 + 140x - 35} \\
 0
 \end{array}$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a .

Answer:

By division algorithm,

Dividend = Divisor $\times$ Quotient + Remainder

Dividend – Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$\begin{array}{r}
 x^2 - 4x + (8 - k) \\
 x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 26x + 10 - a} \\
 \underline{x^4 - 2x^3 + kx^2} \\
 -4x^3 + (16 - k)x^2 - 26x \\
 \underline{-4x^3 + 8x^2 - 4kx} \\
 (8 - k)x^2 - (26 - 4k)x + 10 - a \\
 \underline{(8 - k)x^2 - (16 - 2k)x + (8k - k^2)} \\
 (-10 + 2k)x + (10 - a - 8k + k^2)
 \end{array}$$

It can be observed that $(-10 + 2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10 + 2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10 + 2k) = 0$,

$$2k = 10$$

And thus, $k = 5$

For $(10 - a - 8k + k^2) = 0$

$$10 - a - 8 \times 5 + 25 = 0$$

$$10 - a - 40 + 25 = 0$$

$$-5 - a = 0$$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$