Unit 5 (Arithematic Progressions)

Exercise 5.1 Multiple Choice Questions (MCQs)

Question 1:

In an AP, if d = – 4, n = 7 and a_n = 4, then a is equal to

(a) 6 (b) 7 (c) 20 (d) 28

Solution:

(d) In an AP, $a_n = a + (n-1)d$

$\Rightarrow$ $4 = a + (7-1)(-4)$ [by given conditions]

$\Rightarrow$ $4 = a + 6(-4)$

$\Rightarrow$ $4 + 24 = a$

$\therefore$ $a = 28$

Question 2:

In an AP, if a = 3.5, d = 0 and n = 101, then a_n will be

(a) 0 (b) 3.5 (c) 103.5 (d) 104.5

Solution:

(b) For an AP $a_n = a + (n-1)d = 3.5 + (101-1) \times 0$ [by given conditions]

$\therefore$ $= 3.5$

Question 3:

The list of numbers – 10, – 6, – 2, 2,… is

(a) an AP with d = -16

(b) an AP with d = 4

(c) Fan AP with d = – 4

(d) not an AP

Solution:

(b) The given numbers are -10,-6,-2, 2…………………..

Here, a., = -10, a_2 = -6, a_3 = -2 and a_4 = 2………

Since,
$$
\begin{aligned}
a_2 - a_1 &= -6-(-10) \\
&= -6+10 = 4 \\
a_3 - a_2 &= -2-(-6) \\
&= -2+6 = 4 \\
a_4 - a_3 &= 2-(-2) \\
&= 2+2 = 4 \\
&\dots \dots \dots \dots \dots \dots \\
&\dots \dots \dots \dots \dots \dots \\
&\dots \dots \dots \dots \dots \dots
\end{aligned}
$$

Each successive term of given list has same difference i.e., 4.

So, the given list forms an AP with common difference, d=4.

Question 4:

The 11th term of an AP – 5$\frac{-5}{2}$, 0$\frac{5}{2}$…

(a)-20 (b) 20 (c) -30 (d) 30

Solution:

Given AP,- 5,$\frac{-5}{2}$, 0$\frac{5}{2}$

Here, $a = -5, d = \frac{-5}{2} + 5 = \frac{5}{2}$

$\therefore$ $a_{11} = a + (11-1)d$ $[\because a_n = a + (n-1)d]$

$= -5 + (10) \times \frac{5}{2}$

$= -5 + 25 = 20$

Question 5:

The first four terms of an AP whose first term is – 2 and the common difference is-2 are

(a) -2,0,2, 4

(b) -2, 4, -8,16

(c) -2,-4,-6,-8

(d) -2, – 4, -8, -16

Solution:

(c) Let the first four terms of an AP are a, a + d, a + 2d and a + 3d.

Given, that first term, a = – 2 and common difference, d = – 2, then we have an AP as follows

-2, – 2 – 2, – 2 + 2(-2), – 2 + 3(-2)

= – 2, – 4, – 6, – 8

Question 6:

The 21st term of an AP whose first two terms are – 3 and 4, is

(a) 17 (b) 137 (c) 143 (d)-143

Solution:

(b) Given, first two terms of an AP are a = – 3 and a + d = 4.

⇒ – 3 + d = 4

Question 7:

If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33 (c) 37 (d) 38

Solution:

(b) Given, a_2 = 13 and a_5 = 25

$\Rightarrow \quad a + (2-1)d = 13 \quad [\because a_n = a + (n-1)d]$

and $\quad a + (5-1)d = 25$

$\Rightarrow \quad a + d = 13 \quad \ldots(i)$

and $\quad a + 4d = 25 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$$3d = 25 - 13 = 12 \Rightarrow d = 4$$

From Eq. (i), $a = 13 - 4 = 9$

$\therefore \quad a_7 = a + (7-1)d = 9 + 6 \times 4 = 33$

Question 8:

Which term of an AP : 21, 42, 63, 84,… is 210?

(a) 9th (b) 10th (c)11th (d) 12th

Solution:

(b) Let nth term of the given AP be 210

Here, first term, $a = 21$

and common difference, $d = 42 - 21 = 21$ and $a_n = 210$

$\because \quad a_n = a + (n-1)d$

$\Rightarrow \quad 210 = 21 + (n-1)21$

$\Rightarrow \quad 210 = 21 + 21n - 21$

$\Rightarrow \quad 210 = 21n \Rightarrow n = 10$

Hence, the 10th term of an AP is 210.

Question 9:

If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20 (c) 25 (d) 30

Solution:

(c) Given, the common difference of AP i.e., d = 5

Now, $\quad a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d] \quad [\because a_n = a + (n-1)d]$

$= a + 17 \times 5 - a - 12 \times 5$

$= 85 - 60 = 25$

Question 10:

What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8 (c) – 4 (d) 4

Solution:

(a) Given, $\quad a_{18} - a_{14} = 32 \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + (18-1)d - [a + (14-1)d] = 32$

$\Rightarrow \quad a + 17d - a - 13d = 32$

$\Rightarrow \quad 4d = 32$

$\therefore \quad d = 8$

which is the required common difference of an AP.

Question 11:

Two APs have the same common difference. The first term of one of these is – 1 and that of the other is – 8. The difference between their 4th terms is

(a) -1 (b) -8 (c) 7 (d) -9

Solution:

(c) Let the common difference of two APs are d_1 and d_2, respectively.

Bycondition, $\quad d_1 = d_2 = d \quad \ldots(i)$

Let the first term of first AP (a_1) = -1

and the first term of second AP (a_2) = – 8

We know that, the nth term of an AP, $T_1 = a + (n – 1) d$
∴4th term of first AP, $T_4 = a, + (4 – 1)d = – 1 + 3d$.
and 4th term of second AP, $T'_4 = a_2 + (4 – 1)d = – 8 + 3d$
Now, the difference between their 4th terms is i.e.,
|T4 -T'4|= (-1 + 3d)-(-8+3d)
= -1+ 3d + 8 – 3d = 7
Hence, the required difference is 7.

Question 12:

If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be

(a) 7 (b) 11 (c) 18 (d) 0

Solution:

(d) According to the question,

$$7a_7 = 11a_{11}$$
$$\Rightarrow \quad 7[a + (7-1)d] = 11[a + (11-1)d] \quad [\because a_n = a + (n-1)d]$$
$$\Rightarrow \quad 7(a + 6d) = 11(a + 10d)$$
$$\Rightarrow \quad 7a + 42d = 11a + 110d$$
$$\Rightarrow \quad 4a + 68d = 0$$
$$\Rightarrow \quad 2(2a + 34d) = 0$$
$$\Rightarrow \quad 2a + 34d = 0 \quad [\because 2 \neq 0]$$
$$\Rightarrow \quad a + 17d = 0 \quad \ldots(i)$$
$$\therefore \quad \text{18th term of an AP, } a_{18} = a + (18-1)d$$
$$= a + 17d = 0 \quad [\text{from Eq. (i)}]$$

Question 13:

The 4th term from the end of an AP – 11, -8,-5,…, 49 is

(a) 37 (b) 40 (c)43 (d) 58

Solution:

(b) We know that, the n th term of an AP from the end is

$$a_n = l - (n-1)d \quad \ldots(i)$$

Here, l = Last term and $l = 49$ [given]

Common difference, $d = -8 - (-11)$
$$= -8 + 11 = 3$$

From Eq. (i), $a_4 = 49 - (4-1)3 = 49 - 9 = 40$

From Eq, (i), $a_4 = 49 – (4 -1) 3 = 49 – 9 = 40$

Question 14:

The famous mathematician associated with finding the sum of the first 100 natural numbers is

(a)'Pythagoras
(b) Newton
(c) Gauss
(d) Euclid

Solution:

(c) Gauss is the famous mathematician associated with finding the sum of the first 100 natural numbers i,e., 1,2, 3……………..100.

Question 15:

If the first term of an AP is – 5 and the common difference is 2, then the sum of the first 6 terms is

(a) 0 (b) 5 (c) 6 (d) 15

Solution:

(a) Given, $a = -5$ and $d = 2$

$\therefore$ $$S_6 = \frac{6}{2}[2a + (6-1)d] \qquad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$$
$$= 3[2(-5) + 5(2)]$$
$$= 3(-10 + 10) = 0$$

Question 16:

The sum of first 16 terms of the AP 10, 6, 2, … is

(a)-320 (b) 320 (c)-352 (d)-400

Solution:

(a) Given, AP is 10, 6, 2,…

Here, first term $a = 10$, common difference, $d = -4$

$\therefore$ $$S_{16} = \frac{16}{2}[2a + (16-1)d] \qquad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$$
$$= 8[2 \times 10 + 15(-4)]$$
$$= 8(20 - 60) = 8(-40) = -320$$

Question 17:

In an AP, if a = 1, an = 20 and Sn = 399, then n is equal to

(a) 19 (b) 21 (c) 38 (d) 42

Solution:

(c) $\because$ $$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$399 = \frac{n}{2}[2 \times 1 + (n-1)d]$$
$$798 = 2n + n(n-1)d \qquad \text{...(i)}$$

and $$a_n = 20$$

$\Rightarrow$ $$a + (n-1)d = 20 \qquad [\because a_n = a + (n-1)d]$$

$\Rightarrow$ $$1 + (n-1)d = 20 \Rightarrow (n-1)d = 19 \qquad \text{...(ii)}$$

Using Eq. (ii) in Eq. (i), we get
$$798 = 2n + 19n$$

$\Rightarrow$ $$798 = 21n$$

$\therefore$ $$n = \frac{798}{21} = 38$$

Question 18:

The sum of first five multiples of 3 is

(a) 45 (b) 55 (c) 65 (d) 75

Solution:

(a) The first five multiples of 3 are 3, 6, 9,12 and 15.

Here, first term, a = 3, common difference, d = 6-3 = 3 and number of terms, n = 5

$\therefore$ $$S_5 = \frac{5}{2}[2a + (5-1)d] \qquad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$$
$$= \frac{5}{2}[2 \times 3 + 4 \times 3]$$
$$= \frac{5}{2}(6 + 12) = 5 \times 9 = 45$$

Exercise 5.2 Very Short Answer Type Questions

Question 1:

Which of the following form of an AP ? Justify your answer.

(i) $-1, -1, -1, -1, \ldots$

(ii) $0, 2, 0, 2, \ldots$

(iii) $1, 1, 2, 2, 3, 3, \ldots$

(iv) $11, 22, 33, \ldots$

(v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

(vi) $2, 2^2, 2^3, 2^4$

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Solution:

(i) Here, $t_1 = -1, t_2 = -1, t_3 = -1$ and $t_4 = -1$

$$t_2 - t_1 = -1 + 1 = 0$$
$$t_3 - t_2 = -1 + 1 = 0$$
$$t_4 - t_3 = -1 + 1 = 0$$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

(ii) Here, $t_1 = 0, t_2 = 2, t_3 = 0$ and $t_4 = 2$

$$t_2 - t_1 = 2 - 0 = 2$$
$$t_3 - t_2 = 0 - 2 = -2$$
$$t_4 - t_3 = 2 - 0 = 2$$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iii) Here, $t_1 = 1, t_2 = 1, t_3 = 2$ and $t_4 = 2$

$$t_2 - t_1 = 1 - 1 = 0$$
$$t_3 - t_2 = 2 - 1 = 1$$
$$t_4 - t_2 = 2 - 2 = 0$$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iv) Here, $t_1 = 11, t_2 = 22$ and $t_3 = 33$

$$t_2 - t_1 = 22 - 11 = 11$$
$$t_3 - t_2 = 33 - 22 = 11$$
$$t_4 - t_3 = 33 - 22 = 11$$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

(v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

Here, $t_1 = \frac{1}{2}, t_2 = \frac{1}{3}$ and $t_3 = \frac{1}{4}$

$$t_2 - t_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}$$
$$t_3 - t_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = -\frac{1}{12}$$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vi) $2, 2^2, 2^3, 2^4, \ldots$ *i.e.*, $2, 4, 8, 16, \ldots$

Here, $t_1 = 2, t_2 = 4, t_3 = 8$ and $t_4 = 16$

$$t_2 - t_1 = 4 - 2 = 2$$
$$t_3 - t_2 = 8 - 4 = 4$$
$$t_4 - t_3 = 16 - 8 = 8$$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$ *i.e.*, $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \ldots$

Here, $t_1 = \sqrt{3}, t_2 = 2\sqrt{3}, t_3 = 3\sqrt{3}$ and $t_4 = 4\sqrt{3}$

$$t_2 - t_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$
$$t_3 - t_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$
$$t_4 - t_3 = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

Question 2:

Justify whether it is true to say that -1, $\frac{-3}{2}$, -2, $\frac{5}{2}$… forms an AP as

$a_2 - a_1 = a_3 - a_2$

Solution:

False

Here, $a_1 = -1$, $a_2 = \frac{-3}{2}$, $a_3 = -2$ and $a_4 = \frac{5}{2}$

$$a_2 - a_1 = \frac{-3}{2} + 1 = -\frac{1}{2}$$

$$a_3 - a_2 = -2 + \frac{3}{2} = -\frac{1}{2}$$

$$a_4 - a_3 = \frac{5}{2} + 2 = \frac{9}{2}$$

Clearly, the difference of successive terms is not same, all though, $a_2 - a_1 = a_3 - a_2$ but $a_3 - a_2$ $a_4 - a_3$, therefore it does not form an AP.

Question 3:

For the AP -3, – 7, – 11,… can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reason for your answer.

Solution:

True

$\because$ nth term of an AP, $a_n = a + (n-1)d$

$\therefore \quad a_{30} = a + (30-1)d = a + 29d$

and $\quad a_{20} = a + (20-1)d = a + 19d \quad$ …(i)

Now, $\quad a_{30} - a_{20} = (a + 29d) - (a + 19d) = 10d$

and from given AP common difference, $d = -7 - (-3) = -7 + 3$

$= -4$

$\therefore \quad a_{30} - a_{20} = 10(-4) = -40 \quad$ [from Eq. (i)]

Question 4:

Two AP's have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10th terms is the same as the difference between their 21st terms, which is the same as the difference between any two corresponding terms? Why?

Solution:

Let the same common difference of two AP's isd, Given that, the first term of first AP and second AP are 2 and 7 respectively, then the AP's are

2,2 + d,2 + 2d,2 + 3d,.,.

and 7,7+ d, 7 +2d, 7+3d,…

Now, 10th terms of first and second AP's are 2 + 9d and 7 + 9 d, respectively.

So, their difference is 7 + 9d – (2 + 9d) = 5

Also, 21st terms of first and second AP's are 2 + 20d and 7 + 20d, respectively.

So, their difference is 7 + 20d – (2 + 9d) = 5

Also, if the a_n and b_n are the nth terms of first and second AP.

Then, $b_n - a_n = [7 + (n-1)d)] - [2 + (n-1)d] = 5$

Hence, the difference between any two corresponding terms of such AP's is the same as the difference between their first terms.

Question 5:

Is 0 a term of the AP 31, 28, 25,…? Justify your answer.

Solution:

Let 0 be the nth term of given AP i.e., $a_n = 0$.

Given that, first term a = 31, common difference, d = 28 – 31 = – 3

The nth terms of an AP, is

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 0 = 31 + (n-1)(-3)$$

$$\Rightarrow \quad 3(n-1) = 31$$

$$\Rightarrow \quad n - 1 = \frac{31}{3}$$

$$\therefore \quad n = \frac{31}{3} + 1 = \frac{34}{3} = 11\frac{1}{3}$$

Since, n should be positive integer. So, 0 is not a term of the given AP.

Question 6:

The taxi fare after each km, when the fare is ₹ 15 for the first km ahd ₹ 8 for each additional km, does not form an AP as the total fare (in ₹) after each km is 15, 8, 8, 8, … . Is the statement true? Give reasons.

Solution:

No, because the total fare (in ?) after each km is

15,(15+ 8), (15 + 2 x 8), (15+ 3 x 8),…= 15,23, 31, 39,…

Let $t_1 = 15, t_2 = 23, t_3 = 31$ and $t_4 = 39$

Now, $t_2 - t_1 = 23 - 15 = 8$

$$t_3 - t_2 = 31 - 23 = 8$$

$$t_4 - t_3 = 39 - 31 = 8$$

Since, all the successive terms of the given list have same difference i.e., common difference = 8

Hence, the total fare after each km form an AP.

Question 7:

In which of the following situations, do the lists of numbers involved form an AP? Give reasons for your answers.

(i) The fee charged from a student every month by a school for the whole session, when the monthly fee is ₹ 400.

(ii) The fee charged every month by a school from classes I to XII, When the monthly fee for class I is ₹ 250 and it increase by ₹ 50 for the next higher class.

(iii) The amount of money in the account of Varun at the end of every year when ₹ 1000 is deposited at simple interest of 10% per annum.

(iv) The number of bacteria in a certain food item after each second, when they double in every second.

Solution:

(i) The fee charged from a student every month by a school for the whole session is

400, 400, 400, 400,…

which form an AP, with common difference (d) = 400-400 = 0

(ii) The fee charged month by a school from I to XII is

250, (250 + 50), (250 + 2 x 50), (250 + 3 x 50),…

i.e., 250,300,350,400,…

which form an AP, with common difference (d) = 300 – 250 = 50

(iii) Simple interest $= \dfrac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

$$= \frac{1000 \times 10 \times 1}{100} = 100$$

So, the amount of money in the account of Varun at the end of every year is

$$1000, (1000 + 100 \times 1), (1000 + 100 \times 2), (1000 + 100 \times 3), \ldots$$

i.e., $\quad 1000, 1100, 1200, 1300, \ldots$

which form an AP, with common difference $(d) = 1100 - 1000 = 100$

(iv) Let the number of bacteria in a certain food $= x$

Since, they double in every second.

$\therefore \quad x, 2x, 2(2x), 2(2 \cdot 2 \cdot x), \ldots$

i.e., $\quad x, 2x, 4x, 8x, \ldots$

Now, let $\quad t_1 = x, t_2 = 2x, t_3 = 4x$ and $t_4 = 8x$

$$t_2 - t_1 = 2x - x = x$$
$$t_3 - t_2 = 4x - 2x = 2x$$
$$t_4 - t_3 = 8x - 4x = 4x$$

Since, the difference between each successive term is not same, So, the list does form an AP

Question 8:

Justify whether it is true to say that the following are the nth terms of an AP.

(i) $2n - 3$ (ii) $3n^2 + 5$ (iii) $1 + n + n^2$

Solution:

(i) *Yes,* here $a_n = 2n - 3$

Put $n = 1$, $\quad a_1 = 2(1) - 3 = -1$

Put $n = 2$, $\quad a_2 = 2(2) - 3 = 1$

Put $n = 3$, $\quad a_3 = 2(3) - 3 = 3$

Put $n = 4$, $\quad a_4 = 2(4) - 3 = 5$

List of numbers becomes $-1, 1, 3, \ldots$

Here, $\quad a_2 - a_1 = 1 - (-1) = 1 + 1 = 2$

$$a_3 - a_2 = 3 - 1 = 2$$
$$a_4 - a_3 = 5 - 3 = 2$$

$\because \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots$

Hence, $2n - 3$ is the nth term of an AP.

(ii) *No,* here $a_n = 3n^2 + 5$

Put $n = 1$, $\quad a_1 = 3(1)^2 + 5 = 8$

Put $n = 2$, $\quad a_2 = 3(2)^2 + 5 = 3(4) + 5 = 17$

Put $n = 3$, $\quad a_3 = 3(3)^2 + 5 = 3(9) + 5 = 27 + 5 = 32$

So, the list of number becomes 8, 17, 32, ...

Here, $a_2 - a_1 = 17 - 8 = 9$

$a_3 - a_2 = 32 - 17 = 15$

$\therefore \quad a_2 - a_1 \neq a_3 - a_2$

Since, the successive difference of the list is not same. So, it does not form an AP.

(iii) *No*, here $a_n = 1 + n + n^2$

Put $n = 1$, $\quad a_1 = 1 + 1 + (1)^2 = 3$

Put $n = 2$, $\quad a_2 = 1 + 2 + (2)^2 = 1 + 2 + 4 = 7$

Put $n = 3$, $\quad a_3 = 1 + 3 + (3)^2 = 1 + 3 + 9 = 13$

So, the list of number becomes 3, 7, 13,...

Here, $a_2 - a_1 = 7 - 3 = 4$

$a_3 - a_2 = 13 - 7 = 6$

$\therefore \quad a_2 - a_1 \neq a_3 - a_2$

Since, the successive difference of the list is not same. So, it does not form an AP.

Exercise 5.3 Short Answer Type Questions

Question 1:

Match the AP's given in column A with suitable common differences given in column B.

	Column A		Column B
(A_1)	$2, -2, -6, -10, \ldots$	(B_1)	$\frac{2}{3}$
(A_2)	$a = -18, n = 10, a_n = 0$	(B_2)	-5
(A_3)	$a = 0, a_{10} = 6$	(B_3)	4
(A_4)	$a_2 = 13, a_4 = 3$	(B_4)	-4
		(B_5)	2
		(B_6)	$\frac{1}{2}$
		(B_7)	5

Solution:

A_1. $2, -2, -6, -10, \ldots$

Here, common difference, $d = -2 - 2 = -4$

A_2. $\because \quad a_n = a + (n-1)d$

$\Rightarrow \quad 0 = -18 + (10-1)d$

$18 = 9d$

$\therefore$ Common difference, $d = 2$

A_3. $\because \quad a_{10} = 6$

$\Rightarrow \quad a + (10-1)d = 6$

$\Rightarrow \quad 0 + 9d = 6 \qquad [\because a = 0 \text{ (given)}]$

$\Rightarrow \quad 9d = 6 \Rightarrow d = \frac{2}{3}$

A_4. $\because \quad a_2 = 13$

$\Rightarrow \quad a + (2-1)d = 13 \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + d = 13 \qquad \ldots(i)$

and $\quad a_4 = 3 \Rightarrow a + (4-1)d = 3$

$\therefore \quad a + 3d = 3 \qquad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$2d = -10$

$\Rightarrow \quad d = -5$

$\therefore \quad (A_1) \to B_4, (A_2) \to B_5, (A_3) \to B_1$ and $(A_4) \to B_2$

Question 2:

Verify that each of the following is an AP and then write its next three terms.

(i) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ (ii) $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

(iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ (iv) $a+b, (a+1)+b, (a+1)+(b+1), \ldots$

(v) $a, 2a+1, 3a+2, 4a+3, \ldots$

Solution:

(i) Here, $a_1 = 0,\ a_2 = \frac{1}{4}, a_3 = \frac{1}{2}$ and $a_4 = \frac{3}{4}$

$$a_2 - a_1 = \frac{1}{4}, a_3 - a_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}, a_4 - a_3 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

$\therefore \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Since, the each successive term of the given list has the same difference. So, it forms an AP.

The next three terms are, $a_5 = a_1 + 4d$

$$= a + 4\left(\frac{1}{4}\right) = 1,\ a_6 = a_1 + 5d = a + 5\left(\frac{1}{4}\right) = \frac{5}{4}$$

$$a_7 = a + 6d = 0 + \frac{6}{4} = \frac{3}{2}$$

(ii) Here, $a_1 = 5, a_2 = \frac{14}{3}, a_3 = \frac{13}{3}$ and $a_4 = 4$

$$a_2 - a_1 = \frac{14}{3} - 5 = \frac{14-15}{3} = \frac{-1}{3}, a_3 - a_2 = \frac{13}{3} - \frac{14}{3} = -\frac{1}{3}$$

$$a_4 - a_3 = 4 - \frac{13}{3} = \frac{12-13}{3} = \frac{-1}{3}$$

$\because \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Since, the each successive term of the given list has same difference.

It forms an AP.

The next three terms are,

$$a_5 = a_1 + 4d = 5 + 4\left(-\frac{1}{3}\right) = 5 - \frac{4}{3} = \frac{11}{3}$$

$$a_6 = a_1 + 5d = 5 + 5\left(-\frac{1}{3}\right) = 5 - \frac{5}{3} = \frac{10}{3}$$

$$a_7 = a_1 + 6d = 5 + 6\left(-\frac{1}{3}\right) = 5 - 2 = 3$$

(iii) Here, $a_1 = \sqrt{3}$, $a_2 = 2\sqrt{3}$ and $a_3 = 3\sqrt{3}$

$$a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3},\ a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

$\because \quad a_2 - a_1 = a_3 - a_2 = \sqrt{3} =$ Common difference

Since, the each successive term of the given list has same difference.

So, it forms an AP.

The next three terms are,

$$a_4 = a_1 + 3d = \sqrt{3} + 3(\sqrt{3}) = 4\sqrt{3}$$
$$a_5 = a_1 + 4d = \sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$$
$$a_6 = a_1 + 5d = \sqrt{3} + 5\sqrt{3} = 6\sqrt{3}$$

(iv) Here, $a_1 = a + b$, $a_2 = (a + 1) + b$, $a_3 = (a + 1) + (b + 1)$

$$a_2 - a_1 = (a + 1) + b - (a + b) = a + 1 + b - a - b = 1$$
$$a_3 - a_2 = (a + 1) + (b + 1) - [(a + 1) + b]$$
$$= a + 1 + b + 1 - a - 1 - b = 1$$

$\because \quad a_2 - a_1 = a_3 - a_2 = 1 =$ Common difference

Since, the each successive term of the given list has same difference.

So, it forms an AP.

The next three terms are,

$$a_4 = a_1 + 3d = a + b + 3(1) = (a + 2) + (b + 1)$$
$$a_5 = a_1 + 4d = a + b + 4(1) = (a + 2) + (b + 2)$$
$$a_6 = a_1 + 5d = a + b + 5(1) = (a + 3) + (b + 2)$$

(v) Here, $a_1 = a$, $a_2 = 2a + 1$, $a_3 = 3a + 2$ and $a_4 = 4a + 3$

$$a_2 - a_1 = 2a + 1 - a = a + 1$$
$$a_3 - a_2 = 3a + 2 - 2a - 1 = a + 1$$
$$a_4 - a_3 = 4a + 3 - 3a - 2 = a + 1$$
$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = a + 1 = \text{Common difference}$$

Since, the each successive term of the given list has same difference.

So, it forms an AP.

The next three terms are,

$$a_5 = a + 4d = a + 4(a + 1) = 5a + 4$$
$$a_6 = a + 5d = a + 5(a + 1) = 6a + 5$$
$$a_7 = a + 6d = a + 6(a + 1) = 7a + 6$$

Question 3:

Write the first three terms of the AP's, when a and d are as given below

(i) a $= \frac{1}{2}$,d $= \frac{-1}{6}$ (ii) a = -5,d = -3 (iii) a = √2, d $= \frac{1}{\sqrt{2}}$

Solution:

(i) Given that, first term $(a) = \frac{1}{2}$ and common difference $(d) = -\frac{1}{6}$

$\because$ nth term of an AP, $T_n = a + (n-1)d$

$\therefore$ Second term of an AP, $T_2 = a + d = \frac{1}{2} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

and third term of an AP, $T_3 = a + 2d = \frac{1}{2} - \frac{2}{6} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

Hence, required three terms are $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$.

(ii) Given that, first term (a) = -5 and common difference $(d) = -3$

$\because$ nth term of an AP, $T_n = a + (n-1)d$

$\therefore$ Second term of an AP, $T_2 = a + d = -5 - 3 = -8$

and third term of an AP, $T_3 = a + 2d = -5 + 2(-3)$

$= -5 - 6 = -11$

Hence, required three terms are $-5, -8, -11$.

(iii) Given that, first term $(a) = \sqrt{2}$ and common difference $(d) = \frac{1}{\sqrt{2}}$

$\because$ nth term of an AP, $T_n = a + (n-1)d$

$\therefore$ Second term of an AP, $T_2 = a + d = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

and third term of an AP, $T_3 + a + 2d = \sqrt{2} + \frac{2}{\sqrt{2}} = \frac{2+2}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

Hence, required three terms are $\sqrt{2}, \frac{3}{\sqrt{2}}, \frac{4}{\sqrt{2}}$.

Question 4:

Find a, b and c such that the following numbers are in AP, a, 7, b, 23 and c.

solution :

Since a, 7, b, 23 and c are in AR

$$\therefore \quad \underset{\text{I}}{7-a} = \underset{\text{II}}{b-7} = \underset{\text{III}}{23-b} = \underset{\text{IV}}{c-23} = \text{Common difference}$$

Taking second and third terms, we get

$$b - 7 = 23 - b$$
$$\Rightarrow \quad 2b = 30$$
$$\therefore \quad b = 15$$

Taking first and second terms, we get

$$7 - a = b - 7$$
$$\Rightarrow \quad 7 - a = 15 - 7 \quad [\because b = 15]$$
$$\Rightarrow \quad 7 - a = 8$$
$$\therefore \quad a = -1$$

Taking third and fourth terms, we get

$$23 - b = c - 23$$
$$\Rightarrow \quad 23 - 15 = c - 23 \quad [\because b = 15]$$
$$\Rightarrow \quad 8 = c - 23$$
$$\Rightarrow \quad 8 + 23 = c \Rightarrow c = 31$$

Hence, $a = -1, b = 15, c = 31$

Question 5:

Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

Solution:

Let the first term of an AP be a and common difference d.

Given, $a_5 = 19$and $a_{13} - a_8 = 20$ [given]

$\therefore \quad a_5 = a + (5-1)d = 19$ and $[a + (13-1)d] - [a + (8-1)d] = 20 \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 4d = 19 \quad \ldots(i)$

and $\quad a + 12d - a - 7d = 20 \Rightarrow 5d = 20$

$\therefore \quad d = 4$

On putting $d = 4$ in Eq. (i), we get

$$a + 4(4) = 19$$
$$a + 16 = 19$$
$$a = 19 - 16 = 3$$

So, required AP is $a, a + d, a + 2d, a + 3d, \ldots$ i.e., $3, 3 + 4, 3 + 2(4), 3 + 3(4), \ldots$

i.e., $3, 7, 11, 15, \ldots$

Question 6:

The 26th, 11th and the last terms of an AP are, 0,3 and $\frac{-1}{6}$, respectively.

Find the common difference and the number of terms.

Solution:

Let the first term, common difference and number of terms of an AP are a, d and n, respectively.

We know that, if last term of an AP is known, then

$$l = a + (n-1)d \quad \ldots(i)$$

and nth term of an AP is

$$T_n = a + (n-1)d \quad \ldots(ii)$$

Given that, 26th term of an AP = 0

$\Rightarrow \quad T_{26} = a + (26-1)d = 0 \quad$ [from Eq. (i)]

$\Rightarrow \quad a + 25d = 0 \quad \ldots(iii)$

11th term of an AP = 3

$\Rightarrow \quad T_{11} = a + (11-1)d = 3 \quad$ [from Eq. (ii)]

$\Rightarrow \quad a + 10d = 3 \quad \ldots(iv)$

and last term of an AP $= -1/5$

$\Rightarrow \quad l = a + (n-1)d \quad$ [from Eq. (i)]

$\Rightarrow \quad -1/5 = a + (n-1)d \quad \ldots(v)$

Now, subtracting Eq. (iv) from Eq. (iii),

$$\begin{array}{r} a + 25d = 0 \\ a + 10d = 3 \\ - \quad - \quad\quad - \\ \hline 15d = -3 \end{array}$$

$\Rightarrow \quad d = -\frac{1}{5}$

Put the value of d in Eq. (iii), we get

$$a + 25\left(-\frac{1}{5}\right) = 0$$

$\Rightarrow \quad a - 5 = 0 \Rightarrow a = 5$

Now, put the value of a, d in Eq. (v), we get

$$-1/5 = 5 + (n-1)(-1/5)$$

$\Rightarrow \quad -1 = 25 - (n-1)$

$\Rightarrow \quad -1 = 25 - n + 1$

$\Rightarrow \quad n = 25 + 2 = 27$

Hence, the common difference and number of terms are $-1/5$ and 27, respectively.

Question 7:

The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

Solution:

Let the first term and common difference of AP are a and d, respectively.

According to the question,

$$a_5 + a_7 = 52 \text{ and } a_{10} = 46$$

$\Rightarrow$ $a + (5-1)d + a + (7-1)d = 52$ $\quad [\because a_n = a + (n-1)d]$

and $a + (10-1)d = 46$

$\Rightarrow$ $a + 4d + a + 6d = 52$

and $a + 9d = 46$

$\Rightarrow$ $2a + 10d = 52$

and $a + 9d = 46$

$\Rightarrow$ $a + 5d = 26$ $\quad \ldots(i)$

$a + 9d = 46$ $\quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$$4d = 20 \Rightarrow d = 5$$

From Eq. (i), $a = 26 - 5(5) = 1$

So, required AP is $a, a+d, a+2d, a+3d, \ldots$ i.e., $1, 1+5, 1+2(5), 1+3(5), \ldots$ i.e., $1, 6, 11, 16, \ldots$

Question 8:

Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.

Solution:

Let the first term, common difference and number of terms of an AP are a,d and n, respectively, Given that, first term (a) = 12.

Now by condition,

$$\text{7th term } (T_7) = \text{11th term } (T_{11}) - 24$$

$[\because \text{nth term of an AP, } T_n = a + (n-1)d]$

$\Rightarrow$ $a + (7-1)d = a + (11-1)d - 24$

$\Rightarrow$ $a + 6d = a + 10d - 24$

$\Rightarrow$ $24 = 4d \Rightarrow d = 6$

$\therefore$ 20th term of AP, $T_{20} = a + (20-1)d$

$= 12 + 19 \times 6 = 126$

Hence, the required 20th term of an AP is 126.

Question 9:

If the 9th term of an AP is zero, then prove that its 29th term is twice its 19th term.

Solution:

Let the first term, common difference and number of terms of an AP are a, d and n respectively.

Given that, 9th term of an AP, $T_9 = 0$ $\quad [\because \text{nth term of an AP, } T_n = a + (n-1)d]$

$\Rightarrow$ $a + (9-1)d = 0$

$\Rightarrow$ $a + 8d = 0 \Rightarrow a = -8d$ $\quad \ldots (i)$

Now, its 19th term, $T_{19} = a + (19-1)d$

$= -8d + 18d$ $\quad$ [from Eq. (i)]

$= 10d$ $\quad \ldots (ii)$

and its 29th term, $T_{29} = a + (29-1)d$

$= -8d + 28d$ $\quad$ [from Eq. (i)]

$= 20d = 2 \times (10d)$

$\Rightarrow$ $T_{29} = 2 \times T_{19}$

Hence, its 29th term is twice its 19th term.

Question 10:

Find whether 55 is a term of the AP 7, 10, 13, … or not. If yes, find which term it is.

Solution:

Yes, let the first term, common difference and the number of terms of an AP are a, d and n respectively.

Let the nth term of an AP be 55. i.e., $T_n = 55$.

Let the nth term of an AP be 55. *i.e.*, $T_n = 55$.
We know that, the nth term of an AP, $T_n = a + (n-1)d$ ...(i)
Given that, first term $(a) = 7$ and common difference $(d) = 10 - 7 = 3$
From Eq. (i), $55 = 7 + (n-1) \times 3$
$\Rightarrow 55 = 7 + 3n - 3 \Rightarrow 55 = 4 + 3n$
$\Rightarrow 3n = 51$
$\therefore n = 17$
Since, n is a positive integer. So, 55 is a term of the AP.
Now, put the values of a, d and n in Eq. (i),
$$T_n = 7 + (17-1)(3)$$
$$= 7 + 16 \times 3 = 7 + 48 = 55$$

Hence, 17th term of an AP is 55.

Question 11:

Determine k, so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are three consecutive terms of an AP.

Solution:

Since, $k^2 + 4k + 8, 2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are consecutive terms of an AP

$\therefore$ $2k^2 + 3k + 6 - (k^2 + 4k + 8) = 3k^2 + 4k + 4 - (2k^2 + 3k + 6)$ = *Common difference*

$\Rightarrow 2k^2 + 3k + 6 - k^2 - 4k - 8 = 3k^2 + 4k + 4 - 2k^2 - 3k - 6$
$\Rightarrow k^2 - k - 2 = k^2 + k - 2$
$\Rightarrow -k = k \Rightarrow 2k = 0 \Rightarrow k = 0$

Question 12:

Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

Solution:

Let the three parts of the number 207 are (a – d), a and (a + d), which are in AP

Now, by given condition,

$\Rightarrow$ Sum of these parts = 207
$\Rightarrow a - d + a + a + d = 207$
$\Rightarrow 3a = 207$
$a = 69$
Given that, product of the two smaller parts = 4623
$\Rightarrow a(a - d) = 4623$
$\Rightarrow 69 \cdot (69 - d) = 4623$
$\Rightarrow 69 - d = 67$
$\Rightarrow d = 69 - 67 = 2$
So, first part $= a - d = 69 - 2 = 67$,
second part $= a = 69$
and third part $= a + d = 69 + 2 = 71$,

Hence, required three parts are 67, 69, 71.

Question 13:

The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

Solution:

Given that, the angles of a triangle are in AR

Let A, B and C are angles of a Δ ABC.

$$\therefore \quad B = \frac{A + C}{2}$$

$$\Rightarrow \quad 2B = A + C \quad \text{...(i)}$$

We know that, sum of all interior angles of a $\Delta ABC = 180°$

$$A + B + C = 180°$$

$$\Rightarrow \quad 2B + B = 180° \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad 3B = 180° \Rightarrow B = 60°$$

Let the greatest and least angles are A and C respectively.

$$A = 2C \quad \text{[by condition] ... (ii)}$$

Now, put the values of B and A in Eq. (i), we get

$$2 \times 60 = 2C + C$$

$$\Rightarrow \quad 120 = 3C \Rightarrow C = 40°$$

Put the value of C in Eq. (ii), we get

$$A = 2 \times 40° \Rightarrow A = 80°$$

Hence, the required angles of triangle are 80°, 60° and 40°.

Question 14:

If the nth terms of the two AP's 9,7,5,……….. and 24,21,18,…. are the same, then find the value of n.Also,that term.

Solution:

Let the first term, common difference and number of terms of the AP 9, 7, 5,…are a_1,d_1 and n_1 respectively.

i.e., first term (a_1) = 9and common difference (d_1)= 7 – 9 = – 2.

$\therefore$ Its nth term, $\quad T'_{n_1} = a_1 + (n_1 - 1)d_1$

$$\Rightarrow \quad T'_{n_1} = 9 + (n_1 - 1)(-2)$$

$$\Rightarrow \quad T'_{n_1} = 9 - 2n_1 + 2$$

$$\Rightarrow \quad T'_{n_1} = 11 - 2n_1 \quad [\because n\text{th term of an AP, } T_n = a + (n-1)d] \text{...(i)}$$

Let the first term, common difference and the number of terms of the AP 24, 21, 18, ... are a_2, d_2 and n_2, respectively.

i.e., first term, $(a_2) = 24$ and common difference $(d_2) = 21 - 24 = -3$.

$\therefore$ Its nth term, $\quad T''_{n_2} = a_2 + (n_2 - 1)d_2$

$$\Rightarrow \quad T''_{n_2} = 24 + (n_2 - 1)(-3)$$

$$\Rightarrow \quad T''_{n_2} = 24 - 3n_2 + 3$$

$$\Rightarrow \quad T''_{n_2} = 27 - 3n_2 \quad \text{...(ii)}$$

Now, by given condition,

nth terms of the both APs are same, *i.e.*, $T'_{n_1} = T''_{n_2}$

$$11 - 2n_1 = 27 - 3n_2 \quad \text{[from Eqs. (i) and (ii)]}$$

$$\Rightarrow \quad n = 16$$

$\therefore$ nth term of first AP, $T'n_1 = 11 - 2n_1 = 11 - 2\,(16)$

$$= 11 - 32 = -21$$

and nth term of second AP, $T''n_2 = 27 - 3n_2 = 27 - 3\,(16)$

$$= 27 - 48 = -21$$

Hence, the value of n is 16 and that term i.e., nth term is -21.

Question 15:

If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and 14th terms is -3, then find the 10th term.

Solution:

Let the first term and common difference of an AP are a and d, respectively.

According to the question,

$$a_3 + a_8 = 7 \text{ and } a_7 + a_{14} = -3$$

$\Rightarrow \quad a + (3-1)d + a + (8-1)d = 7 \quad [\because a_n = a + (n-1)d]$

and $\quad a + (7-1)d + a + (14-1)d = -3$

$$a + 2d + a + 7d = 7$$

and $\quad a + 6d + a + 13d = -3$

$$2a + 9d = 7 \quad \ldots(i)$$

and $\quad 2a + 19d = -3 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$$10d = -10 \Rightarrow d = -1$$

$$2a + 9(-1) = 7 \quad \text{[from Eq. (i)]}$$

$\Rightarrow \quad 2a - 9 = 7$

$\Rightarrow \quad 2a = 16 \Rightarrow a = 8$

$\therefore \quad a_{10} = a + (10-1)d$

$$= 8 + 9(-1)$$

$$= 8 - 9 = -1$$

Question 16:

Find the 12th term from the end of the AP – 2, – 4, – 6,……….,-100.

Solution:

Given AP -2, -4, -6,…,-100

Here, first term (a) = – 2, common difference (d) = – 4 – (-2) = -2 and the last term (l) = -100.

We know,that, the nth term a„ of an AP from the end is a„ = l – (n – 1)d, where l is the last term and d is the common difference,

∴12th term from the end,

a12 =-100-(12-1)(-2)

= -100+ (11) (2) = – 100 + 22 = – 78.

Hence, the 12th term from the end is -78

Question 17:

Which term of the AP 53, 48, 43, … is the first negative term?

Solution:

Given AP is 53, 48, 43, …

Whose, first term (a) = 53 and common difference (d) = 48 – 53 = – 5

Let nth term of the AP be the first negative term.

i.e., $\quad T_n < 0 \quad [\because n\text{th term of an AP, } T_n = a + (n-1)d]$

$$(a + (n-1)d) < 0$$

$\Rightarrow \quad 53 + (n-1)(-5) < 0$

$\Rightarrow \quad 53 - 5n + 5 < 0$

$\Rightarrow \quad 58 - 5n < 0 \Rightarrow 5n > 58$

$\Rightarrow \quad n > 11.6 \Rightarrow n = 12$

i.e., 12th term is the first negative term of the given AP.

$\therefore \quad T_{12} = a + (12-1)d = 53 + 11(-5)$

$$= 53 - 55 = -2 < 0$$

Question 18:

How many numbers lie between 10 and 300, which divided by 4 leave a remainder 3?

Solution:

Here, the first number is 11, which divided by 4 leave remainder 3 between 10 and 300. Last term before 300 is 299, which divided by 4 leave remainder 3.

∴ 11,15,19,23…. 299

Here, first term (a) = 11, common difference d = 15 -11 = 4

$\because$ $\quad$ nth term, $a_n = a + (n-1)d = l$ $\quad$ [last term]

$$\Rightarrow \quad 299 = 11 + (n-1)4$$
$$\Rightarrow \quad 299 - 11 = (n-1)4$$
$$\Rightarrow \quad 4(n-1) = 288$$
$$\Rightarrow \quad (n-1) = 72$$
$$\therefore \quad n = 73$$

Question 19:

Find the sum of the two middle most terms of an AP

$\frac{-4}{3}$,-1,$\frac{-2}{3}$,………..,$4\frac{1}{3}$

Solution:

Here, first term $(a) = -\frac{4}{3}$, common difference $(d) = -1 + \frac{4}{3} = \frac{1}{3}$

and the last term $(l) = 4\frac{1}{3} = \frac{13}{3}$

$\because$ nth term of an AP, $l = a_n = a + (n-1)d$

$$\Rightarrow \quad \frac{13}{3} = -\frac{4}{3} + (n-1)\frac{1}{3}$$
$$\Rightarrow \quad 13 = -4 + (n-1)$$
$$\Rightarrow \quad n - 1 = 17$$
$$\Rightarrow \quad n = 18 \quad \text{[even]}$$

So, the two middle most terms are $\left(\frac{n}{2}\right)$th and $\left(\frac{n}{2}+1\right)$th. *i.e.*, $\left(\frac{18}{2}\right)$th and $\left(\frac{18}{2}+1\right)$th terms

i.e., 9th and 10th terms.

$$\therefore \quad a_9 = a + 8d = -\frac{4}{3} + 8\left(\frac{1}{3}\right) = \frac{8-4}{3} = \frac{4}{3}$$

and
$$a_{10} = a + 9d = \frac{-4}{3} + 9\left(\frac{1}{3}\right) = \frac{9-4}{3} = \frac{5}{3}$$

So, sum of the two middle most terms $= a_9 + a_{10} = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$

Question 20:

The first term of an AP is – 5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

Solution:

Let the first term, common difference and the number of terms of an AP are a, d and n respectively.

Given that, first term (a) = – 5 and last term (l) = 45

Sum of the terms of the AP = 120 ⇒ S_n = 120

We know that, if last term of an AP is known, then sum of n terms of an AP is,

$$S_n = \frac{n}{2}(a + l)$$
$$\Rightarrow \quad 120 = \frac{n}{2}(-5 + 45) \Rightarrow 120 \times 2 = 40 \times n$$
$$\Rightarrow \quad n = 3 \times 2 \Rightarrow n = 6$$

$\therefore$ Number of terms of an AP is known, then the nth term of an AP is,

$$l = a + (n-1)d \Rightarrow 45 = -5 + (6-1)d$$
$$\Rightarrow \quad 50 = 5d \Rightarrow d = 10$$

So, the common difference is 10.

Hence, number of terms and the common difference of an AP are 6 and 10 respectively.

Question 21:

Find the sum

(i) $1 + (-2) + (-5) + (-8) + \ldots + (-236)$

(ii) $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots$ upto n terms.

(iii) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \ldots$ to 11 terms.

Solution:

(i) Here, first term $(a) = 1$ and common difference $(d) = (-2) - 1 = -3$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_n = \frac{n}{2}[2 \times 1 + (n-1) \times (-3)]$

$\Rightarrow \quad S_n = \frac{n}{2}(2 - 3n + 3) \Rightarrow S_n = \frac{n}{2}(5 - 3n)$ ...(i)

We know that, if the last term (l) of an AP is known, then

$$l = a + (n-1)d$$

$\Rightarrow \quad -236 = 1 + (n-1)(-3)$ [$\because l = -236$, given]

$\Rightarrow \quad -237 = -(n-1) \times 3$

$\Rightarrow \quad n - 1 = 79 \Rightarrow n = 80$

Now, put the value of n in Eq. (i), we get

$$S_n = \frac{80}{2}[5 - 3 \times 80] = 40(5 - 240)$$

$$= 40 \times (-235) = -9400$$

Hence, the required sum is -9400.

Alternate Method

Given, $a = 1, d = -3$ and $l = -236$

$\therefore$ Sum of n terms of an AP, $S_n = \frac{n}{2}[a + l]$

$= \frac{80}{2}(1 + (-236))$ [$\because n = 80$]

$= 40 \times (-235) = -9400$

(ii) Here, first term, $a = 4 - \frac{1}{n}$

Common difference, $d = \left(4 - \frac{2}{n}\right) - \left(4 - \frac{1}{n}\right) = \frac{-2}{n} + \frac{1}{n} = \frac{-1}{n}$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \quad S_n = \frac{n}{2}\left[2\left(4 - \frac{1}{n}\right) + (n-1)\left(\frac{-1}{n}\right)\right]$$

$$= \frac{n}{2}\left\{8 - \frac{2}{n} - 1 + \frac{1}{n}\right\}$$

$$= \frac{n}{2}\left(7 - \frac{1}{n}\right) = \frac{n}{2} \times \left(\frac{7n-1}{n}\right) = \frac{7n-1}{2}$$

(iii) Here, first term $(A) = \frac{a-b}{a+b}$

and common difference, $D = \frac{3a-2b}{a+b} - \frac{a-b}{a+b} = \frac{2a-b}{a+b}$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \quad S_n = \frac{n}{2}\left\{2\,\frac{(a-b)}{(a+b)} + (n-1)\,\frac{(2a-b)}{(a+b)}\right\}$$

$$= \frac{n}{2}\left\{\frac{2a - 2b + 2an - 2a - bn + b}{a+b}\right\}$$

$$= \frac{n}{2}\left(\frac{2an - bn - b}{a+b}\right)$$

$$\therefore \quad S_{11} = \frac{11}{2}\left\{\frac{2a(11) - b(11) - b}{a+b}\right\}$$

$$= \frac{11(11a - 6b)}{a+b} = \frac{11}{2}\left(\frac{22a - 12b}{a+b}\right)$$

Question 22:

Which term of the AP – 2, – 7, – 12,… will be – 77 ? Find the sum of this AP upto the term – 77.

Solution:

Given, AP -2,-7,-12,…

Let the nth term of an AP is – 77.

Then, first term (a) = – 2 and common difference (d) = – 7 – (- 2) = – 7 + 2 = – 5.

$\therefore$nth term of an AP, $T_n = a + (n-1)d$

$$\Rightarrow \quad -77 = -2 + (n-1)(-5)$$

$$\Rightarrow \quad -75 = -(n-1) \times 5$$

$$\Rightarrow \quad (n-1) = 15 \Rightarrow n = 16.$$

So, the 16th term of the given AP will be – 77.

Now, the sum of n terms of an AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

So, sum of 16 terms *i.e.*, upto the term – 77.

i.e., $$S_{16} = \frac{16}{2}[2 \times (-2) + (n-1)(-5)]$$

$$= 8[-4 + (16-1)(-5)] = 8(-4-75)$$

$$= 8 \times -79 = -632$$

Hence, the sum of this AP upto the term – 77 is – 632.

Question 23:

If $a_n = 3 - 4n$, then show that $a_1, a_2, a_3, \ldots$ form an AP. Also, find S_{20}.

Solution:

Given that, nth term of the series is an = 3 –
4n …(i)

Put $n = 1$, $a_1 = 3 - 4(1) = 3 - 4 = -1$
Put $n = 2$, $a_2 = 3 - 4(2) = 3 - 8 = -5$
Put $n = 3$, $a_3 = 3 - 4(3) = 3 - 12 = -9$
Put $n = 4$, $a_4 = 3 - 4(4) = 3 - 16 = -13$

So, the series becomes $-1, -5, -9, -13, \ldots$

We see that,

$$a_2 - a_1 = -5 - (-1) = -5 + 1 = -4,$$
$$a_3 - a_2 = -9 - (-5) = -9 + 5 = -4,$$
$$a_4 - a_3 = -13 - (-9) = -13 + 9 = -4$$

i.e., $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = -4$

Since, the each successive term of the series has the same difference. So, it forms an AP.

We know that, sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ Sum of 20 terms of the AP, $S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)]$

$$= 10(-2 + (19)(-4)) = 10(-2 - 76)$$
$$= 10 \times -78 = -780$$

the required sum of 20 terms i.e., S_{20} is – 780.

Question 24:

In an AP, if s_n =n (4n + 1), then find the AP.

Solution:

We know that, the n th term of an AP is

$$a_n = S_n - S_{n-1}$$
$$a_n = n(4n+1) - (n-1)\{4(n-1)+1\} \quad [\because S_n = n(4n+1)]$$
$$\Rightarrow \quad a_n = 4n^2 + n - (n-1)(4n-3)$$
$$= 4n^2 + n - 4n^2 + 3n + 4n - 3 = 8n - 3$$

Put $n = 1$, $a_1 = 8(1) - 3 = 5$
Put $n = 2$, $a_2 = 8(2) - 3 = 16 - 3 = 13$
Put $n = 3$, $a_3 = 8(3) - 3 = 24 - 3 = 21$

Hence, the required AP is 5,13, 21,…

Question 25:

In an AP, if $s_n = 3n^2 + 5n$ and $a_k = 164$, then find the value of k.

Solution:

$\because$ nth term of an AP,

$$a_n = S_n - S_{n-1}$$
$$= 3n^2 + 5n - 3(n-1)^2 - 5(n-1) \quad [\because S_n = 3n^2 + 5n \text{ (given)}]$$
$$= 3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5$$
$$a_n = 6n + 2 \quad \ldots(i)$$
$$\text{or} \quad a_k = 6k + 2 = 164 \quad [\because a_k = 164 \text{ (given)}]$$
$$\Rightarrow \quad 6k = 164 - 2 = 162$$
$$\therefore \quad k = 27$$

Question 26:

If s_n denotes the sum of first n terms of an AP, then prove that $s_{12} = 3(s_8 - s_4)$

Solution:

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$ ...(i)

$\therefore$ $S_8 = \frac{8}{2}[2a + (8-1)d] = 4(2a + 7d) = 8a + 28d$

and $S_4 = \frac{4}{2}[2a + (4-1)d] = 2(2a + 3d) = 4a + 6d$

Now, $S_8 - S_4 = 8a + 28d - 4a - 6d = 4a + 22d$ ...(ii)

and $S_{12} = \frac{12}{2}[2a + (12-1)d] = 6(2a + 11d)$

$= 3(4a + 22d) = 3(S_8 - S_4)$ [from Eq. (ii)]

$\therefore$ $S_{12} = 3(S_8 - S_4)$ **Hence proved.**

Question 27:

Find the sum of first 17 terms of an AP whose 4th and 9th terms are – 15 and – 30, respectively.

Solution:

Let the first term, common difference and the number of terms in an AP are a, d and n,respectively

We know that, the nth term of an AP, $T_n = a + (n-1)d$ ... (i)

$\therefore$ 4th term of an AP, $T_4 = a + (4-1)d = -15$ [given]

$\Rightarrow$ $a + 3d = -15$ ...(ii)

and 9th term of an AP, $T_9 = a + (9-1)d = -30$ [given]

$\Rightarrow$ $a + 8d = -30$ ...(iii)

Now, subtract Eq. (ii) from Eq. (iii), we get

$$\begin{array}{r} a + 8d = -30 \\ a + 3d = -15 \\ \underline{- \quad - \qquad +} \\ 5d = -15 \end{array}$$

$\Rightarrow$ $d = -3$

Put the value of d in Eq. (ii), we get

$a + 3(-3) = -15 \Rightarrow a - 9 = -15$

$\Rightarrow$ $a = -15 + 9 \Rightarrow a = -6$

$\because$ Sum of first n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ Sum of first 17 terms of an AP, $S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$

$= \frac{17}{2}[-12 + (16)(-3)]$

$= \frac{17}{2}(-12 - 48) = \frac{17}{2} \times (-60)$

$= 17 \times (-30) = -510$

Hence,the required sum of first 17 terms of an AP is -510

Question 28:

If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, then find the sum of first 10 terms.

Solution:

Let a and d be the first term and common difference, respectively of an AP

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$ ...(i)

Now, $S_6 = 36$ [given]

$\Rightarrow \frac{6}{2}[2a + (6-1)d] = 36$

$\Rightarrow 2a + 5d = 12$ ...(ii)

and $S_{16} = 256$

$\Rightarrow \frac{16}{2}[2a + (16-1)d] = 256$

$\Rightarrow 2a + 15d = 32$ ...(iii)

On subtracting Eq. (ii) from Eq. (iii), we get

$10d = 20 \Rightarrow d = 2$

From Eq. (ii), $2a + 5(2) = 12$

$\Rightarrow 2a = 12 - 10 = 2$

$\Rightarrow a = 1$

$\therefore S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 5[2(1) + 9(2)] = 5(2 + 18)$

$= 5 \times 20 = 100$

Hence, the required sum of first 10 terms is 100.

Question 29:

Find the sum of all the 11 terms of an AP whose middle most term is 30.

Solution:

Since, the total number of terms

(n)=11 [odd]

$\therefore$ Middle most term $= \frac{(n+1)}{2}$th term $= \left(\frac{11+1}{2}\right)$th term = 6th term

Given that, $a_6 = 30$

$\Rightarrow a + (6-1)d = 30$

$\Rightarrow a + 5d = 30$ ...(i)

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{11} = \frac{11}{2}[2a + (11-1)d]$

$= \frac{11}{2}(2a + 10d) = 11(a + 5d)$ [from Eq. (i)]

$= 11 \times 30 = 330$

Question 30:

Find the sum of last ten terms of the AP 8, 10, 12,…, 126.

solution :

For finding, the sum of last ten terms, we write the given AP in reverse order.

i.e., 126,124,122… 12,10,8

Here, first term (a) = 126, common difference, (d) = 124-126=-2

$\therefore S_{10} = \frac{10}{2}[2a + (10-1)d]$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 5\{2(126) + 9(-2)\} = 5(252 - 18)$

$= 5 \times 234 = 1170$

Question 31:

Find the sum of first seven numbers which are multiples of 2 as well as of 9.

Solution:

For finding, the sum of first seven numbers which are multiples of 2 as well as of 9. Take LCM of 2 and 9 which is 18.

So, the series becomes 18, 36, 54,…

Here, first term (a) = 18, common difference (of) = 36 -18 = 18

$$\therefore \quad S_7 = \frac{n}{2}[2a + (n-1)d] = \frac{7}{2}[2(18) + (7-1)18]$$
$$= \frac{7}{2}[36 + 6 \times 18] = 7(18 + 3 \times 18)$$
$$= 7(18 + 54) = 7 \times 72 = 504$$

Question 32:

How many terms of the AP – 15, – 13, -11, … are needed to make the sum 55?

Solution:

Let n number of terms are needed to make the sum – 55.

Here, first term (a) = -15, common difference (d) = -13+15 = 2

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \quad -55 = \frac{n}{2}[2(-15) + (n-1)2] \qquad [\because S_n = -55 \text{ (given)}]$$
$$\Rightarrow \quad -55 = -15n + n(n-1)$$
$$\Rightarrow \quad n^2 - 16n + 55 = 0$$
$$\Rightarrow \quad n^2 - 11n - 5n + 55 = 0 \qquad \text{[by factorisation method]}$$
$$\Rightarrow \quad n(n-11) - 5(n-11) = 0$$
$$\Rightarrow \quad (n-11)(n-5) = 0$$
$$\therefore \quad n = 5, 11$$

Hence, either 5 and 11 terms are needed to make the sum – 55.

Question 33:

The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2n terms of another AP whose first term is – 30 and the. common difference is 8. Find n.

Solution:

Given that, first term of the first AP (a) = 8

and common difference of the first AP (d) = 20

Let the number of terms in first AP be n.

$\because$ Sum of first n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_n = \frac{n}{2}[2 \times 8 + (n-1)20]$$
$$\Rightarrow \quad S_n = \frac{n}{2}(16 + 20n - 20)$$
$$\Rightarrow \quad S_n = \frac{n}{2}(20n - 4)$$
$$\therefore \quad S_n = n(10n - 2) \qquad \ldots\text{(i)}$$

Now, first term of the second AP $(a') = -30$

and common difference of the second AP $(d') = 8$

$\therefore$ Sum of first $2n$ terms of second AP, $S_{2n} = \frac{2n}{2}[2a' + (2n-1)d']$

$$\Rightarrow \quad S_{2n} = n[2(-30) + (2n-1)(8)]$$
$$\Rightarrow \quad S_{2n} = n[-60 + 16n - 8]$$
$$\Rightarrow \quad S_{2n} = n[16n - 68] \qquad \ldots\text{(ii)}$$

Now, by given condition,

Sum of first n terms of the first AP = Sum of first $2n$ terms of the second AP

$$\Rightarrow \quad S_n = S_{2n} \qquad \text{[from Eqs. (i) and (ii)]}$$
$$\Rightarrow \quad n(10n - 2) = n(16n - 68)$$
$$\Rightarrow \quad n[(16n - 68) - (10n - 2)] = 0$$
$$\Rightarrow \quad n(16n - 68 - 10n + 2) = 0$$
$$\Rightarrow \quad n(6n - 66) = 0$$
$$\therefore \quad n = 11 \qquad [\because n \neq 0]$$

Hence, the required value of n is 11.

Question 34:

Kanika was given her pocket money on Jan 1st, 2008. She puts ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day 3 and continued doing so till the end of the month, from this money into her piggy bank she also spent ₹ 204 of her pocket money, and found that at the end of the month she still had ₹ 100 with her. How much was her pocket money for the month?

Solution:

Let her pocket money be ₹ x.

Now, she takes ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day 3 and so on till the end of the month, from this money.

i.e., $1 + 2 + 3 + 4 + \dots + 31$.

which form an AP in which terms are 31 and first term (a) = 1, common difference (d) = 2 — 1 = 1 .

Sum of first 31 terms = S_{31}

$\therefore$ Sum of first 31 terms $= S_{31}$

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_{31} = \frac{31}{2}[2 \times 1 + (31-1) \times 1]$$

$$= \frac{31}{2}(2 + 30) = \frac{31 \times 32}{2}$$

$$= 31 \times 16 = 496$$

So, Kanika takes ₹ 496 till the end of the month from this money.

Also, she spent ₹ 204 of her pocket money and found that at the end of the month she still has ₹ 100 with her.

Now, according to the condition,

$$(x - 496) - 204 = 100$$

$$\Rightarrow \quad x - 700 = 100$$

$$\therefore \quad x = ₹\ 800$$

Hence, ₹ 800 was her poket money for the month

Question 35:

Yasmeen saves ₹ 32 during the first month, ₹ 36 in the second month and ₹ 40 in the third month. If she continues to save in this manner, in how many months will she save ₹ 2000 ?

Solution:

Given that,

Yasmeen, during the first month, saves = ₹ 32

During the second month, saves = ₹ 36

During the third month, saves = ₹ 40

Let Yasmeen saves ₹ 2000 during the n months.

Here, we have arithmetic progression 32, 36, 40,…

First term (a) = 32, common difference (d) = 36 – 32 = 4

and she saves total money, i.e., S_n = ₹ 2000

We know that, sum of first n terms of an AP is,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 2000 = \frac{n}{2}[2 \times 32 + (n-1) \times 4]$$

$$\Rightarrow \quad 2000 = n(32 + 2n - 2)$$

$$\Rightarrow \quad 2000 = n(30 + 2n)$$

$$\Rightarrow \quad 1000 = n(15 + n)$$

$$\Rightarrow \quad 1000 = 15n + n^2$$

$$\Rightarrow \quad n^2 + 15n - 1000 = 0$$

$$\Rightarrow \quad n^2 + 40n - 25n - 1000 = 0$$

$$\Rightarrow \quad n(n + 40) - 25(n + 40) = 0 \Rightarrow (n + 40)(n - 25) = 0$$

$$\therefore \quad n = 25 \qquad [\because n \neq -40]$$

Hence, in 25 months will she save ₹ 2000. [Since, month cannot be negative]

Exercise 5.4 Long Answer Type Questions

Question 1:

The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

Solution:

Let the first term, common difference and the number of terms of an AP are a, d and n, respectively.

$\because$ Sum of first n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$ ...(i)

$\therefore$ Sum of first five terms of an AP, $S_5 = \frac{5}{2}[2a + (5-1)d]$ [from Eq. (i)]

$$= \frac{5}{2}(2a + 4d) = 5(a + 2d)$$

$\Rightarrow$ $S_5 = 5a + 10d$ ...(ii)

and sum of first seven terms of an AP, $S_7 = \frac{7}{2}[2a + (7-1)d]$

$$= \frac{7}{2}[2a + 6d] = 7(a + 3d)$$

$\Rightarrow$ $S_7 = 7a + 21d$ ...(iii)

Now, by given condition,

$$S_5 + S_7 = 167$$

$\Rightarrow$ $5a + 10d + 7a + 21d = 167$

$\Rightarrow$ $12a + 31d = 167$ ...(iv)

Given that, sum of first ten terms of this AP is 235.

$\therefore$ $S_{10} = 235$

$\Rightarrow$ $\frac{10}{2}[2a + (10-1)d] = 235$

$\Rightarrow$ $5(2a + 9d) = 235$

$\Rightarrow$ $2a + 9d = 47$ ...(v)

On multiplying Eq. (v) by 6 and then subtracting it into Eq. (vi), we get

$$\begin{array}{r} 12a + 54d = 282 \\ 12a + 31d = 167 \\ - \quad - \quad - \\ \hline 23d = 115 \end{array}$$

$\Rightarrow$ $d = 5$

Now, put the value of d in Eq. (v), we get

$$2a + 9(5) = 47 \Rightarrow 2a + 45 = 47$$

$\Rightarrow$ $2a = 47 - 45 = 2 \Rightarrow a = 1$

Sum of first twenty terms of this AP, $S_{20} = \frac{20}{2}[2a + (20-1)d]$

$$= 10[2 \times (1) + 19 \times (5)] = 10(2 + 95)$$
$$= 10 \times 97 = 970$$

Hence, the required sum of its first twenty terms is 970.

Question 2:

Find the

(i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

(iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.

Solution:

(i) Since, multiples of 2 as well as of 5 = LCM of (2, 5) = 10

$\therefore$ Multiples of 2 as well as of 5 between 1 and 500 is 10, 20, 30. 490

which form an AP with first term (a) = 10 and common difference (d) = 20 -10 = 10

nth term a_n =Last term (l) = 490

$\therefore$ Sum of n terms between 1 and 500,

$$S_n = \frac{n}{2}[a + l] \quad \ldots(i)$$

$\because \quad a_n = a + (n-1)d = l$

$\Rightarrow \quad 10 + (n-1)10 = 490$

$\Rightarrow \quad (n-1)10 = 480$

$\Rightarrow \quad n - 1 = 48 \Rightarrow n = 49$

From Eq. (i), $\quad S_{49} = \frac{49}{2}(10 + 490)$

$= \frac{49}{2} \times 500$

$= 49 \times 250 = 12250$

(ii) Same as part (i),

but multiples of 2 as well as of 5 from 1 to 500 is 10, 20, 30, ..., 500.

$\therefore \quad a = 10, d = 10, a_n = l = 500$

$\because \quad a_n = a + (n-1)d = l$

$\Rightarrow \quad 500 = 10 + (n-1)10$

$\Rightarrow \quad 490 = (n-1)10$

$\Rightarrow \quad n - 1 = 49 \Rightarrow n = 50$

$\because \quad S_n = \frac{n}{2}(a + l)$

$\Rightarrow \quad S_{50} = \frac{50}{2}(10 + 500) = \frac{50}{2} \times 510$

$= 50 \times 255 = 12750$

(iii) Since, multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (2, 5) *i.e.*, 10.

$\therefore$ Multiples of 2 or 5 from 1 to 500

= List of multiple of 2 from 1 to 500 + List of multiple of 5 from 1 to 500 – List of multiple of 10 from 1 to 500

$= (2, 4, 6, \ldots, 500) + (5, 10, 15, \ldots, 500) - (10, 20, \ldots, 500) \quad \ldots(i)$

All of these list form an AP.

Now, number of terms in first list,

$500 = 2 + (n_1 - 1)2 \Rightarrow 498 = (n_1 - 1)2$

$\Rightarrow \quad n_1 - 1 = 249 \Rightarrow n_1 = 250$

Number of terms in second list,

$500 = 5 + (n_2 - 1)5 \Rightarrow 495 = (n_2 - 1)5$

$\Rightarrow \quad 99 = (n_2 - 1) \Rightarrow n_2 = 100$

and number of terms in third list,

$500 = 10 + (n_3 - 1)10 \Rightarrow 490 = (n_3 - 1)10$

$\Rightarrow \quad n_3 - 1 = 49 \Rightarrow n_3 = 50$

From Eq. (i), Sum of multiples of 2 or 5 from 1 to 500

$= \text{Sum of } (2, 4, 6, \ldots, 500) + \text{Sum of } (5, 10, \ldots, 500) - \text{Sum of } (10, 20, \ldots, 500)$

$= \frac{n_1}{2}[2 + 500] + \frac{n_2}{2}[5 + 500] - \frac{n_3}{2}[10 + 500] \quad \left[\because S_n = \frac{n}{2}(a + l)\right]$

$= \frac{250}{2} \times 502 + \frac{100}{2} \times 505 - \frac{50}{2} \times 510$

$= 250 \times 251 + 505 \times 50 - 25 \times 510 = 62750 + 25250 - 12750$

$= 88000 - 12750 = 75250$

Question 3:

The eighth term of an AP is half its second term and the eleventh term exceeds one-third of its fourth term by 1. Find the 15th term.

Solution:

Let a and d be the first term and common difference of an AP respectively.

Now, by given condition, $a_8 = \frac{1}{2} a_2$

$\Rightarrow$ $a + 7d = \frac{1}{2}(a + d)$ $[\because a_n = a + (n-1)d]$

$\Rightarrow$ $2a + 14d = a + d$

$\Rightarrow$ $a + 13d = 0$ ...(i)

and $a_{11} = \frac{1}{3} a_4 + 1$

$\Rightarrow$ $a + 10d = \frac{1}{3}[a + 3d] + 1$

$\Rightarrow$ $3a + 30d = a + 3d + 3$

$\Rightarrow$ $2a + 27d = 3$ ...(ii)

From Eqs. (i) and (ii),

$2(-13d) + 27d = 3$

$\Rightarrow$ $-26d + 27d = 3$

$\Rightarrow$ $d = 3$

From Eq. (i),

$a + 13(3) = 0$

$\Rightarrow$ $a = -39$

$\therefore$ $a_{15} = a + 14d = -39 + 14(3)$

$= -39 + 42 = 3$

Question 4:

An AP consists of 37 terms. The sum of the three middle most terms and the sum of the last three terms is 429. Find the AP.

Solution:

Since, total number of terms [Odd]

$\therefore$ Middle term $= \left(\frac{37+1}{2}\right)$th term = 19th term

So, the three middle most terms = 18th, 19th and 20th.

By given condition,

Sum of the three middle most terms = 225

$a_{18} + a_{19} + a_{20} = 225$

$\Rightarrow$ $(a + 17d) + (a + 18d) + (a + 19d) = 225$

$\Rightarrow$ $3a + 54d = 225$

$\Rightarrow$ $a + 18d = 75$ ...(i)

and sum of the last three terms = 429

$\Rightarrow$ $a_{35} + a_{36} + a_{37} = 429$

$\Rightarrow$ $(a + 34d) + (a + 35d) + (a + 36d) = 429$

$\Rightarrow$ $3a + 105d = 429$

$\Rightarrow$ $a + 35d = 143$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$17d = 68$

$\Rightarrow$ $d = 4$

From Eq. (i), $a + 18(4) = 75$

$\Rightarrow$ $a = 75 - 72$

$\Rightarrow$ $a = 3$

$\therefore$ Required AP is $a, a + d, a + 2, a + 3d, \ldots$

i.e., $3, 3 + 4, 3 + 2(4), 3 + 3(4), \ldots$

i.e., $3, 7, 3 + 8, 3 + 12, \ldots$

i.e., $3, 7, 11, 15, \ldots$

Question 5:

Find the sum of the integers between 100 and that are

(i) divisible by 9. (ii) not divisible by 9.

Solution:

(i) The numbers (integers) between 100 and 200 which is divisible by 9 are 108, 117, 126, …

198.

Let n be the number of terms between 100 and 200 which is divisible by 9.

Here, $a = 108, d = 117 - 108 = 9$ and $a_n = l = 198$

$\because \quad a_n = l = a + (n-1)d$

$\Rightarrow \quad 198 = 108 + (n-1)9$

$\Rightarrow \quad 90 = (n-1)9$

$\Rightarrow \quad n - 1 = 10$

$\Rightarrow \quad n = 11$

$\therefore$ Sum of terms between 100 and 200 which is divisible by 9,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad S_{11} = \frac{11}{2}[2(108) + (11-1)9] = \frac{11}{2}[216 + 90]$$

$$= \frac{11}{2} \times 306 = 11 \times 153 = 1683$$

Hence, required sum of the integers between 100 and 200 that are divisible by 9 is 1683.

(ii) The sum of the integers between 100 and 200 which is not divisible by 9 = (sum of total numbers between 100 and 200) – (sum of total numbers between 100 and 200 which is divisible by 9).

Here, $a = 101, d = 102 - 101 = 1$ and $a_n = l = 199$

$\because \quad a_n = l = a + (n-1)d$

$\Rightarrow \quad 199 = 101 + (n-1)1$

$\Rightarrow \quad (n-1) = 98 \Rightarrow n = 99$

$\therefore$ Sum of terms between 100 and 200,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad S_{99} = \frac{99}{2}[2(101) + (99-1)1] = \frac{99}{2}[202 + 98]$$

$$= \frac{99}{2} \times 300 = 99 \times 150 = 14850$$

From Eq. (i), sum of the integers between 100 and 200 which is not divisible by 9

$$= 14850 - 1683 \quad \text{[from part (i)]}$$

$$= 13167$$

Hence, the required sum is 13167.

Question 6:

The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

Solution:

Let a arid d be the first term and common difference of an AP

Given that, $\quad a_{11} : a_{18} = 2 : 3$

$$\Rightarrow \quad \frac{a + 10d}{a + 17d} = \frac{2}{3}$$

$$\Rightarrow \quad 3a + 30d = 2a + 34d$$

$$\Rightarrow \quad a = 4d \qquad \ldots(i)$$

Now, $a_5 = a + 4d = 4d + 4d = 8d$ [from Eq. (i)]

and $a_{21} = a + 20d = 4d + 20d = 24d$ [from Eq. (i)]

$\therefore$ $a_5 : a_{21} = 8d : 24d = 1 : 3$

Now, sum of the first five terms, $S_5 = \frac{5}{2}[2a + (5-1)d]$

$$= \frac{5}{2}[2(4d) + 4d] \quad \text{[from Eq. (i)]}$$

$$= \frac{5}{2}(8d + 4d) = \frac{5}{2} \times 12d = 30d$$

and sum of the first 21 terms, $S_{21} = \frac{21}{2}[2a + (21-1)d]$

$$= \frac{21}{2}[2(4d) + 20d] \quad \text{[from Eq. (i)]}$$

$$= \frac{21}{2}(28d) = 294d$$

So, ratio of the sum of the first five terms to the sum of the first 21 terms

S5 :S21 = 30 d :294 d = 5:49

Question 7:

Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Solution:

Given that, the AP is a, b………..c

Here, first term = a, common difference = b – a

and last term, $l = a_n = c$

$\because$ $a_n = l = a + (n-1)d$

$\Rightarrow$ $c = a + (n-1)(b-a)$

$\Rightarrow$ $(n-1) = \frac{c-a}{b-a}$

$\Rightarrow$ $n = \frac{c-a}{b-a} + 1$

$\Rightarrow$ $n = \frac{c-a+b-a}{b-a} = \frac{c+b-2a}{b-a}$ …(i)

$\therefore$ Sum of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{(b+c-2a)}{2(b-a)}\left[2a + \left\{\frac{b+c-2a}{b-a} - 1\right\}(b-a)\right]$$

$$= \frac{(b+c-2a)}{2(b-a)}\left[2a + \frac{c-a}{b-a}\cdot(b-a)\right]$$

$$= \frac{(b+c-2a)}{2(b-a)}(2a + c - a)$$

$$= \frac{(b+c-2a)}{2(b-a)}\cdot(a+c)$$

Hence proved.

Question 8:

Solve the equation – 4 + (-1) + 2 +… + x = 437.

Solution:

Given equation is, – 4 -1 + 2 + … + x =

437 …(i)

Here, – 4-1 + 2 + …+ x forms an AP with first term =-4, common difference =3,

$a_n = l = x$

$\because$ nth term of an AP, $a_n = l = a + (n-1)d$

$\Rightarrow \quad x = -4 + (n-1)3$

$\Rightarrow \quad \frac{x+4}{3} = n - 1 \Rightarrow n = \frac{x+7}{3}$ ...(ii)

$\therefore$ Sum of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_n = \frac{x+7}{2 \times 3}\left[2(-4) + \left(\frac{x+4}{3}\right) \cdot 3\right]$$

$$= \frac{x+7}{2 \times 3}(-8 + x + 4) = \frac{(x+7)(x-4)}{2 \times 3}$$

From Eq. (i),

$$S_n = 437$$

$\Rightarrow \quad \frac{(x+7)(x-4)}{2 \times 3} = 437$

$\Rightarrow \quad x^2 + 7x - 4x - 28 = 874 \times 3$

$\Rightarrow \quad x^2 + 3x - 2650 = 0$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2650)}}{2}$$ [by quadratic formula]

$$= \frac{-3 \pm \sqrt{9 + 10600}}{2}$$

$$= \frac{-3 \pm \sqrt{10609}}{2} = \frac{-3 \pm 103}{2} = \frac{100}{2}, \frac{-106}{2}$$

$$= 50, -53$$

Here, x cannot be negative, i.e., x ≠- 53

also, for x = -53, n will be negatuve which is not possible

Hence, the required value of x is 50.

Question 9:

Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first installment of ₹ 1000. If he increases the installment by ₹ 100 every month, what amount will be paid by him in the 30th installment? What amount of loan does he still have to pay after the 30th installment?

Solution:

Given that,

Jaspal singh takes total loan = ₹ 118000 He repays his total loan by paying every month.

His first installment = ₹ 1000

Second installment = 1000 + 100 = ₹ 1100

Third installment = 1100 + 100 = ₹ 1200 and so on

Question 10:

The students of a school decided to beautify the school on the annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags.

Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance she did cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

Solution:

Given that, the students of a school decided to beautify the school oh the annua] day by fixing colourful flags on the straight passage of the school.

Given that, the number of flags = 27 and distance between each flag = 2 m.

Also, the flags are stored at the position of the middle most flag i.e., 14th flag and Ruchi wds given the responsibility of placing the flags. Ruchi kept her books, where the flags were stored i.e., 14th flag and she could carry only one flag at a time.

Let she placed 13 flags into her left position from middle most flag i.e., 14th flag. For placing second flag and return his initial position distance travelled = 2+ 2= 4 m.

Similarly, for placing third flag and return his initial position, distance travelled =t 4+ 4= 8 m

For placing fourth flag and return his initial position, distance travelled = 6+ 6= 12 m.

For placing fourteenth flag and return his initial position, distance travelled ,

= 26+ 26= 52 m

Proceed same manner into her right position from middle most flag

i.e., 14th flag. Total distance travelled in that case = 52 m

Also, when Ruchi placed the last flag she return his middle most position and collect her books. This distance also included in placed the last flag.

So, these distances form a series.

4+ 8+ 12 + 16+ …+ 52 [for left]

and 4+8+12 +16+… + 52 [for right]

$\therefore$ Total distance covered by Ruchi for placing these flags

$$= 2 \times (4 + 8 + 12 + \ldots + 52)$$

$$= 2 \times \left[\frac{13}{2}\{2 \times 4 + (13 - 1) \times (8 - 4)\}\right] \qquad \left\{\frac{\because \text{ Sum of } n \text{ terms of an AP}}{S_n = \dfrac{n}{2}\ [2a + (n-1)d}\right\}$$

$$= 2 \times \left[\frac{13}{2}(8 + 12 \times 4)\right]$$

[$\because$ both sides of Ruchi number of flags *i.e.*, $n = 13$]

$$= 2 \times [13(4 + 12 \times 2)] = 2 \times 13(4 + 24)$$

$$= 2 \times 13 \times 28 = 728 \text{ m}$$

Hence, the required is 728 m in which she did cover in completing this job and returning back to collect her books.

Now, the maximum distance she travelled carrying a flag = Distance travelled by Ruchi during placing the 14th flag in her left position or 27th flag in her right position

= (2 + 2 + 2 + … + 13 times)

= 2 x 13 = 26m

Hence, the required maximum distance she travelled carrying a flag is 26 m.