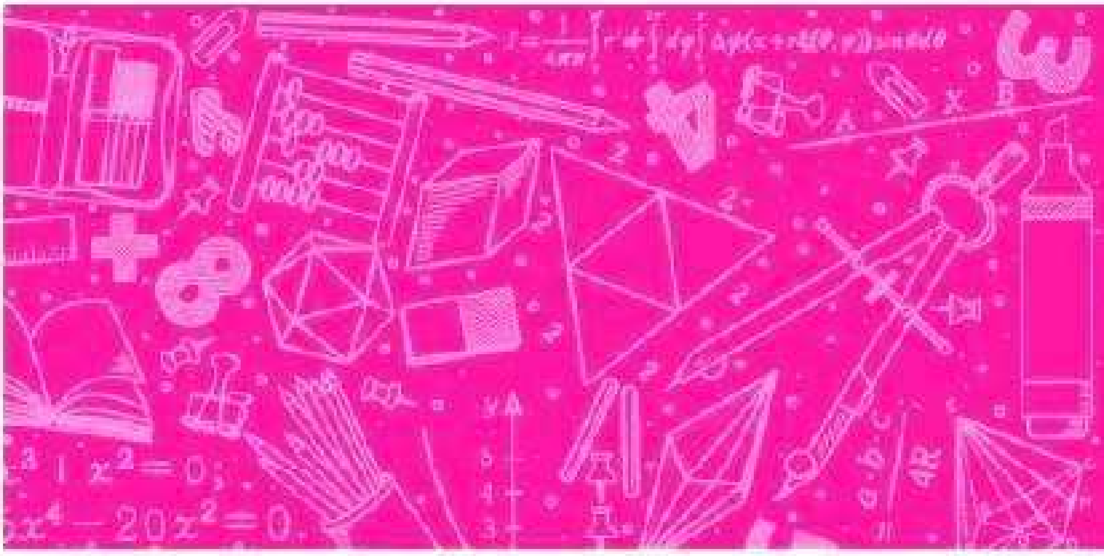

C
DIF

BASIC CONCEPTS

1. **Continuity and Discontin**
point a of its domain if

$$\lim_{x \to a^-} f(x), \ \lim_{x \to a^+} f(x), \ f(a)$$

A function $f(x)$ is said to b

2. **Properties of Continuous**

(ii) $\dfrac{d}{dx}\cos^{-1}x = \dfrac{-1}{\sqrt{1-x^2}}$

(iii) $\dfrac{d}{dx}\tan^{-1}x = \dfrac{1}{1+x^2}$

(iv) $\dfrac{d}{dx}\cot^{-1}x = \dfrac{-1}{1+x^2}$

(v) $\dfrac{d}{dx}\sec^{-1}x = \dfrac{1}{x\sqrt{x^2-1}}$

(vi) $\dfrac{d}{dx}\operatorname{cosec}^{-1}x = \dfrac{-1}{x\sqrt{x^2-}}$

5. Chain Rule: Chain rule is

if y is a function of x, then

6. Parametric Form: Someti

terms of another variable

form and t is called the pa

(iv) Limits at infinity: This

Procedure to solve th

(a) Write the given e

(b) Divide the numer

(c) Use the result $\lim_{x \to}$

(d) Simplify and get

MULTIPLE CHOICE QU

Choose and write the correct

1. The function $f : R \to R$

(a) continuous as well

(b) not continuous but

(c) continuous but not

(d) neither continuous

9. The function $f(x) = \frac{x}{x(x}$

(a) exactly one point

(c) exactly three points

10. If $f(x) = x^2 \sin\frac{1}{x}$, wher

continuous at $x = 0$, is

(a) 0

11. The function $f(x) = |x|$

(a) continuous at $x = 0$

(c) discontinuous at $x =$

12. The function $f(x) = \frac{4 -}{4x}$

(a) discontinuous at on

(c) discontinuous at ex

13. The set of points wher

(a) R

22. **If $f(x) = x - 3$ and $g(x)$**

(*a*) $f(x) + g(x)$

23. **The set of points where**

(*a*) R

24. **Let $f(x) = \begin{cases} \cos[x], & x \geq \\ |x| + a, & x < \end{cases}$**

is equal to

(*a*) 1

25. **Let $f(x) = \begin{cases} x[x-1], \\ [x](x-1), \end{cases}$**

(*a*) $f(x)$ is continuous ev

(*b*) $f(x)$ is not continuou

(*c*) $f(x)$ is not differenti

(*d*) none of these

32. **The function $f(x) = \begin{cases} \text{si} \\ \end{cases}$**

(*a*) is continuous.

(*c*) has jump discontinu

33. **The function $f(x) = \begin{cases} 2 \\ 2 \end{cases}$**

(*a*) is continuous.

(*c*) has jump discontinu

34. **The function $f(x) = \begin{cases} \frac{\text{si}}{} \\ \end{cases}$**

(*a*) is continuous.

(*c*) has jump discontinu

35. **If $f(x) = |x| + |x-2|$, th**

(*a*) $f(x)$ is continuous at

42. If $f(x) = x^n$, then the val

$$f(1) + \frac{f(1)}{1!} + \frac{f^2(1)}{2!} + \frac{f}{}$$

is the r^{th} derivative of f

(*a*) 1

43. If $f(x)$, $g(x)$, $h(x)$ are poly

$$F(x) = \begin{vmatrix} f(x) & g(x) & h \\ f'(x) & g'(x) & h \\ f''(x) & g''(x) & h \end{vmatrix}$$

(*a*) -1

44. The derivative of $f(\tan$

(*a*) $\sqrt{2}$

45. If g is inverse function

(*a*) $\sin(g(x))$

(c) differentiable at $x =$

(d) differentiable at $x =$

54. Let $f(x) = x - |x|$ then $f($

(a) differentiable $\forall\, x \in$

(b) continuous $\forall\, x \in \mathbb{R}$

(c) neither continuous

(d) discontinuous at $x =$

55. Let $f(x) = \begin{cases} -1, & -2 \\ x^2 - 1, & 0 \end{cases}$

then the number of poi

(a) at most one point (

56. Let $f(x)$ be differentiabl

$$f\left(\frac{x+y}{1-xy}\right) = f(x) + f(y)$$

If $\lim_{x \to 0} \frac{f(x)}{x} = \frac{1}{3}$ then $f($

62. **Let** $f(x) = x^{3/2} - \sqrt{x^3 + x^2}$

(a) LHD at $x = 0$ exists

(b) $f(x)$ is differentiable

(c) RHD at $x = 0$ exists

(d) None of these

63. **Number of points at w**

(a) 2

64. **If** $f(x) = \dfrac{\sin 4\pi [\pi^2 x]}{7 + [x]^2}, [$

(a) continuous for all x

(b) discontinuous at so

(c) $f''(x)$ exists for all x

(d) $f'(\mathrm{x})$ exists but $f''(x)$

65. **The function** $f(x) = \{ \mathbf{si}$

70. Let $f(x) = [x]^2 + \sqrt{\{x\}}$, w
part functions, then

(*a*) $f(x)$ is continuous at

(*b*) $f(x)$ is non differenti

(*c*) $f(x)$ is discontinuou

(*d*) $f(x)$ is continuous ar

71. Find the value of *a* if th

$$f(x) = \begin{cases} 2x - 1, & x < \\ a, & x = \\ x + 1, & x > \end{cases}$$

(*a*) 3

72. If the function $f(x)$ defi

$$f(x) = \begin{cases} \dfrac{\log(1 + ax) - \mathrm{l}}{x} \\ k \end{cases}$$

(*a*) *a*

(c) 0

80. If $y = x^x$ find $\frac{d^2y}{dx^2}$.

(a) $x^x\left\{(1+\log x)^2 - \frac{1}{x}\right\}$

(c) 0

81. Find $\frac{d^2y}{dx^2}$, if $x = at^2$, y

(a) $\frac{-1}{2at^3}$

82. Determine the value of

(a) $\frac{3}{2}$

Answers

1. (c) 2. (a)

7. (a) 8. (b)

(ii) Will the slope vary

(a) Yes

(c) may or may not

(iii) What is $\frac{dy}{dx}$ at $x =$

(a) 2

(c) Function is not

(iv) When the x value li

(a) $2x - 5$

(v) If the potter is tryi
Why?

(a) Yes, because it i

(b) Yes, because it i

(c) No , because it i

(d) No , because it i

Sol. (i) We have, $f(x) = |x$

∴ Option (c) is co

(v) We have the functi

It is not a continuo

∴ Option (d) is co

ASSERTION-REASON

HINTS/S

1. We have,

$$f(x) = -|x-1| = \Bigg\{$$

At $x = 1$

LHL $= \lim_{h \to 0} f(1-h) = \text{li}$

RHL $= \lim_{h \to 0} f(1+h) = \text{li}$

$f(1) = 1 - 1 = 0$

$\therefore$ LHL = RHL = $f(0)$

Now, at $x = 1$

$$\text{LHD} = \frac{d}{dx}(x -$$

$$\text{LHD} \neq \text{RHD}$$

$\therefore$ $f(x)$ is not differentia

$\therefore \quad f(x) = |\sin x|$ is not c

$\therefore$ Option (b) is correct.

10. $\because f(x) = x^2 \sin\left(\frac{1}{x}\right)$, whe

Hence, value of the fun

$\therefore$ Option (a) is correct.

12. $f(x) = \dfrac{4 - x^2}{4x - x^3}$

$f(x)$ is discontinuous wh

i.e., $x(4 - x^2) = 0$ i.e., $x(2$

Hence $f(x)$ is discontinu

$\therefore$ Option (c) is correct.

13. $f(x) = |x - 3| \cos x = g(x$

$h(x) = \cos x$ is differenti

But $g(x) = |x - 3|$ is diff

$$\Rightarrow \quad \frac{dy}{dx} = \sec^2 x \text{ and } \frac{dt}{dx}$$

Now, derivative of tan

$\therefore$ Option (d) is correct.

22. $$\because \quad \frac{g(x)}{f(x)} = \frac{\frac{x^2}{3}+1}{x-3} = \frac{x^2}{3(x}$$

$\Rightarrow$ Function $\frac{g(x)}{f(x)}$ is c

$\therefore$ Option (d) is correct.

24. $\because \lim_{x \to 0} f(x)$ exists $\Rightarrow$ li

$$\Rightarrow \quad \lim_{h \to 0} f(0-h) = \lim_{h \to 0}$$

$$\Rightarrow \quad \lim_{h \to 0} f(-h) = \lim_{h \to 0} f$$

$$\Rightarrow \quad \lim\{|-h| + a\} = \lim$$

29. Given expression is

$$x = e^{y + e^{y + \cdots\cdots \text{ to } \infty}}$$

Taking log on both side

$$\log x = \log e^{y+x}$$

Differentiating, w.r.t. x,

$$\frac{1}{x} = \frac{dy}{dx} + 1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-x}{x}$$

$\therefore$ Option (c) is correct.

32. $$f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \sin\left(\frac{1}{x}\right)$$

None of the options (a),

$\therefore$ Option (d) is correct

42. $f(x) = x^n$

$$f(1) + \frac{f^1(1)}{\underline{|1}} + \frac{f^2(1)}{\underline{|2}} + $$

$$= 1 + \frac{n}{\underline{|1}} + \frac{n(n-1)}{\underline{|2}} + \frac{n(}{}$$

$= (1+1)^n = 2^n$ [By using

Hence option (c) is corre

43. $$F(x) = \begin{vmatrix} f(x) & g(x) & h \\ f'(x) & g'(x) & h \\ f''(x) & g''(x) & h \end{vmatrix}$$

where $f(x)$, $g(x)$, $h(x)$ are

$\therefore \quad f'''(x) = 0 = g'''(x) = h$

$$= \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow \quad f'(x) = \begin{vmatrix} 3x^2 & \cos x \\ 6 & -1 \\ p & p^2 \end{vmatrix}$$

$$\Rightarrow \quad f''(x) = \begin{vmatrix} 6x & -\sin x \\ 6 & -1 \\ p & p^2 \end{vmatrix}$$

$$= \begin{vmatrix} 6x & -\sin x & -\cos \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\text{Similarly, } f'''(x) = \begin{vmatrix} 6 & - \\ 6 & \\ p & \end{vmatrix}$$

56. We have, $f\left(\frac{x+y}{1-xy}\right)=f($

Since it is of the form ta

Let $f(x)=A\tan^{-1}x$

$$\lim_{x\to 0}\frac{f(x)}{x}=\lim_{x\to 0}\frac{A\tan}{x}$$

$$\Rightarrow A\lim_{x\to 0}\frac{\tan^{-1}x}{x}=\frac{1}{3}$$

$$\therefore f(x)=\frac{1}{3}\tan^{-1}x$$

$$\Rightarrow f(1)=\frac{1}{3}\tan^{-1}(1)=\frac{1}{3}$$

$\therefore$ Option (b) is correct.

57. LHL $=\lim\ f(x)=\lim$

$\therefore$ Option (c) is correct.

62. $f(x) = x^{3/2} - \sqrt{x^3 + x^2}$

Domain of $f = [0, \infty)$

So, LHD at $x = 0$ does n

$\therefore$ Option (c) is correct.

63. $f(x) = \dfrac{1}{\log|x|}$

$f(x)$ is not defined for x

$\therefore f(x)$ is not continuous

$\therefore$ Option (b) is correct.

64. We have $f(x) = \dfrac{\sin 4\pi [\pi}{7 + [x]}$

We know that $[\pi^2 x]$ is a

$\therefore 4\pi[\pi^2 x]$ is an integral

69.

Given $g(x) = \begin{cases} e^{2x}, & x \ldots \\ e^{-2x}, & x \ldots \end{cases}$

$$g'(x) = \begin{cases} 2e^{2x}, & x < 0 \\ -2e^{-2x}, & x \geq 0 \end{cases}$$

$\therefore$ LHD at $x = 0$, $g'(0) =$

$$\frac{dy}{dx} = \frac{(1 + \log x) \times 1 - x\Big(0}{(1 + \log x)^2}$$

$\therefore$ Option (a) is correct.

75. $x^y = y^x \Rightarrow y \log x = x \log$

$$\frac{y}{x} + \log x \frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log$$

$$\Rightarrow \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \Big(\log$$

$$\Rightarrow \frac{dy}{dx} = \frac{x \log y - y}{y \log x - x} \times \frac{y}{x}$$

$\therefore$ Option (b) is correct.

76. Let $y = \sqrt{\sin x + \sqrt{\sin x +}}$

$\Rightarrow y = \sqrt{\sin x + y}$ should

$\Rightarrow y^2 = \sin x + y \Rightarrow y^2 -$