

Continuity and Differentiability

Question 1.

The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x = \frac{\pi}{4}$, where $f(1) = 2$ and $g'(\sqrt{2}) = 4$, is

- (a) $\frac{1}{\sqrt{2}}$
- (b) $\sqrt{2}$
- (c) 1
- (d) 0

Answer:

- (a) $\frac{1}{\sqrt{2}}$

Question 2.

The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to

$\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

- (a) 1
- (b) $\frac{3}{2}$
- (c) $\frac{2}{3}$
- (d) $\frac{1}{2}$

Answer:

- (c) $\frac{2}{3}$

Question 3.

The derivative of

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is

- (a) 0 (b) 1
(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

Answer:

(b) 1

Question 4.

img src="https://live.staticflickr.com/65535/50354653758_a00e3fc2ee_o.png" width="374" height="162" alt="Maths MCQs for Class 12 with Answers Chapter 5 Continuity and Differentiability Q34">

Answer:

(c) $\frac{5}{16t^6}$

Question 5.

If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

- (a) n^2y (b) $-n^2y$
(c) $-y$ (d) $2x^2y$

Answer:

(a) n^2y

Question 6.

If $x = a \sin \theta$ and $y = b \cos \theta$, then $\frac{d^2y}{dx^2}$ is equal to

- (a) $\frac{a}{b^2} \sec^2 \theta$ (b) $\frac{b}{a} \sec^2 \theta$
(c) $\frac{b}{a^2} \sec^3 \theta$ (d) $-\frac{b}{a^2} \sec^3 \theta$

Answer:

(d) $-\frac{b}{a^2} \sec^3 \theta$

Question 7.

If $y = a^x, b^{2x-1}$, then $\frac{d^2 y}{dx^2}$ is

(a) $y^2 \cdot \log ab^2$ (b) $y \cdot \log ab^2$

(c) $y \cdot (\log ab^2)^2$ (d) $y \cdot (\log a^2 b)^2$

Answer:

(c) $y \cdot (\log ab^2)^2$

Question 8.

If $y = \frac{\ln x}{x}$, then the value of $y''(e)$ is

(a) 1 (b) $-\frac{1}{e}$

(c) $-\frac{1}{e^2}$ (d) $-\frac{1}{e^3}$

Answer:

(d) $-\frac{1}{e^3}$

Question 9.

If $x = a(\cos \theta + \theta \sin \theta)$, y

$= a(\sin \theta - \theta \cos \theta)$, then $\frac{d^2 y}{dx^2} =$

(a) $\frac{\sec^3 \theta}{a\theta}$ (b) $\frac{\sec^2 \theta}{\theta}$

(c) $a\theta \cos^3 \theta$ (d) $\frac{\sec^2 \theta}{a}$

Answer:

(a) $\frac{\sec^3 \theta}{a\theta}$

Question 10.

If $y^2 = ax^2 + bx + c$, then $\frac{d}{dx}(y^3 y_z) =$

- (a) 1 (b) -1
(c) $\frac{4ac - b^2}{a^2}$ (d) 0

Answer:

(d) 0

Question 11.

If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then the value of $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

- (a) $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\frac{\pi}{6}}$
(c) $\frac{1}{\sqrt{6}}$ (d) $\frac{\pi}{\sqrt{6}}$

Answer:

(b) $-\sqrt{\frac{\pi}{6}}$

Question 12.

If $\sqrt{x+y} + \sqrt{y-x} = a$, then $\frac{dy}{dx} =$

- (a) $\frac{\sqrt{x+y} - \sqrt{y-x}}{\sqrt{y-x} + \sqrt{x+y}}$ (b) $\frac{2\sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}}$
(c) $\frac{x+y+\sqrt{xy}}{\sqrt{x+y}}$ (d) $\frac{x^2+y^2+2xy}{x^2+y^2}$

Answer:

(a) $\frac{\sqrt{x+y} - \sqrt{y-x}}{\sqrt{y-x} + \sqrt{x+y}}$

Question 13.

If $xy^2 = ax^2 + bxy + y^2$, then find $\frac{dy}{dx}$

- (a) $\frac{2ax + by + y^2}{2xy + bx + 2y}$ (b) $\frac{2ax + by - y^2}{2xy - bx - 2y}$
- (c) $\frac{ax + by - xy}{xy + x^2 + y^2}$ (d) $\frac{2x^2 + axy + y^2}{x^2 + y^2 + 2xy}$

Answer:

(b) $\frac{2ax + by - y^2}{2xy - bx - 2y}$

Question 14.

If $y = \tan^{-1} \left[\frac{\sin x + \cos x}{\cos x - \sin x} \right]$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{1}{2}$ (b) $\frac{\pi}{4}$
- (c) 0 (d) 1

Answer:

(d) 1

Question 15.

The differential coefficient of $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$ is

- (a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$
- (c) $\frac{1}{2\sqrt{1-x^2}}$ (d) x

Answer:

(c) $\frac{1}{2\sqrt{1-x^2}}$

Question 16.

If $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $0 \leq x < \frac{\pi}{2}$, then $f'\left(\frac{\pi}{6}\right)$ is

- (a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Answer:

(d) $\frac{1}{2}$

Question 17.

$\frac{d}{dx}\left\{\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)\right\}$ is equal to

- (a) $-\frac{2}{1+x^2}, x \neq 0$ (b) $\frac{2(1+x)}{1+x^2}, x \neq 0$
(c) $\frac{2(1-x^2)}{(1+x^2)|1-x^2|}, x \neq \pm 1, 0$
(d) None of these

Answer:

(c) $\frac{2(1-x^2)}{(1+x^2)|1-x^2|}, x \neq \pm 1, 0$

Question 18.

If $y = \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$, $x > 0$, then $\frac{dy}{dx}$ is
equal to

- (a) 1 (b) 0
(c) $\frac{\pi}{2}$ (d) None of these

Answer:

(b) 0

Question 19.

If $y = e^{\frac{1}{2} \log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{1}{2} \sec^2 x$ (b) $\sec^2 x$
(c) $\sec x \tan x$ (d) $e^{\frac{1}{2} \log(1+\tan^2 x)}$

Answer:

(c) $\sec x \tan x$

Question 20.

If $y = e^{3x+7}$, then the value of $\left. \frac{dy}{dx} \right|_{x=0}$ is

- (a) 1 (b) 0
(c) -1 (d) $3e^7$

Answer:

(d) $3e^7$

Question 21.

If $x^2 + y^2 = 1$, then

- (a) $yy'' - (2y')^2 + 1 = 0$
(b) $yy'' + (y')^2 + 1 = 0$
(c) $yy'' - (y')^2 - 1 = 0$
(d) $yy'' + (2y')^2 + 1 = 0$

Answer:

(b) $yy'' + (y')^2 + 1 = 0$

Question 22.

If $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$, then $\frac{d^2y}{dx^2}$ is

- (a) $-3\sqrt{1-y^2}$ (b) $9y$
(c) $-9y$ (d) $3\sqrt{1-y^2}$

Answer:

(c) $-9y$

Question 23.

The value of c in Rolle's theorem for the function, $f(x) = \sin 2x$ in $[0, \frac{\pi}{2}]$ is

- (a) $\frac{\pi}{2}$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{3}$
- (d) $\frac{\pi}{6}$

Answer:

- (b) $\frac{\pi}{4}$

Question 24.

The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x$, $x \in [0, \pi]$ is

- (a) $\frac{\pi}{6}$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{2}$
- (d) $\frac{3\pi}{4}$

Answer:

- (d) $\frac{3\pi}{4}$

Question 25.

A value of c for which the Mean value theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

- (a) $2\log_3 e$
- (b) $\frac{1}{2} \log_e 3$
- (c) $\log_3 e$
- (d) $\log_e 3$

Answer:

- (a) $2\log_3 e$

Question 26.

The value of c in mean value theorem for the function $f(x) = (x - 3)(x - 6)(x - 9)$ in $[3, 5]$ is

- (a) $6 \pm \sqrt{13/3}$
- (b) $6 + \sqrt{13/3}$
- (c) $6 - \sqrt{13/3}$
- (d) None of these

Answer:

- (c) $6 - \sqrt{13/3}$

Question 27.

The value of c in Mean value theorem for the function $f(x) = x(x - 2)$, $x \in [1, 2]$ is

- (a) $\frac{3}{2}$
- (b) $\frac{2}{3}$
- (c) $\frac{1}{2}$
- (d) $\frac{5}{2}$

Answer:

- (a) $\frac{3}{2}$

Question 28.

Let $f(x) = \frac{\ln(1+ax) - \ln(1-bx)}{x}$, $x \neq 0$. If $f(x)$ is continuous at $x = 0$, then $f(0) =$

- (a) $a - b$
- (b) $a + b$
- (c) $b - a$
- (d) $\ln a + \ln b$

Answer:

- (b) $\ln a + \ln b$

Question 29.

If $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & x > 0 \end{cases}$ is continuous at $x =$

0, then $a =$

- (a) 4
- (b) 6
- (c) 8
- (d) none of these

Answer:

- (c) 8

Question 30.

The number of discontinuous functions $y(x)$ on $[-2, 2]$ satisfying $x^2 + y^2 = 4$ is

- (a) 0
- (b) 1
- (c) 2
- (d) > 2

Answer:

- (a) 0

Question 31.

Let $f(x) = \frac{1 - \tan x}{4x - \pi}$, $x \neq \frac{\pi}{4}$, $x \in \left(0, \frac{\pi}{2}\right)$.

If $f(x)$ is continuous in $\left(0, \frac{\pi}{2}\right)$, then $f\left(\frac{\pi}{4}\right) =$

- (a) 1 (b) $\frac{1}{2}$
(c) $-\frac{1}{2}$ (d) -1

Answer:

(c) $-\frac{1}{2}$

Question 32.

If $f(x) = \frac{\sqrt{4+x} - 2}{x}$, $x \neq 0$ be continuous at $x = 0$, then

$f(0) =$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) 2 (d) $\frac{3}{2}$

Answer:

(b) $\frac{1}{4}$

Question 33.

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx} =$

- (a) $\frac{x+1}{x}$ (b) $\frac{1}{1+x}$
(c) $\frac{-1}{(1+x)^2}$ (d) $\frac{x}{1+x}$

Answer:

(c) $\frac{-1}{(1+x)^2}$

Question 34.

If $y = (1 + x)(1 + x^2)(1 + x^4) \dots (1 + x^{2^n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is

- (a) 0
- (b) -1
- (c) 1
- (d) None of these

Answer:

- (c) 1

Question 35.

If $f(x) = -\sqrt{25 - x^2}$, then $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$ is equal to

- (a) $\frac{1}{24}$
- (b) $\frac{1}{5}$
- (c) $-\sqrt{24}$
- (d) $\frac{1}{\sqrt{24}}$

Answer:

- (d) $\frac{1}{\sqrt{24}}$

Question 36.

If $y = ax^2 + b$, then $\frac{dy}{dx}$ at $x = 2$ is equal to

- (a) $4a$
- (b) $3a$
- (c) $2a$
- (d) None of these

Answer:

- (a) $4a$

Question 37.

If $\sec\left(\frac{x^2 - 2x}{x^2 + 1}\right) = y$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y^2}{x^2}$

(b) $\frac{2y\sqrt{y^2 - 1}(x^2 + x - 1)}{(x^2 + 1)^2}$

(c) $\frac{(x^2 + x - 1)}{y(y^2 - 1)}$

(d) $\frac{x^2 - y^2}{x^2 + y^2}$

Answer:

(b) $\frac{2y\sqrt{y^2 - 1}(x^2 + x - 1)}{(x^2 + 1)^2}$

Question 38.

If $f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1} \frac{4x}{4 - x^2}$,

then $f'(2)$ is equal to

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 1

(d) -1

Answer:

(a) $\frac{1}{2}$

Question 39.

If $y = \log_{10} x + \log_e y$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y}{y-1}$

(b) $\frac{y}{x}$

(c) $\frac{\log_{10} e}{x} \left(\frac{y}{y-1} \right)$

(d) None of these

Answer:

(c) $\frac{\log_{10} e}{x} \left(\frac{y}{y-1} \right)$

Question 40.

If $y = \log \left[e^x \left(\frac{x-1}{x+2} \right)^{1/2} \right]$, then $\frac{dy}{dx}$ is equal to

- (a) 7 (b) $\frac{3}{x-2}$
(c) $\frac{3}{(x-1)}$ (d) None of these

Answer:

(d) None of these

Question 41.

If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{x+y}{xy}$ (b) xy
(c) $\frac{x}{y}$ (d) $\frac{y}{x}$

Answer:

(d) $\frac{y}{x}$

Question 42.

If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$ with $c = (2 + \frac{1}{\sqrt{3}})$, find the value of a and b.

- (a) $a = 11, b = -6$
(b) $a = 10, b = 6$
(c) $a = -11, b = 6$
(d) $a = 11, b = 6$

Answer:

(a) $a = 11, b = -6$

Question 43.

If $y = (\tan x)^{\sin x}$, then $\frac{dy}{dx}$ is equal to

- (a) $\sec x + \cos x$
(b) $\sec x + \log \tan x$
(c) $(\tan x)^{\sin x}$
(d) None of these

Answer:

(d) None of these

Question 44.

If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

(a) $\frac{1+x}{1+\log x}$

(b) $\frac{1-\log x}{1+\log x}$

(c) not defined

(d) $\frac{\log x}{(1+\log x)^2}$

Answer:

(d) $\frac{\log x}{(1+\log x)^2}$

Question 45.

The derivative of $y = (1-x)(2-x) \dots (n-x)$ at $x = 1$ is equal to

(a) 0

(b) $(-1)(n-1)!$

(c) $n! - 1$

(d) $(-1)^{n-1}(n-1)!$

Answer:

(b) $(-1)(n-1)!$

Question 46.

If $x^y \cdot y^x = 16$, then the value of $\frac{dy}{dx}$ at $(2, 2)$ is

(a) -1

(b) 0

(c) 1

(d) none of these

Answer:

(a) -1

Question 47.

If $y = e^{x+e^{x+e^{x+\dots \text{to } \infty}}}$, find $\frac{dy}{dx} =$

(a) $\frac{y^2}{1-y}$

(b) $\frac{y^2}{y-1}$

(c) $\frac{y}{1-y}$

(d) $\frac{-y}{1-y}$

Answer:

(c) $\frac{y}{1-y}$