A
D

BASIC CONCEPTS

1. Tangents and Normals:

Equation of tangent to the

$$(y - y_1) = \left(\frac{dy}{dx}\right)$$

If the sign of
from +ve to –v
through c (from

$x = c$ is relativ
$f(c)$ is relative n

(*ii*) Second order derivati

Step I: Find $f'(x) = 0$.

Step II: The equation

Step III: $f''(x)$ is obtai

If $f''(c) >$

7. **The tangent to the curv**

(*a*) $(0, 1)$ (

8. **The line $y = x + 1$ is a t**

(*a*) $(1, 2)$ (

9. **The point on the curve**

(*a*) $(2\sqrt{2}, 4)$ (

10. **The slope of tangent to**

(*a*) $\frac{22}{7}$ (

11. **The greatest of the num**

(*a*) $2^{1/2}$ (

12. **The minimum value of**

(*a*) 1 (

13. **The values of a for whi**

23. **The least value of the f**

(a) $\frac{a}{b}$ (

24. **The condition that the**
is

(a) $a_1 + a = b_1 + b$

(c) $\frac{1}{a_1} - \frac{1}{a} = \frac{1}{b_1} - \frac{1}{b}$

25. **If m be the slope of a ta**

(a) $|m| > 1$ (

26. **If at each point of the**
positive direction of th

(a) $a > 0$ (

27. **Slope of tangent to the**
quadrant is

(a) 2 (

38. **If $f(x) = a\log_e|x| + bx^2$**

(a) $a = -\frac{3}{4}, b = -\frac{1}{8}$

39. **Let $f(x)$ be a function su**

(a) cannot have a maxi

(b) cannot have a minir

(c) must have neither a

(d) None of these

40. **The function $f(x) = \sin$**

(a) $0 < x < \frac{\pi}{8}$

41. **If $f(x) = \frac{x}{\sin x}$ and $g(x)$**

(a) both $f(x)$ and $g(x)$ ar

(b) both $f(x)$ and $g(x)$ ar

(c) $f(x)$ is an increasing

47. **The equation of norma**

(*a*) $3x - y = 8$

48. **The equation of tanger**

(*a*) $x + 5y = 2$

49. **The interval on which**

(*a*) $[-1, \infty)$

50. **The function $f(x) = 4$ si**

(*a*) Increasing in $\left(\pi, \frac{3\pi}{2}\right.$

(*c*) decreasing in $\left(\frac{-\pi}{2}, \right.$

51. **The function $f(x) = \tan$**

(*a*) always increases

(*c*) never increases

(*d*) sometimes increase

63. **The angle between the**

(*a*) 90°

64. **Find the maximum and**

(*a*) Maximum value = 2

Minimum value = 0

(*c*) Maximum value = 0

Minimum value = 0

65. **If the curve $a\,y + x^2 = 7$**

(*a*) 1

66. **The total revenue recei**
the marginal revenue o

(*a*) 5860

67. **Find the equations of a**

(*a*) $2x - y = 3$

68. **The point on the curve**

(*a*) (3, 9)

CASE-BASED QUESTI

Choose the correct option in

1. **Read the following and**

Based on the above inf

(*i*) The rate of growth

(*a*) $4x - \frac{1}{2}x^2$

(*ii*) What is the numbe

$$y = 4 \times 4 -$$

$\therefore$ Option (c) is corr

(iv) Height of the plant

$$y = 4 \times 2 -$$

$\therefore$ Option (b) is corr

(v) Given height of the

$\therefore \qquad \dfrac{7}{2} = 4x - \dfrac{1}{2}$

$\Rightarrow \qquad x^2 - 8x + 7 =$

(v) The maximum valu

(a) $\frac{P^2}{8}$

Sol. (i) Perimeter of rectan

Option (c) is correct

(ii) $\because A = x.y$

$= x.\frac{P-2x}{2}$

$= \frac{Px - 2x^2}{2}$

Option (b) is correct

(iii) $\because \quad A = \frac{Px - 2x^2}{2}$

$\Rightarrow \frac{dA}{} = \frac{P - 4x}{}$

Based on above inform

(*i*) If x any y represen
between the variab

$$(a)\quad x + y + \frac{x}{2} = 10$$

(ii) The area of the wi

(*iii*) For maximum valu

$$\frac{dA}{dx} = 0$$

$\Rightarrow$ $5 - x -$

$\Rightarrow$ $4x + \pi x$

$\Rightarrow$ $x = \frac{20}{4 +}$

Option (*d*) is correc

(*iv*) $\because y = 5 - \frac{x}{2} - \frac{\pi x}{4}$

$= 5 - \left(\frac{2x + \pi x}{4}\right)$

$= 5 - \frac{20}{4 + \pi} \cdot \frac{2 +}{4}$

$= \frac{80 + 20\pi - 40 -}{4(4 + \pi)}$

(*ii*) If construction of t
sides, then making

(*a*) $C = 80 + 80\left(x + \right.$

(*c*) $C = 280 + 180\left(x\right.$

(*iii*) The owner of a con
for this to happen t

(*a*) 4 m

(*iv*) For minimum cost

(*a*) 1 m

(*v*) The Pradhan of vil

(*a*) ₹2000

Sol. (*i*) Volume of tank = le

$8 = x$

Hence, to minimize

Option (d) is correct

(iv) $\because$ $xy = 4$

$\Rightarrow$ $y = \frac{4}{x}$

Option (c) is correct

(v) $\because$ $C = 280$

$= 280$

Option (d) is correct

5. These days chinese and

locations near the disp

One day a helicopter o

placed at (3, 7) wants to

$D = \sqrt{(}$

$\because \quad (x_1, y_1)$

$\Rightarrow \quad y_1 = x_1^2$

$\therefore \quad D^2 = (x$

$D^2 = x_1^2$

Option (c) is correct

(iii) We have $\quad D^2 = x_1^2$

$\therefore \quad \dfrac{d(D^2)}{dx_1}$

For minimum value

$\dfrac{d(D^2)}{dx_1}$

$\Rightarrow \quad 2x_1 + 4$

$\Rightarrow \quad 4x_1^2(x_1$

ASSERTION-REASON

In the following questions, a
Choose the correct answer ou

(*a*) *Both A and R are true a*
(*b*) *Both A and R are true b*
(*c*) *A is true but R is false.*
(*d*) *A is false and R is also*

1. **Assertion (A) :** The rat
12π cm
Reason (R) : Rate of
area of
2. **Assertion (A) :** $f(x) =$ ta
Reason (R) : Any fu
3. **Assertion (A) :** $f(x) = x$

and $\quad \log y = x log\, x$

$\therefore \quad \frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{x} +$

$\Rightarrow \quad \frac{dy}{dx} = (1 + \log x)$

$\because \quad \frac{dy}{dx} = 0 \Rightarrow (1 +$

$\Rightarrow \quad \log x = -1 \Rightarrow$ log

$\Rightarrow \quad x = e^{-1} \Rightarrow x = \frac{1}{e}$

Hence, $f(x)$ has a station

Option (b) is correct.

4. From first equation of th

$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} = (m_1)$ s

$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} =$

$\Rightarrow \quad x = e$ is maxima o

$\Rightarrow \quad e^{1/e}$ is greatest val

Also, $f(4) = 4^{1/4} = 2^{2 \times 1}$

And $1 < 2 < e < 3 < 4 < 5$

Hence, $3^{1/3}$ is the greate

Option (b) is correct.

13. Given, $y = x^2 + ax + 25$

The curve (i) touches th

$\Rightarrow \quad \frac{dy}{dx} = 0 \quad \Rightarrow \quad 2x$

$\Rightarrow \quad x = -\frac{a}{2}$

$\Rightarrow$ The co-ordinate of

21. $f(x) = \sin x - ax + b$

$\Rightarrow \quad f'(x) = \cos x - a$

For increasing function

$f'(x) \geq 0$

$\cos x - a \geq 0 \Rightarrow \cos x \geq$

i.e. $a \leq \cos x \qquad a \leq$

$\therefore \;\; a \in (-\infty, -1)$.

So option (c) is correct.

22. $f(x) = \dfrac{2x^2 - 1}{x^4}$

$f'(x) = \dfrac{x^4(4x) - (2x^2 - 1}{(x^4)^2}$

$= \dfrac{-4x^5 + 4x^3}{x^8} =$

f is decreasing if $f'(x) \leq$

Slope of tangent $m = \frac{dy}{dx}$

Since the line $y = 2$ cuts

$\Rightarrow \quad 2 = x^2 - x \Rightarrow x^2 - x$

$\Rightarrow \quad x^2 + x - 2x - 2 = 0$

$\Rightarrow \quad (x + 1)(x - 2) = 0$

$\Rightarrow \quad x = -1$ or 2

Point of intersection of t

$y = x^2 - x$ are $(-1, 2)$,

As point (2, 2) lies in fir

$\therefore$ Slope of tangent at (2

Option (b) is correct.

29. $y = \cos^{-1}(\cos x)$

$\Rightarrow \quad \cos y = \cos x$

$\Rightarrow \quad -\sin y \frac{dy}{dx} = -\sin x$

$\Rightarrow \quad 2x^2(x-1)+2x(x-$

$\Rightarrow \quad (x-1)(2x^2+2x+1$

$\Rightarrow \quad x = 1 \text{ as } 2x^2+2x+$

$\therefore \; f''(x) = 12x^2 - 2$

$\therefore \; f''(1) = 12 - 2 = 10 >$

Global minimum value

Option (c) is correct.

36. $f(x) = \cos x + \cos(\sqrt{2}\,x)$

$\therefore \; f(x) = 2\cos\frac{\sqrt{2}+1}{2}x$

and it is 2 when $\cos\frac{\sqrt{2}}{}$

only when $x = 0$.

Option (b) is correct.

39. Let $f(x) = |x|$ is not diffe

$\therefore g(x)$ is decreasing in

Option (c) is correct.

42. $h(x) = f(x) - \{f(x)\}^2 + \{f($

$h'(x) = f'(x) - 2(f(x)f'($

$= \{1 - 2(f(x) + 3\{(f($

$= 3\left\{\frac{1}{3} - \frac{2}{3}f(x) + (f\right.$

$= \left(3\left[(f(x))^2 - \frac{2}{3}f(x\right.\right.$

$= \left[3\left\{f(x) - \frac{1}{3}\right\}^2 + \frac{2}{3}\right.$

If $f(x)$ is increasing, $f'(x$

i.e., $h(x)$ is increasing

If $f(x)$ is decreasing, $f'($

Option (a) is correct.

$= 6\cos x\,\{\sin^2 x - \sin x$

$= 6\cos x\left\{\sin^2 x - 2\sin\right.$

$= 6\cos x\left\{\left(\sin x - \frac{1}{2}\right)^2 +\right.$

$\therefore f(x)$ is increasing in

Option (b) is correct.

46. $x = e^t\cos t, \frac{dx}{dt} = e^t\cos t$

$y = e^t\sin t, \frac{dy}{dt} = e^t\sin t +$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t + \text{s}}{\cos t - \text{s}}$

$\therefore$ Slope of tangent to c

Tangent makes an angl

Option (c) is correct.

53. $f(x) = x^3 - 18x^7 + 96x$

$f(x) \geq 0 \ \forall \ x \in [0, 9]$

$\Rightarrow$ Minimuim value of

Option (b) is correct.

55. $f(x) = \sin x \cos x = \frac{1}{2}\sin$

$\because -1 \leq \sin 2x \leq 1.$

$\therefore -\frac{1}{2} \leq \frac{1}{2}\sin 2x \leq \frac{1}{2}$

Hence maximum value

Option (b) is correct.

56. $y = -x^3 + 3x^2 + 9x - 27$

$y'(x) = -3x^2 + 6x + 9 =$ sl

$\therefore \quad m'(x) = -6x + 6 = 0 =$

62. We are given curve $2y^2$

and point $(1, -1)$ is $2 = a$

Differentiating (A), w.r.

$$4y\frac{dy}{dx} = 2ax \Rightarrow \frac{dy}{dx} = \frac{ax}{2y}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(1,-1)} = -\frac{a}{2} \quad \text{Giv}$$

$$\Rightarrow a = 2$$

$$(i) \Rightarrow b = 2 - 2 = 0$$

$$\therefore a = 2, b = 0$$

Option (a) is correct.

64. $f(x) = x + \sin 2x$

$f(0) = 0$ and $f(2\pi) = 2\pi$

Hence $f(x)$ has maximu

Option (a) is correct