

CHAPTER

6

Syllabus

Explanation: Given

curve is $y = 5x - 2x^2$

$$\text{or} \quad \frac{dy}{dx} = 5 - 4x$$

$$\text{or} \quad m = 5 - 4x$$

$$\begin{aligned} \frac{dm}{dt} &= -4 \frac{dx}{dt} \\ &= -4 \times 18 \\ &= -72 \end{aligned}$$

$$\left. \frac{dm}{dt} \right|_{x=3} = -72$$

Q. 3. The contentment obtained after a new dish at a trial function is $f(x) = x^3 + 6x^2 + 5x + 3$. The rate of change of contentment when 3 units of dish are consumed is

(A) 60

(B) 72

(C) 24

(D) 36

Ans. Option (B) is correct.

$$\Rightarrow x = \pm 2$$

$$\text{For } x = 2$$

$$3(2)^2 - y^2 = 8$$

$$\Rightarrow y^2 = 4$$

$$\Rightarrow y = \pm 2$$

$$\text{and for } x = -2,$$

$$3(-2)^2 - y^2 = 8$$

$$\Rightarrow y^2 = 4$$

$$\Rightarrow y = \pm 2$$

So, the points at which normal to the given line are $(\pm 2, \pm 2)$.

Hence, the equation of normal is

$$\Rightarrow y - (\pm 2) = -\frac{1}{3}(x - (\pm 2))$$

$$\Rightarrow 3[y - (\pm 2)] = -(x - (\pm 2))$$

$$\therefore x + 3y \pm 8 = 0$$

Q. 10. The tangent to the curve $y = e^{2x}$ meets x -axis at :

(A) (0, 1) (B)

(C) (2, 0) (D)

Ans. Option (B) is correct.

Explanation : The equation of the curve is given by $y = e^{2x}$

Since, it passes through the point (0, 1)

$$\begin{aligned}\therefore \frac{dy}{dx} &= e^{2x} \cdot 2 \\ &= 2 \cdot e^{2x}\end{aligned}$$

$$\begin{aligned}\Rightarrow \left(\frac{dy}{dx} \right)_{(0,1)} &= 2 \cdot e^{2 \cdot 0} \\ &= 2\end{aligned}$$

= Slope of tangent

$\therefore$ Equation of tangent is

$$y - 1 = 2(x - 0)$$

(A) $\sin 2x$ (B)

(C) $\cos x$ (D)

Ans. Option (C) is correct.

Explanation : In the given in

$$f(x) = \cos x$$

On differentiating with resp

$$f'(x) = -\sin x$$

which gives $f'(x) < 0$ in $\left(0, \frac{\pi}{2}\right)$

Hence, $f(x) = \cos x$ is decre

Q. 15. The function $f(x) = \tan x - x$

(A) always increases

(B) always decreases

(C) never increases

(D) sometimes increases and

Ans. Option (A) is correct

Explanation : We have,

$$f(x) = 2x^3 - 3x^2 -$$

$$f'(x) = 6x^2 - 6x -$$

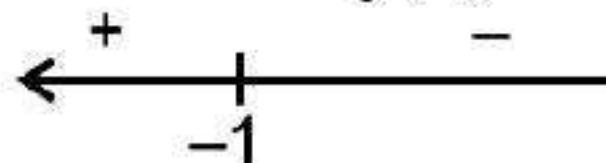
$$\text{Now, } f'(x) = 0$$

$$\Rightarrow 6(x^2 - x - 2) = 0$$

$$\Rightarrow 6(x + 1)(x - 2) = 0$$

$$\Rightarrow x = -1 \text{ and } x =$$

On number line for $f'(x)$, we



Hence, $x = -1$ is point of local
is point of local minima.

So, $f(x)$ has one maxima and

Q. 20. The maximum value of $\sin x$.

(A) $\frac{1}{4}$

(B)

(C) $\sqrt{2}$

(D)



ASSERTION

Directions : In the following question, an Assertion (A) is followed by a Reason (R). Mark the correct choice.

- (A) Both A and R are true and R is a correct explanation of A
- (B) Both A and R are true but R is not a correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is True

Q. 1. The total revenue received from the sale of a product is given by $R(x)$ rupees.

Assertion (A): The marginal revenue is 66.

Reason (R): Marginal revenue

Q. 5. Assertion (A): At $x = \frac{\pi}{6}$, the curve has a vertical tangent.

Reason (R): The slope of tangent

to the curve $y = 2\cos^2(3x)$ at $x = \frac{\pi}{6}$ is zero.

Ans. Option (D) is correct.

Explanation:

Given $y = 2\cos^2(3x)$

$$\frac{dy}{dx} = 2 \times 2 \times \cos(3x) \times (-\sin 3x)$$

$$\frac{dy}{dx} = -4\cos(3x)\sin(3x)$$

$$\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{6}} = -4\cos\left(3 \times \frac{\pi}{6}\right)\sin\left(3 \times \frac{\pi}{6}\right)$$

$$= -4 \times 0 \times 1$$

$$= 0$$

$\therefore$ R is true.

$$4 = \frac{6a}{\pm\sqrt{9}}$$

$$\therefore 4 = \frac{6a}{\pm 3}$$

$$\therefore 4 = \frac{6a}{3} \text{ or } 4 = \frac{6a}{-3}$$

$$\text{either, } a = 2 \quad \text{or}$$

$$\text{For } a = 2,$$

$$9 = 8(2) + b$$

$$\text{or } b = -7$$

$$\therefore a = 2 \text{ and } b = -7$$

$$\text{and for } a = -2,$$

$$9 = 8(-2) + b$$

$$\text{or } b = 25$$

$$\text{or } a = -2 \text{ and } b = 25$$

Hence A is true and R is false

Q. 9. Assertion (A): The function $f(x)$ is strictly increasing on the set $\mathbb{R}$.

Hence R is true.

$$y = \log(1+x) - \frac{2x}{2+x}$$

Diff. w.r.t. 'x',

$$\frac{dy}{dx} = \frac{1}{1+x}$$

$$= \frac{1}{1+x}$$

$$= \frac{1}{1+x}$$

$$= \frac{(2-x)}{(2-x)}$$

$$= \frac{4-x}{(2-x)}$$

$$= \frac{4-x}{(2-x)}$$

For increasing function

$$= \frac{1}{4} x^2 (4r^2$$

$$= \frac{1}{4} (4r^2 x^2$$

$$\therefore \frac{dS}{dx} = \frac{1}{4} [8r^2 x -$$

$$\text{or } \frac{dS}{dx} = 0$$

$$\text{or } x^2 = 2r^2 \text{ or } x$$

$$\text{and } y^2 = 4r^2 - 2r^2$$

$$\text{or } y = \sqrt{2}r$$

$$\text{i.e., } x = y \text{ and } \frac{d^2}{dx^2}$$

$$= 2r^2 - 6r^2$$

or Area is maximum, when

Hence A is true.

Angle in a semicircle is a right

∴

$$\begin{aligned}\frac{d^2V}{dh^2} &= \frac{\pi}{3}(-6h + 4r) \\ &= \frac{\pi}{3}\left(-6\left(\frac{4r}{3}\right)\right) \\ &= -\frac{4\pi r}{3} < 0 \\ \text{at } h &= \frac{4r}{3}, \text{ Volume}\end{aligned}$$



CASE-BASE

Attempt any four sub-parts
Each sub-part carries 1 mark

I. Read the following text and

Q. 5. If the height of the plant is 7
days it has been exposed to the

- (A) 2 (B)
(C) 4 (D)

Ans. Option (D) is correct.

Explanation:

Given, $y = \frac{7}{2}$

i.e., $4x - \frac{1}{2}x^2 = \frac{7}{2}$

$$8x - x^2 - 7 = 0$$

$$x^2 - 8x + 7 = 0$$

$$x^2 - 7x - x + 7 = 0$$

$$x(x - 7) - (x - 7) = 0$$

$$x = 1,$$

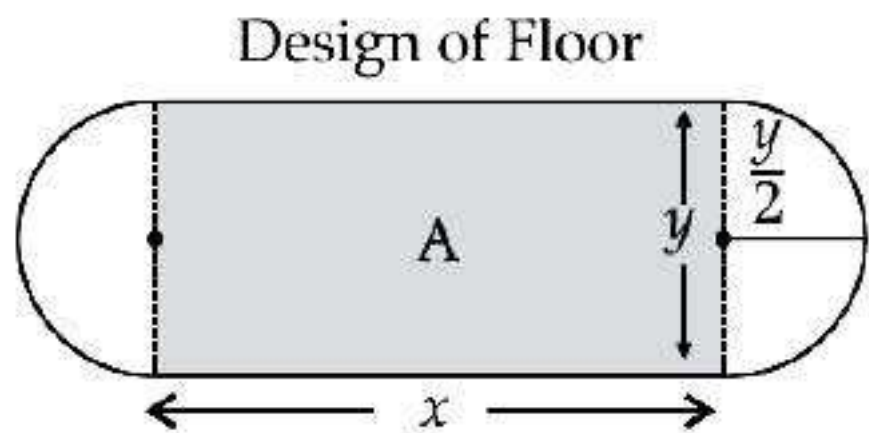
We will take $x = 1$, because
for the plant to grow to the

III. Read the following text and
questions on the basis of the
The shape of a toy is given
To make the toy beautiful
perpendicular to each other w
(2, 3), above the toy.



Q. 1. Which value from the following
critical point?
(A) $\pm 1/4$

with semicircular ends having
as shown below:



Q. 1. If x and y represents the length
rectangular region, then the
variables is :

- (A) $x + \pi y = 100$ (B)
(C) $\pi x + y = 50$ (D)

Ans. Option (B) is correct.

Explanation:

Explanation:

Volume of open box = length

$$= (24x^3 - 96x^2 + 144x - 48)$$

$$= (4x^3 - 16x^2 + 24x - 8)$$

Q. 2. Find the value of $\frac{dV}{dx}$?

(A) $12(x^2 + 16x - 48)$ (B)

(C) $6(x^2 + 8x - 24)$ (D)

Ans. Option (B) is correct.

Explanation:

$$\frac{dV}{dx} = \frac{d}{dx} [4x^3 - 96x^2 + 144x - 48]$$

$$= 12x^2 - 2 \times 96x + 144$$

$$= 12 [x^2 - 16x + 12]$$



$$= \frac{2\pi r}{r_1}(r_1 - r)h_1$$

$$= 2\pi r h_1 \times \frac{h}{h_1}$$

$$\frac{2\pi r_1 (h_1 - h).h}{h_1}$$

$\therefore$

$$S = \frac{2\pi r_1}{h_1} (h_1 - h).h$$

Q. 3. What is the value of $\frac{dS}{dh}$?

(A) $\frac{2\pi r_1}{h}(h_1 - 2h)$ (B)

(C) $\frac{2\pi r}{h}(h_1 - 2h)$ (D)

Ans. Option (D) is correct.