

CHAPTER

7

Syllabus

Q. 3. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(A) $\tan^{-1}(e^x) + C$

(B) $\tan^{-1}(e^{-x}) + C$

(C) $\log(e^x - e^{-x}) + C$

(D) $\log(e^x + e^{-x}) + C$

Ans. Option (A) is correct.

Explanation:

Let

$$\begin{aligned} I &= \int \frac{dx}{e^x + e^{-x}} \\ &= \int \frac{e^x}{e^{2x} + 1} \end{aligned}$$

Also, let

$$e^x = t$$

$\Rightarrow$

$$e^x dx = dt$$

$\Rightarrow$

$$\begin{aligned} I &= \int \frac{dt}{1 + t^2} \\ &= \tan^{-1} t + C \end{aligned}$$

Ans. Option (C) is correct.

Explanation : Let,

$$\begin{aligned}
 I &= \int \frac{dx}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a-x+b)}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin\{(x-a)-(b-x)\}}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(b-x) - \cos(x-a)\sin(b-x)}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(b-a)} \int \frac{\cos(x-b) - \cos(x-a)}{\sin(x-a)} \\
 &= \frac{1}{\sin(b-a)} \int [\cot(x-b) - \cot(x-a)] \\
 &= \frac{1}{\sin(b-a)} [\log|\sin(x-b)| - \log|\sin(x-a)|]
 \end{aligned}$$

$$\Rightarrow dx = 2t dt$$

$$\begin{aligned} \therefore I &= x \tan^{-1} \sqrt{x} - \int \frac{t}{t(1+t^2)} dx \\ &= x \tan^{-1} \sqrt{x} - \int \frac{t^2}{1+t^2} dt \\ &= x \tan^{-1} \sqrt{x} - \int \left(1 - \frac{1}{1+t^2} \right) dt \\ &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} \\ &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} \\ &= (x+1) \tan^{-1} \sqrt{x} - \sqrt{x} \end{aligned}$$

Q. 12. $\int x^2 e^{x^3} dx$ is equal to

(A) $\frac{1}{3} e^{x^3} + C$ (B)

(C) $\frac{1}{2} e^{x^3} + C$ (D)

Ans. Option (A) is correct

$$= \frac{1}{10} \left[\frac{1}{t^5} \right] + C$$

$$= \frac{1}{10} \left(4 + \frac{1}{x^2} \right)^{-5}$$

Q. 17. $\int \frac{x^3}{x+1}$ is equal to

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$

(B) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$

(C) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

(D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

Ans. Option (D) is correct.

Explanation : Let,

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx &= \int x^{1/2}dx + \int x^{-1/2}dx \\&= \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + C \\&= \frac{2}{3}x^{3/2} + 2x^{1/2} + C\end{aligned}$$

Q. 23. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

- (A) $\frac{\pi}{3}$ (B)
- (C) $\frac{\pi}{6}$ (D)

Ans. Option (D) is correct.

Explanation:



ASSERTION

Directions : In the following question, an Assertion (A) is followed by a Reason (R). Mark the correct option.

- (A) Both A and R are true and R is a correct explanation of A
- (B) Both A and R are true but R is not a correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is True

Q. 1. Assertion (A): $\int \frac{dx}{x^2 + 2x + 3} = \frac{1}{2} \tan^{-1} \frac{x+1}{\sqrt{2}}$

Reason (R): $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$

Ans. Option (A) is correct.

Explanation:

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$$

Adding equations (i) + (ii),

$$\begin{aligned} \Rightarrow 2I &= \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx \\ &= \int_0^{\pi/2} 1 dx \end{aligned}$$

$$= \left[x \right]_0^{\pi/2}$$

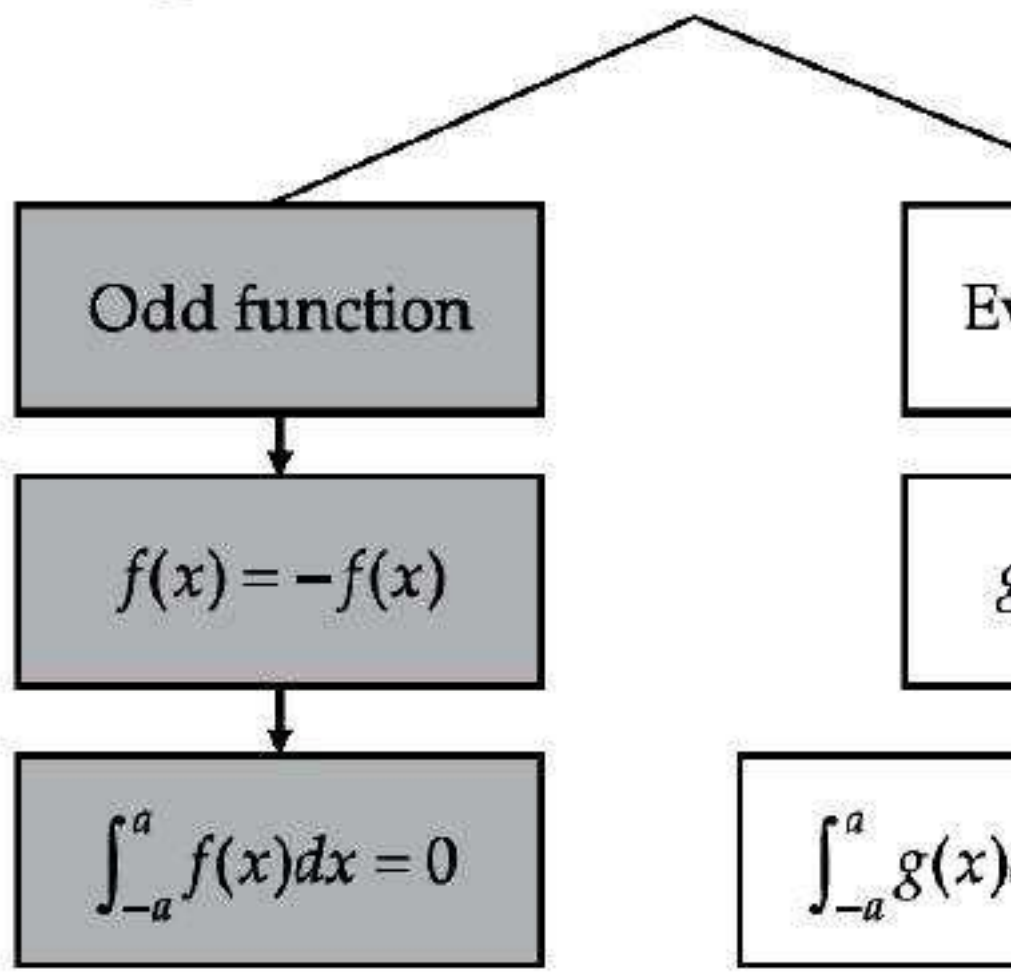


CASE-BASE

Attempt any four sub-parts

Each sub-part carries 1 mark

I. Read the following text and answer the following questions on the basis of the text.



Q. 3. $\int e^x (x + 1) dx = \underline{\hspace{2cm}}$.

(A) $xe^x + c$ (B)

(C) $e^{-x} + c$ (D)

Ans. Option (A) is correct.

Explanation:

$$\int e^x \left(\underbrace{x}_{f(x)} + \underbrace{1}_{f'(x)} \right) dx$$

Q. 4. $\int_0^{\pi} e^x (\tan x + \sec^2 x) dx = \underline{\hspace{2cm}}$

(A) 0 (B)

(C) -1 (D)

Ans. Option (A) is correct.

Explanation:

$$\int_0^{\pi} e^x (\tan x + \sec^2 x) dx =$$

Equating the coefficients of x and constant terms on both sides, we obtain

$$A + B = 1$$

$$2A + B = 0$$

On solving, we obtain

$$A = -1 \text{ and } B = 2$$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{(x+1)} dx + \int \frac{2}{(x+2)} dx$$
$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log(x+1) + C$$

$$= \log \frac{(x+2)^2}{(x+1)} + C$$

Q. 3. $\int \frac{1}{x^2 + 1} dx$