CHAPTER

7

Syllabus

Q. 3. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(A) $\tan^{-1}(e^x) + C$

(B) $\tan^{-1}(e^{-x}) + C$

(C) $\log(e^x - e^{-x}) + C$

(D) $\log(e^x + e^{-x}) + C$

Ans. Option (A) is correct.

Explanation:

Let $I = \int \frac{dx}{e^x + e^{-}$

$= \int \frac{e^x}{e^{2x} + 1}$

Also, let $e^x = t$

$\Rightarrow e^x dx = dt$

$\Rightarrow I = \int \frac{dt}{1 + t^2}$

$= \tan^{-1} t +$

Ans. Option (C) is correct.

Explanation : Let,

$$I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a}{\sin(x-a)\sin($$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a-x}{\sin(x-a)\sin($$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin\{(x-a)-($$

$$\sin(x-a)\cos($$

$$= \frac{1}{\sin(b-a)} \int \frac{\cos(x}{\sin(x-a)}$$

$$= \frac{1}{\sin(b-a)} \int [\cot(x-b)-c$$

$$= \frac{1}{\sin(b-a)} [\log|\sin(x-b)| -$$

$$|\sin(x-b$$

$$\Rightarrow dx = 2t\,dt$$

$$\therefore \quad I = x\tan^{-1}\sqrt{x} - \int \frac{t}{t(1+t^2}$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}$$

$$= x\tan^{-1}\sqrt{x} - \int\left(1 - \frac{}{1}\right.$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan$$

$$- (x+1)\tan^{-1}\sqrt{x} - \sqrt{x}$$

Q. 12. $\int x^2 e^{x^3}\,dx$ is equal to

(A) $\frac{1}{3}e^{x^3} + C$ **(B)**

(C) $\frac{1}{2}e^{x^3} + C$ **(D)**

Ans. Option (A) is correct.

$$= \frac{1}{10}\left[\frac{1}{t^5}\right] + C$$

$$= \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5}$$

Q. 17. $\int \frac{x^3}{x+1}$ is equal to

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$

(B) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$

(C) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

(D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

Ans. Option (D) is correct.

Explanation : Let,

Ans. Option (C) is correct.

Explanation:

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx = \int x^{1/2}dx + \int x^{-}$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} +$$

$$= \frac{2}{3}x^{3/2} + 2x^{1/2}$$

Q. 23. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(A) $\frac{\pi}{3}$ **(B)**

(C) $\frac{\pi}{6}$ **(D)**

Ans. Option (D) is correct.

Explanation:

ASSERTION

Directions : In the following q
of Assertion (A) is followe
Reason (R). Mark the correct

(A) Both A and R are true a
explanation of A

(B) Both A and R are true but
explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $\int \frac{dx}{x^2+2x+3} =$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-}$

Ans. Option (A) is correct.

Explanation:

Let $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + c}$

$\int_0^a f(x)dx = \int_0^a f(a-x)d$

$\therefore \quad I = \int_0^{\pi/2} \frac{\sin}{\sin\left(\frac{\pi}{2} -\right.}$

$I = \int \frac{\cos x}{\cos x + \sin}$

Adding equations (i) + (ii),

$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sin x + c}{\sin x + c}$

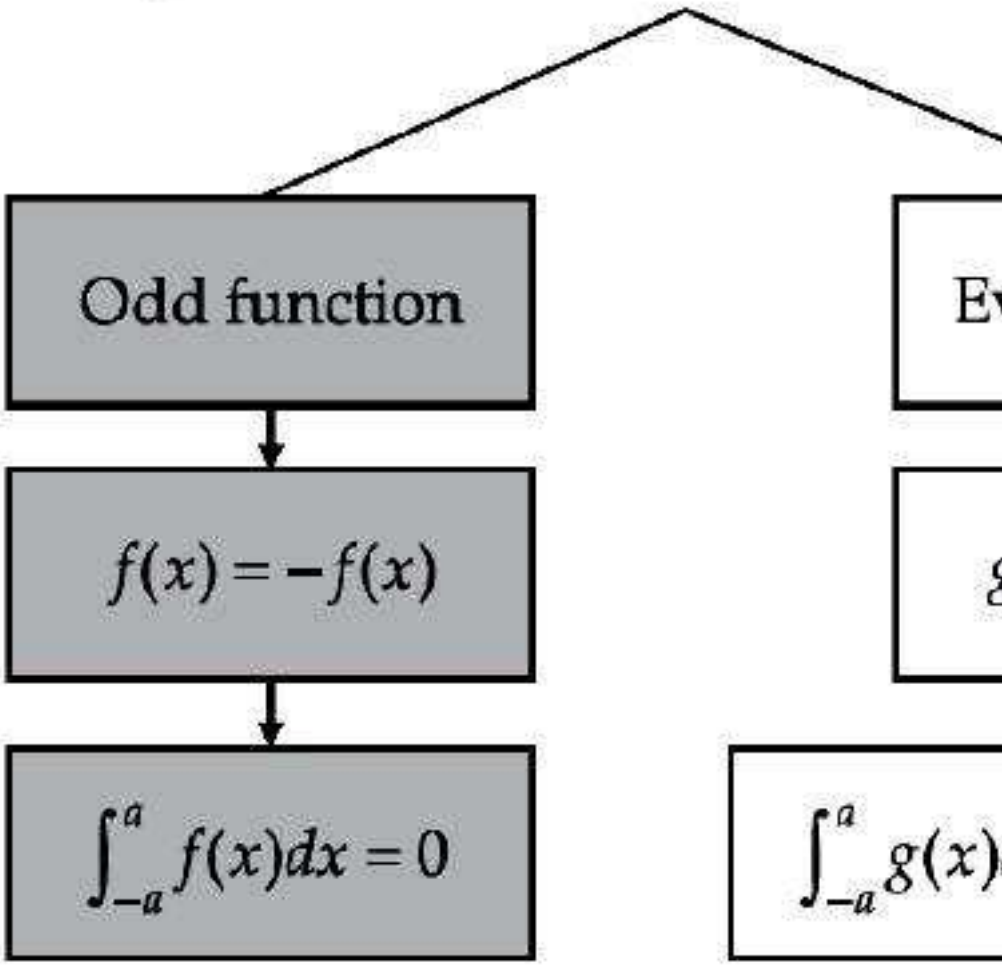
$= \int_0^{\pi/2} 1\, dx$

$= [\quad]_0^{\pi/2}$

CASE-BASE

Attempt any four sub-parts
Each sub-part carries 1 mark

I. Read the following text and
questions on the basis of the



Q. 3. $\int e^x(x+1)dx =$ _____.

(A) $xe^x + c$ **(B)**

(C) $e^{-x} + c$ **(D)**

Ans. Option (A) is correct.

Explanation:

$$\int e^x(\underbrace{x}_{f(x)} + \underbrace{1}_{f'(x)})dx$$

Q. 4. $\int_0^{\pi} e^x(\tan x + \sec^2 x)dx =$ _____

(A) 0 **(B)**

(C) –1 **(D)**

Ans. Option (A) is correct.

Explanation:

$$\int_0^{\pi} e^x(\tan x + \sec^2 x)dx =$$

Equating the coefficients of
we obtain

$$A + B = 1$$

$$2A + B = 0$$

On solving, we obtain

$$A = -1 \text{ and}$$

$$\therefore \quad \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{(x+1}$$

$$= -\log|x$$

$$= \log(x$$

$$= \log \frac{(x}{(x}$$

Q. 3. $\int \frac{1}{} dx$