Unit 9 (Circles)

Exercise 9.1 Multiple Choice Questions (MCQs)

Question 1:

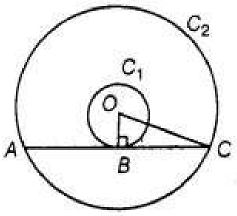
If radii of two concentric circles are 4 cm and 5 cm, then length of each chord of one circle which is tangent to the other circle, is

(a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm

Solution:

(b) Let 0 be the centre of two concentric circles C_1 and C_2, whose radii are r_1 = 4 cm and r_2 = 5 cm. Now, we draw a chord AC of circle C_2, which touches the circle C_1 at B.

Also, join OB, which is perpendicular to AC. [Tangent at any point of circle is perpendicular to radius throughly the point of contact]



Now, in right angled ΔOBC, by using Pythagoras theorem,

$$OC^2 = BC^2 + BO^2$$

[$\because$ (hypotenuse)^2 = (base)^2 + (perpendicular)^2]

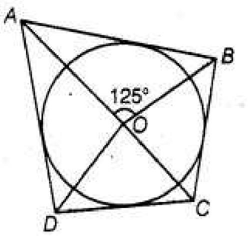
$\Rightarrow$ $5^2 = BC^2 + 4^2$

$\Rightarrow$ $BC^2 = 25 - 16 = 9 \Rightarrow BC = 3\,\text{cm}$

$\therefore$ Length of chord $AC = 2\,BC = 2 \times 3 = 6\,\text{cm}$

Question 2:

In figure, if ∠AOB = 125°, then ∠COD is equal to



(a) 62.5° (b) 45° (c) 35° (d) 55°

Solution:

(d) We know that, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

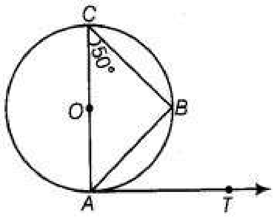
i.e., $\angle AOB + \angle COD = 180°$

$\Rightarrow$ $\angle COD = 180° - \angle AOB$

$= 180° - 125° = 55°$

Question 3:

In figure, AB is a chord of the circle and AOC is its diameter such that ∠ACB = 50°. If AT is the tangent to the circle at the point A, then ∠BAT is equal to



(a) 45° (b) 60° (c) 50° (d) 55°

Solution:

(c) In figure, AOC is a diameter of the circle. We know that, diameter subtends an angle 90° at the circle.

So, $\angle ABC = 90°$

In ΔACB, $\angle A + \angle B + \angle C = 180°$

[since, sum of all angles of a triangle is 180°]

$\Rightarrow$ $\angle A + 90° + 50° = 180°$

$\Rightarrow$ $\angle A + 140 = 180$

$\Rightarrow$ $\angle A = 180° - 140° = 40°$

$\angle A$ or $\angle OAB = 40°$

Now, AT is the tangent to the circle at point A. So, OA is perpendicular to AT.

$\therefore$ $\angle OAT = 90°$ [from figure]

$\Rightarrow$ $\angle OAB + \angle BAT = 90°$

On putting $\angle OAB = 40°$, we get

$\Rightarrow$ $\angle BAT = 90° - 40° = 50°$

Hence, the value of $\angle BAT$ is 50°.

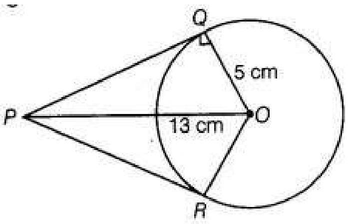
Question 4:

From a point P which is at a distance of 13 cm from the centre 0 of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is

(a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2

Solution:

(a) Firstly, draw a circle of radius 5 cm having centre O. P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn.



Thus, quadrilateral POOR is formed.

$\because$ $OQ \perp QP$ [since, AP is a tangent line]

In right angled ΔPQO, $OP^2 = OQ^2 + QP^2$

$\Rightarrow$ $13^2 = 5^2 + QP^2$

$\Rightarrow$ $QP^2 = 169 - 25 = 144$

$\Rightarrow$ $QP = 12$ cm

Now, area of $\Delta OQP = \frac{1}{2} \times QP \times QO$

$= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$

$\therefore$ Area of quadrilateral $QORP = 2\ \Delta OQP$

$= 2 \times 30 = 60 \text{ cm}^2$

Question 5:

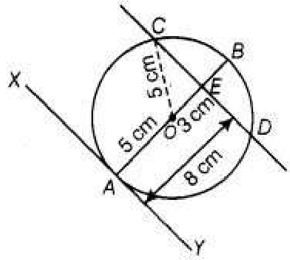
At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A, is

(a) 4 cm (b) 5 cm

(c) 6 cm (d) 8 cm

Solution:

(d) First, draw a circle of radius 5 cm having centre 0. A tangent XY is drawn at point A.



A chord CD is drawn which is parallel to XY and at a distance of 8 cm from A.

Now, $\angle OAY = 90°$

[Tangent and any point of a circle is perpendicular to the radius through the point of contact]

$\Delta OAY + \Delta OED = 180°$ [∵ sum of cointerior is 180°]

⇒ $\Delta OED = 180°$

Also, $AE = 8$ cm. Join OC

Now, in right angled ΔOEC, $OC^2 = OE^2 + EC^2$

⇒ $EC^2 = OC^2 - OE^2$ [by Pythagoras theorem]

$= 5^2 - 3^2$

[∵ OC = radius = 5 cm, $OE = AE - AO = 8 - 5 = 3$ cm]

$= 25 - 9 = 16$

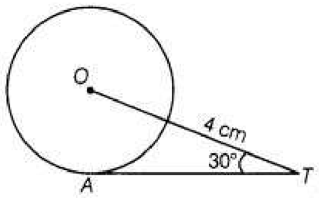
⇒ $EC = 4$ cm

Hence, length of chord $CD = 2\,CE = 2 \times 4 = 8$ cm

[since, perpendicular from centre to the chord bisects the chord]

Question 6:

In figure, AT is a tangent to the circle with centre 0 such that OT = 4 cm and ∠OTA = 30°. Then, AT is equal to



(a) 4 cm (b) 2 cm (c) 2√3 *cm* (d) 4√3 cm

Solution:

(c) Join OA

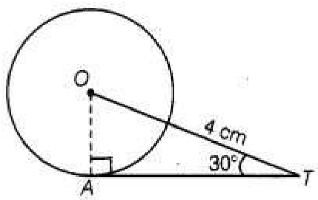
We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ $\angle OAT = 90°$

In ΔOAT, $\cos 30° = \dfrac{AT}{OT}$

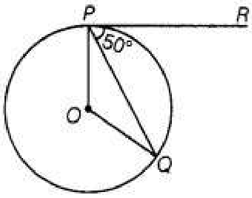
⇒ $\dfrac{\sqrt{3}}{2} = \dfrac{AT}{4}$

⇒ $AT = 2\sqrt{3}$ cm



Question 7:

In figure, if 0 is the centre of a circle, PQ is a chord and the tangent PR at P , ; makes an angle of 50° with PQ, then ∠POQ is equal to



(a) 100° (b) 80° (c) 90° (d) 75°

Solution:

(a) Given, ∠QPR = 50°

We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ $\angle OPR = 90°$

⇒ $\angle OPQ + \angle QPR = 90°$ [from figure]

⇒ $\angle OPQ = 90° - 50° = 40°$ [$\because \angle QPR = 50°$]

Now, $OP = OQ$ = Radius of circle

∴ $\angle OQP = \angle OPQ = 40°$

[since, angles opposite to equal sides are equal]

In ΔOPQ, $\angle O + \angle P + \angle Q = 180°$

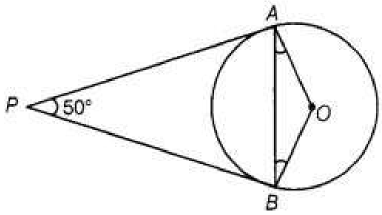
[since, sum of angles of a triangle = 180°]

⇒ $\angle O = 180° - (40° + 40°)$ [$\because \angle P = 40° = \angle Q$]

$= 180° - 80° = 100°$

Question 8:

In figure, if PA and PB are tangents to the circle with centre 0 such that ∠APB = 50°, then ∠OAB is equal to



(a) 25° (b) 30° (c) 40° (d) 50°

Solution:

(a) Given, PA and PB are tangent lines.

∴ $PA = PB$

[since, the length of tangents drawn from an external point to a circle is equal]

⇒ $\angle PBA = \angle PAB = \theta$ [say]

In ΔPAB, $\angle P + \angle A + \angle B = 180°$

[since, sum of angles of a triangle = 180°]

⇒ $50° + \theta + \theta = 180°$

⇒ $2\theta = 180° - 50° = 130°$

⇒ $\theta = 65°$

Also, $OA \perp PA$

[since, tangent at any point of a circle is perpendicular to the radius through the point of contact]

∴ $\angle PAO = 90°$

⇒ $\angle PAB + \angle BAO = 90°$

⇒ $65° + \angle BAO = 90°$

⇒ $\angle BAO = 90° - 65° = 25°$

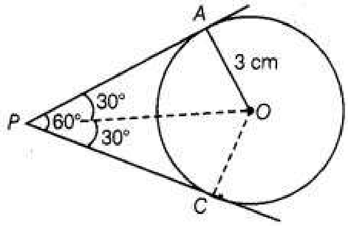
Question 9:

If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is

(a) $\frac{3}{2}\sqrt{3}$ cm (b) 6 cm (c) 3 cm (d) $3\sqrt{3}$ cm

Solution:

(d) Let P be an external point and a pair of tangents is drawn from point P and angle between these two tangents is 60°.



Join OA and OP.
Also, OP is a bisector line of $\angle APC$.

$\therefore \quad \angle APO = \angle CPO = 30°$

Also, $\quad OA \perp AP$

Tangent at any point of a circle is perpendicular to the radius through the point of contact.

In right angled ΔOAP, $\quad \tan 30° = \dfrac{OA}{AP} = \dfrac{3}{AP}$

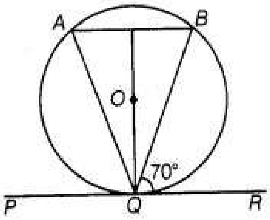
$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{3}{AP}$

$\Rightarrow \quad AP = 3\sqrt{3}$ cm

Hence, the length of each tangent is 3√3 cm.

Question 10:

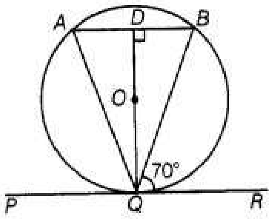
In figure, if PQR is the tangent to a circle at Q whose centre is 0, AB is a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to



(a) 20° (b) 40° (c) 35° (d) 45°

Solution:

(b) Given, $AB \parallel PR$



$\therefore \quad \angle ABQ = \angle BQR = 70°$ [alternate angles]

Also, QD is perpendicular to AB and QD bisects AB.

In ΔQDA and ΔQDB, $\quad \angle QDA = \angle QDB$ [each 90°]

$AD = BD$

$QD = QD$ [common side]

$\therefore \quad \Delta ADQ \cong \Delta BDQ$ [by SAS similarity criterion]

Then $\quad \angle QAD = \angle QBD$ [CPCT] ...(i)

Also $\quad \angle ABQ = \angle BQR$ [alternate interior angle]

$\therefore \quad \angle ABQ = 70°$ [$\because \angle BQR = 70°$]

Hence, $\quad \angle QAB = 70°$ [from Eq. (i)]

Now, in ΔABQ, $\quad \angle A + \angle B + \angle Q = 180°$

$\Rightarrow \quad \angle Q = 180° - (70° + 70°) = 40°$

Exercise 9.2 Very Short Answer Type Questions

Question 1:

If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60°.

Solution:

False

Since a chord AB subtends an angle of 60° at the centre of a circle.

i.e., $\angle AOB = 60°$

As $OA = OB =$ Radius of the circle

$\therefore$ $\angle OAB = \angle OBA = 60°$

The tangent at points A and B is drawn, which intersect at C.

We know, $OA \perp AC$ and $OB \perp BC$.

$\therefore$ $\angle OAC = 90°, \angle OBC = 90°$

$\Rightarrow$ $\angle OAB + \angle BAC = 90°$

and $\angle OBA + \angle ABC = 90°$

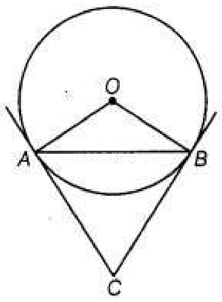
$\Rightarrow$ $\angle BAC = 90° - 60° = 30°$

and $\angle ABC = 90° - 60° = 30°$

In ΔABC, $\angle BAC + \angle CBA + \angle ACB = 180°$

[since, sum of all interior angles of a triangle is 180°]

$\Rightarrow$ $\angle ACB = 180° - (30° + 30°) = 120°$



Question 2:

The length of tangent from an external point P on a circle is always greater than the radius of the circle.

Solution:

False

Because the length of tangent from an external point P on a circle may or may not be greater than the radius of the circle.

Question 3:

The length of tangent from an external point P on a circle with centre 0 is always less than OP.

Solution:

True

PT is a tangent drawn from external point P. Join OT.

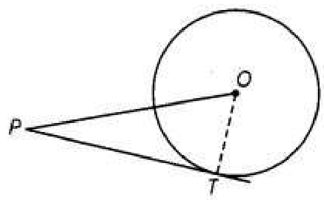
$\because$ $OT \perp PT$

So, OPT is a right angled triangle formed.

In right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.

$\therefore$ $OP > PT$

or $PT < OP$



Question 4:

The angle between two tangents to a circle may be 0°.

Solution:

True

'This may be possible only when both tangent lines coincide or are parallel to each other.

Question 5:

If angle between two tangents drawn from a point P to a circle of radius a and centre 0 is 90°, then OP = a √2.

Solution:

True

From point P, two tangents are drawn.

Given, $OT = a$

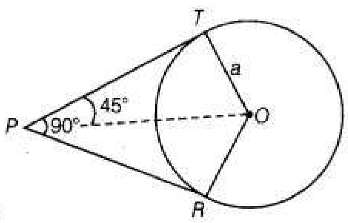
Also, line OP bisects the $\angle RPT$.

$\therefore$ $\angle TPO = \angle RPO = 45°$

Also, $OT \perp PT$

In right angled ΔOTP, $\sin 45° = \dfrac{OT}{OP}$

$\Rightarrow$ $\dfrac{1}{\sqrt{2}} = \dfrac{a}{OP} \Rightarrow OP = a\sqrt{2}$



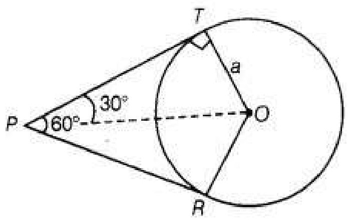
Question 6:

If angle between two tangents drawn from a point P to a circle of radius a and centre 0 is 60°, then OP = a√3.

Solution:

True

From point P, two tangents are drawn.



Given, $OT = a$

Also, line OP bisects the $\angle RPT$.

$\therefore$ $\angle TPO = \angle RPO = 30°$

Also, $OT \perp PT$

In right angled ΔOTP,

$$\sin 30° = \frac{OT}{OP}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{a}{OP}$$

$$\Rightarrow \quad OP = 2a$$

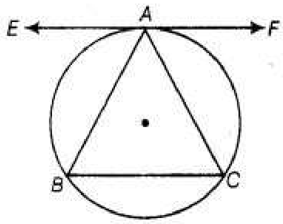
Question 7:

The tangent to the circumcircle of an isosceles ΔABC at A, in which AB = AC, is parallel to BC.

Solution:

True

Let EAF be tangent to the circumcircle of ΔABC.



To prove $EAF \parallel BC$

$\angle EAB = \angle ABC$

Here, $AB = AC$

$\Rightarrow \quad \angle ABC = \angle ACB \quad \ldots(i)$

[angle between tangent and is chord equal to angle made by chord in the alternate segment]

$\therefore$ Also, $\angle EAB = \angle BCA \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$\angle EAB = \angle ABC$

$\Rightarrow \quad EAF \parallel BC$

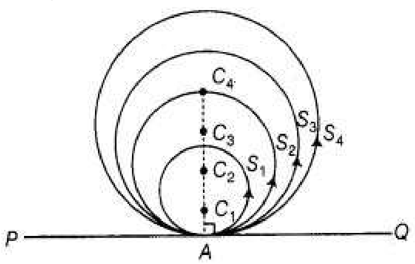
Question 8:

If a number of circles touch a given line segment PQ at a point A, then their centres lie on the perpendicular bisector of PQ.

Solution:

False

Given that PQ is any line segment and S_1, S_2, S_3, S_4,… circles are touch a line segment PQ at a point A. Let the centres of the circlesS_1,S_2, S_3, S_4,… be C_1 C_2, C_3, C_4,… respectively.



To prove centres of these circles lie on the perpendicular bisector PQ

Now, joining each centre of the circles to the point A on the line segment PQ by a line segment i.e., C_1A, C_2A, C_3A, C_4A… so on.

We know that, if we draw a line from the centre of a circle to its tangent line, then the line is always perpendicular to the tangent line. But it not bisect the line segment PQ.

So, $C_1A \perp PQ$ [for S_1]
$C_2A \perp PQ$ [for S_2]
$C_3A \perp PQ$ [for S_3]
$C_4A \perp PQ$ [for S_4]
... so on.

Since, each circle is passing through a point A. Therefore, all the line segments C_1A, C_2A, C_3A, C_4A…. so on are coincident.

So, centre of each circle lies on the perpendicular line of PQ but they do not lie on the perpendicular bisector of PQ.

Hence, a number of circles touch a given line segment PQ at a point A, then their centres lie

Question 9:

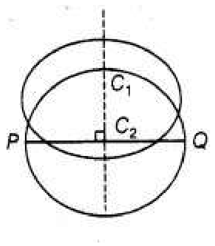
If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Solution:

True

We draw two circle with centres C_1 and C_2 passing through the end points P and Q of a line segment PQ. We know, that perpendicular bisectors of a chord of a circle always passes through the centre of circle.

Thus, perpendicular bisector of PQ passes through C_1 and C_2. Similarly, all the circle passing through PQ will have their centre on perpendiculars bisectors of PQ.



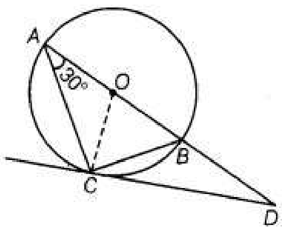
Question 10:

AB is a diameter of a circle and AC is its chord such that ∠BAC – 30°. If the tangent at C intersects AB extended at D, then BC = BD.

Solution:

True

To Prove, BC = BD



Join BC and OC.

Given, $\angle BAC = 30°$

$\Rightarrow$ $\angle BCD = 30°$

[angle between tangent and chord is equal to angle made by chord in the alternate segment]

$\therefore$ $\angle ACD = \angle ACO + \angle OCD = 30° + 90° = 120°$

[$\because OC \perp CD$ and $OA = OC$ = radius $\Rightarrow \angle OAC = \angle OCA = 30°$]

In ΔACD, $\angle CAD + \angle ACD + \angle ADC = 180°$

[since, sum of all interior angles of a triangle is 180°]

$\Rightarrow$ $30° + 120° + \angle ADC = 180°$

$\Rightarrow$ $\angle ADC = 180° - (30° + 120°) = 30°$

Now, in ΔBCD $\angle BCD = \angle BDC = 30°$

$\Rightarrow$ $BC = BD$

[since, sides opposite to equal angles are equal]

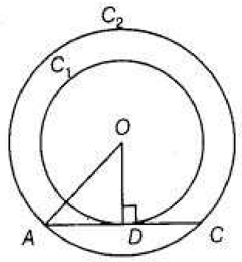
Exercise 9.3 Short Answer Type Questions

Question 1:

Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Solution:

Let C_1 and C_2 be the two circles having same centre O. AC is a chord which touches the C_1 at point D.



Join OD.

Also, $\quad OD \perp AC$

$\therefore \quad AD = DC = 4\text{ cm}$ [perpendicular line OD bisects the chord]

In right angled ΔAOD, $\quad OA^2 = AD^2 + DO^2$

[by Pythagoras theorem, *i.e.*, $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$]

$\Rightarrow \quad DO^2 = 5^2 - 4^2$

$= 25 - 16 = 9$

$\Rightarrow \quad DO = 3\text{ cm}$

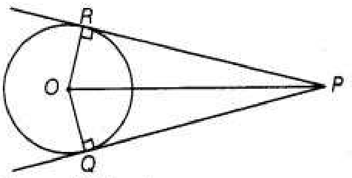
$\therefore$ Radius of the inner circle $OD = 3\text{ cm}$

Question 2:

Two tangents PQ and PR are drawn from an external point to a circle with centre 0. Prove that QORP is a cyclic quadrilateral.

Solution:

Given Two tangents PQ and PR are drawn from an external point to a circle with centre 0.



To prove $QORP$ is a cyclic quadrilateral.

proof Since, PR and PQ are tangents.

So, $\quad OR \perp PR$ and $OQ \perp PQ$

[since, if we drawn a line from centre of a circle to its tangent line. Then, the line always perpendicular to the tangent line]

$\therefore \quad \angle ORP = \angle OQP = 90°$

Hence, $\quad \angle ORP + \angle OQP = 180°$

So, $QOPR$ is cyclic quadrilateral.

[If sum of opposite angles is quadrilateral in 180°, then the quadrilateral is cyclic]

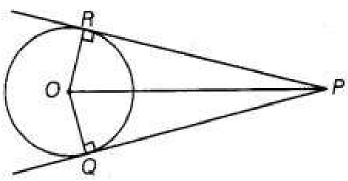
Hence proved.

Question 3:

Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Solution:

Given Two tangents PQ and PR are drawn from an external point P to a circle with centre 0.



To prove Centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

In ∠RPQ.

Construction Join OR, and OQ.

In ΔPOP and ΔPOO

$\angle PRO = \angle PQO = 90°$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$OR = OQ$ [radii of some circle]

Since OP is common

∴ $\Delta PRO \cong \Delta PQO$ [RHS]

Hence, $\angle RPO = \angle QPO$ [by CPCT]

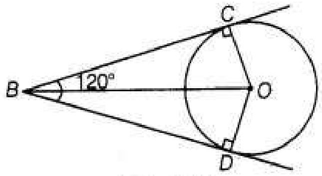
Thus, O lies on angle bisecter of PR and PQ. Hence proved.

Question 4:

If from an external point B of a circle with centre 0, two tangents BC and BD are drawn such that ∠DBC = 120°, prove that BC + BD = B0 i.e., BO = 2 BC.

Solution:

Two tangents BD and BC are drawn from an external point B.



To prove $BO = 2BC$

Given, $\angle DBC = 120°$

Join OC, OD and BO.

Since, BC and BD are tangents.

∴ $OC \perp BC$ and $OD \perp BD$

We know, OB is a angle bisector of $\angle DBC$.

∴ $\angle OBC = \angle DBO = 60°$

In right angled ΔOBC, $\cos 60° = \dfrac{BC}{OB}$

⇒ $\dfrac{1}{2} = \dfrac{BC}{OB}$

⇒ $OB = 2\,BC$

Also, $BC = BD$

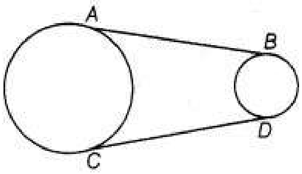
[tangent drawn from internal point to circle are equal]

∴ $OB = BC + BC$

⇒ $OB = BC + BD$

Question 5:

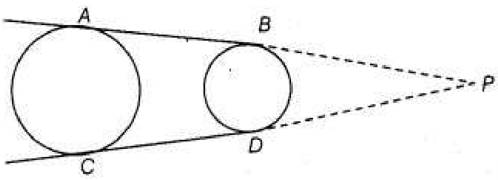
In figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD



Solution:

Given AS and CD are common tangent to two circles of unequal radius

To prove AB = CD



Construction Produce AB and CD, to intersect at P.

Proof $PA = PC$

[the length of tangents drawn from an internal point to a circle are equal]

Also, $PB = PD$

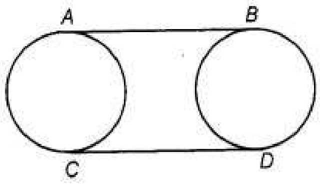
[the lengths of tangents drawn from an internal point to a circle are equal]

∴ $PA - PB = PC - PD$

$AB = CD$ **Hence proved.**

Question 6:

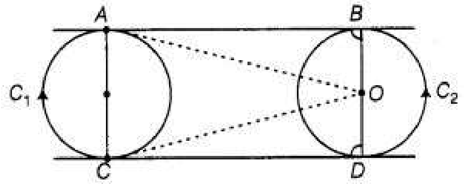
In figure, AB and CD are common tangents to two circles of equal radii. Prove that AB = CD.



Solution:

Given AB and CD are tangents to two circles of equal radii.

To prove AB = CD



Construction Join OA, OC, $O'B$ and $O'D$

Proof Now, $\angle OAB = 90°$

[tangent at any point of a circle is perpendicular to radius through the point of contact]

Thus, AC is a straight line.

Also, $\angle OAB + \angle OCD = 180°$

$\therefore$ $AB \parallel CD$

Similarly, BD is a straight line

and $\angle O'BA = \angle O'DC = 90°$

Also, $AC = BD$ [radii of two circles are equal]

In quadrilateral $ABCD$, $\angle A = \angle B = \angle C = \angle D = 90°$

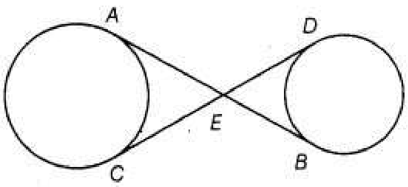
and $AC = BD$

$ABCD$ is a rectangle

Hence, $AB = CD$ [opposite sides of rectangle are equal]

Question 7:

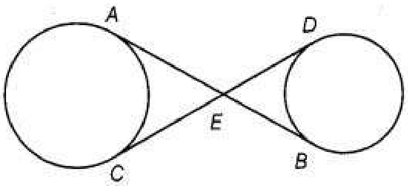
In figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



Solution:

Given Common tangents AB and CD to two circles intersecting at E.

To prove AB = CD



Proof $EA = EC$ ...(i)

[the lengths of tangents drawn from an internal point to a circle are equal]

$EB = ED$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$EA + EB = EC + ED$$

$\Rightarrow$ $AB = CD$ **Hence proved.**

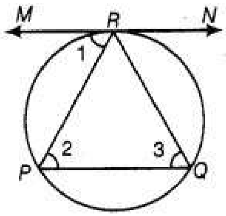
Question 8:

A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Solution:

Given Chord PQ is parallel to tangent at R.

To prove R bisects the arc PRQ



Proof $\angle 1 = \angle 2$ [alternate interior angles]

$\angle 1 = \angle 3$

[angle between tangent and chord is equal to angle made by chord in alternate segment]

$\therefore$ $\angle 2 = \angle 3$

$\Rightarrow$ $PR = QR$ [sides opposite to equal angles are equal]

$\Rightarrow$ $PR = QR$

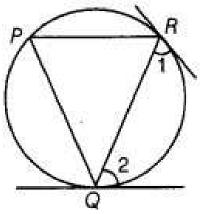
So, R bisects PQ.

Question 9:

Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution:

To prove ∠1 = ∠2, let PQ be a chord of the circle. Tangents are drawn at the points R and Q.



Let P be another point on the circle, then, join PQ and PR.

Since, at point Q, there is a tangent.

$\therefore$ $\angle 2 = \angle P$ [angles in alternate segments are equal]

Since, at point R, there is a tangent.

$\therefore$ $\angle 1 = \angle P$ [angles in alternate segments are equal]

$\therefore$ $\angle 1 = \angle 2 = \angle P$ **Hence proved.**

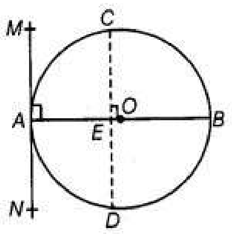
Question 10:

Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

Solution:

Given, AB is a diameter of the circle.

A tangent is drawn from point A. Draw a chord CD parallel to the tangent MAN.



So, CD is a chord of the circle and OA is a radius of the circle.

$\angle MAO = 90°$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\angle CEO = \angle MAO$ [corresponding angles]

$\therefore$ $\angle CEO = 90°$

Thus, OE bisects CD, [perpendicular from centre of circle to chord bisects the chord]

Similarly, the diameter AB bisects all. Chords which are parallel to the tangent at the point A.

Exercise 9.4 Long Answer Type Questions

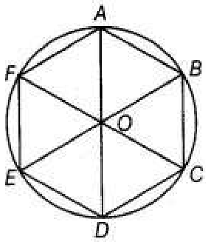
Question 1:

If a hexagon ABCDEF circumscribe a circle, prove that

AB + CD + EF =BC + DE + FA

Solution:

Given A hexagon ABCDEF circumscribe a circle.



To prove $AB + CD + EF = BC + DE + FA$

Proof $AB + CD + EF = (AQ + QB) + (CS + SD) + (EU + UF)$

$= AP + BR + CR + DT + ET + FP$

$= (AP + FP) + (BR + CR) + (DT + ET)$

$$AB + CD + EF = AF + BC + DE$$

$\because$ $AQ = AP$

$QB = BR$

$CS = CR$

$DS = DT$

$EU = ET$

[tangents drawn from an external point to a circle are equal]

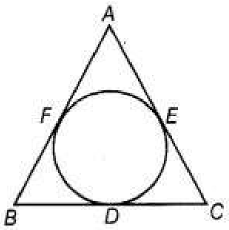
Hence proved.

Question 2:

Let s denotes the semi-perimeter of a Δ ABC in which BC = a, CA = b and AB = c. If a circle touches the sides BC, CA, AB at D, E, F, respectively. Prove that BD = s – b.

Solution:

A circle is inscribed in the A ABC, which touches the BC, CA and AB.



Given, $BC = a, CA = b$ and $AB = c$

By using the property, tangents are drawn from an external point to the circle are equal in length.

$\therefore$ $BD = BF = x$ [say]

$DC = CE = y$ [say]

and $AE = AF = z$ [say]

Now, $BC + CA + AB = a + b + c$

$\Rightarrow$ $(BD + DC) + (CE + EA) + (AF + FB) = a + b + c$

$\Rightarrow$ $(x + y) + (y + z) + (z + x) = a + b + c$

$\Rightarrow$ $2(x + y + z) = 2s$

[$\because 2s = a + b + c =$ perimeter of ΔABC]

$\Rightarrow$ $s = x + y + z$

$\Rightarrow$ $x = s - (y + z)$

$\Rightarrow$ $BD = s - b$ [$\because b = AE + EC = z + y$]

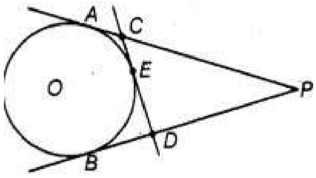
Hence proved.

Question 3:

From an external point P, two tangents, PA and PB are drawn to a circle with centre 0. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the trianlge PCD.

Solution:

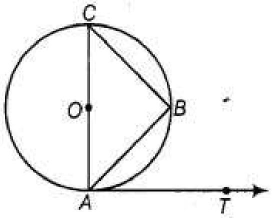
Two tangents PA and PB are drawn to a circle with centre 0 from an external point P



Perimeter of $\Delta PCD = PC + CD + PD$
$= PC + CE + ED + PD$
$= PC + CA + DB + PD$
$= PA + PB$
$= 2PA = 2(10)$
$= 20\text{ cm}$
[$\because$ $CE = CA$, $DE = DB$, $PA = PB$ tangents from internal point to a circle are equal]

Question 4:

If AB is a chord of a circle with centre 0, AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB.



Solution:

Since, AC is a diameter line, so angle in semi-circle makes an angle 90°.

$\therefore$ $\angle ABC = 90°$ [by property]

In ΔABC, $\angle CAB + \angle ABC + \angle ACB = 180°$

[$\because$ sum of all interior angles of any triangle is 180°]

$\Rightarrow$ $\angle CAB + \angle ACB = 180° - 90° = 90°$ ...(i)

Since, diameter of a circle is perpendicular to the tangent.

i.e. $CA \perp AT$

$\therefore$ $\angle CAT = 90°$

$\Rightarrow$ $\angle CAB + \angle BAT = 90°$ ...(ii)

From Eqs. (i) and (ii),

$\angle CAB + \angle ACB = \angle CAB + \angle BAT$

$\Rightarrow$ $\angle ACB = \angle BAT$ **Hence proved.**

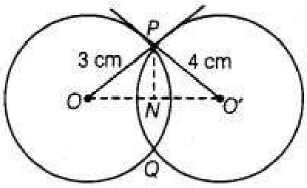
Question 5:

Two circles with centres 0 and 0′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and 0’P are tangents to the two circles. Find the length of the common chord PQ.

Solution:

Here, two circles are of radii OP = 3 cm and PO’ = 4 cm

These two circles intersect at P and Q.



Here, OP and PO' are two tangents drawn at point P.

$\angle OPO' = 90°$

[tangent at any point of circle is perpendicular to radius through the point of contact]

Join OO' and PN.

In right angled $\Delta OPO'$,

$$(OO')^2 = (OP)^2 + (PO')^2 \quad \text{[by Pythagoras theorem]}$$

i.e., $$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$= (3)^2 + (4)^2 = 25$$

$\Rightarrow$ $$OO' = 5 \text{ cm}$$

Also, $$PN \perp OO'$$

Let $ON = x$, then $NO' = 5 - x$

In right angled ΔOPN,

$$(OP)^2 = (ON)^2 + (NP)^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$(NP)^2 = 3^2 - x^2 = 9 - x^2 \quad \ldots\text{(i)}$$

and in right angled $\Delta PNO'$,

$$(PO')^2 = (PN)^2 + (NO')^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$(4)^2 = (PN)^2 + (5 - x)^2$$

$\Rightarrow$ $$(PN)^2 = 16 - (5 - x)^2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$9 - x^2 = 16 - (5 - x)^2$$

$\Rightarrow$ $$7 + x^2 - (25 + x^2 - 10x) = 0$$

$\Rightarrow$ $$10x = 18$$

$\therefore$ $$x = 1.8$$

Again, in right angled ΔOPN,

$$OP^2 = (ON)^2 + (NP)^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$3^2 = (1.8)^2 + (NP)^2$$

$\Rightarrow$ $$(NP)^2 = 9 - 3.24 = 5.76$$

$\therefore$ $$(NP) = 2.4$$

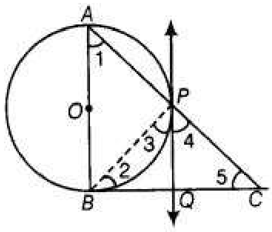
$\therefore$ Length of common chord, $$PQ = 2\,PN = 2 \times 2.4 = 4.8 \text{ cm}$$

Question 6:

In a right angle ΔABC is which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at PQ bisects BC.

Solution:

Let O be the centre of the given circle. Suppose, the tangent at P meets BC at 0. Join BP.



To prove $$BQ = QC \quad \text{[angles in alternate segment]}$$

Proof $$\angle ABC = 90^\circ$$

[tangent at any point of circle is perpendicular to radius through the point of contact]

$\therefore$ In ΔABC, $$\angle 1 + \angle 5 = 90^\circ \quad [\text{angle sum property, } \angle ABC = 90^\circ]$$

$$\angle 3 = \angle 1$$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

$\therefore$ $$\angle 3 + \angle 5 = 90^\circ \quad \ldots\text{(i)}$$

Also, $$\angle APB = 90^\circ \quad \text{[angle in semi-circle]}$$

$\Rightarrow$ $$\angle 3 + \angle 4 = 90^\circ \quad [\angle APB + \angle BPC = 180^\circ, \text{linear pair}]$$

From Eqs. (i) and (ii), we get

$$\angle 3 + \angle 5 = \angle 3 + \angle 4$$

$\Rightarrow$ $$\angle 5 = \angle 4$$

$\Rightarrow$ $$PQ = QC \quad \text{[sides opposite to equal angles are equal]}$$

Also, $$QP = QB$$

[tangents drawn from an internal point to a circle are equal]

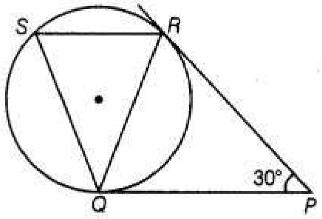
$\Rightarrow$ $$QB = QC$$ **Hence proved.**

Question 7:

In figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find the ∠RQS.

Solution:

PQ and PR are two tangents drawn from an external point P.



$\therefore \quad PQ = PR$

[the lengths of tangents drawn from an external point to a circle are equal]

$\Rightarrow \quad \angle PQR = \angle QRP$

[angles opposite to equal sides are equal]

Now, in $\Delta PQR \quad \angle PQR + \angle QRP + \angle RPQ = 180°$

[sum of all interior angles of any triangle is 180°]

$\Rightarrow \quad \angle PQR + \angle PQR + 30° = 180°$

$\Rightarrow \quad 2\angle PQR = 180° - 30°$

$\Rightarrow \quad \angle PQR = \dfrac{180° - 30°}{2} = 75°$

Since, $\quad SR \parallel QP$

$\therefore \quad \angle SRQ = \angle RQP = 75°$ [alternate interior angles]

Also, $\quad \angle PQR = \angle QSR = 75°$ [by alternate segment theorem]

In ΔQRS, $\quad \angle Q + \angle R + \angle S = 180°$

[sum of all interior angles of any triangle is 180°]

$\Rightarrow \quad \angle Q = 180° - (75° + 75°)$

$= 30°$

$\therefore \quad \angle RQS = 30°$

Question 8:

AB is a diameter and AC is a chord of a circle with centre 0 such that ∠BAC = 30°. The tangent at C intersects extended AB at a point D. Prove that BC = BD.

Solution:

A circle is drawn with centre O and AB is a diameter.

AC is a chord such that ∠BAC = 30°.

Given AB is a diameter and AC is a chord of circle with certre O, ∠BAC = 30°.

Question 9:

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Solution:

Let mid-point of an arc AMB be M and TMT’ be the tangent to the circle.

Join AB, AM and MB.

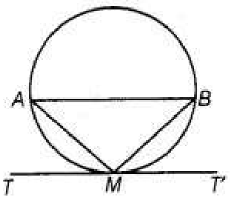
Since, $\quad$ arc AM = arc MB

$\Rightarrow \quad$ Chord AM = Chord MB

In ΔAMB, $\quad AM = MB$

$\Rightarrow \quad \angle MAB = \angle MBA$

[equal sides corresponding to the equal angle] ...(i)



Since, TMT' is a tangent line.

$\therefore \quad \angle AMT = \angle MBA$

[angles in alternate segments are equal]

$= \angle MAB$ [from Eq. (i)]

But ∠AMT and ∠MAB are alternate angles, which is possible only when

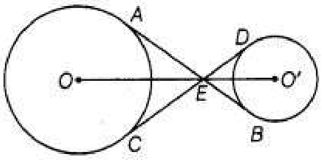
AB||TMT’

Hence, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc

Hence proved.

Question 10:

In a figure the common tangents, AB and CD to two circles with centres 0 and O' intersect at E. Prove that the points 0, E and O' are collinear.



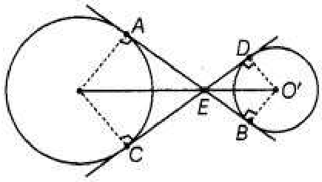
Solution:

Joint AO, OC and $O'D, O'B$.

Now, in $\Delta EO'D$ and $\Delta EO'B$,

$$O'D = O'B \quad \text{[radius]}$$
$$O'E = O'E \quad \text{[common side]}$$
$$ED = EB$$

[since, tangents drawn from an external point to the circle are equal in length]



$\therefore$ $\Delta EO'D \cong \Delta EO'B$ [by SSS congruence rule]

$\Rightarrow$ $\angle O'ED = \angle O'EB$

$O'E$ is the angle bisector of $\angle DEB$. ...(i)

Similarly, OE is the angle bisector of $\angle AEC$.

Now, in quadrilateral $DEBO'$,

$$\angle O'DE = \angle O'BE = 90^\circ$$

[since, CED is a tangent to the circle and $O'D$ is the radius, i.e., $O'D \perp CED$]

$\Rightarrow$ $\angle O'DE + \angle O'BE = 180^\circ$

$\therefore$ $\angle DEB + \angle DO'B = 180^\circ$ [since, $DEBO'$ is cyclic quadrilateral] ...(ii)

Since, AB is a straight line.

$\therefore$ $\angle AED + \angle DEB = 180^\circ$

$\Rightarrow$ $\angle AED + 180^\circ - \angle DO'B = 180^\circ$ [from Eq. (ii)]

$\Rightarrow$ $\angle AED = \angle DO'B$ ...(iii)

Similarly, $\angle AED = \angle AOC$ ...(iv)

Again from Eq. (ii), $\angle DEB = 180^\circ - \angle DO'B$

Divided by 2 on both sides, we get

$$\frac{1}{2}\angle DEB = 90^\circ - \frac{1}{2}\angle DO'B$$

$\Rightarrow$ $\angle DEO' = 90^\circ - \frac{1}{2}\angle DO'B$ ...(v)

[since, $O'E$ is the angle bisector of $\angle DEB$ i.e., $\frac{1}{2}\angle DEB = \angle DEO'$]

Similarly, $\angle AEC = 180^\circ - \angle AOC$

Divided by 2 on both sides, we get

$$\frac{1}{2}\angle AEC = 90^\circ - \frac{1}{2}\angle AOC$$

$\Rightarrow$ $\angle AEO = 90^\circ - \frac{1}{2}\angle AOC$ ...(vi)

[since, OE is the angle bisector of $\angle AEC$ i.e., $\frac{1}{2}\angle AEC = \angle AEO$]

Now, $\angle AED + \angle DEO' + \angle AEO = \angle AED + \left(90^\circ - \frac{1}{2}\angle DO'B\right) + \left(90^\circ - \frac{1}{2}\angle AOC\right)$

$$= \angle AED + 180^\circ - \frac{1}{2}(\angle DO'B + \angle AOC)$$

$$= \angle AED + 180^\circ - \frac{1}{2}(\angle AED + \angle AED) \quad \text{[from Eqs. (iii) and (iv)]}$$

$$= \angle AED + 180^\circ - \frac{1}{2}(2 \times \angle AED)$$

$$= \angle AED + 180^\circ - \angle AED = 180^\circ$$

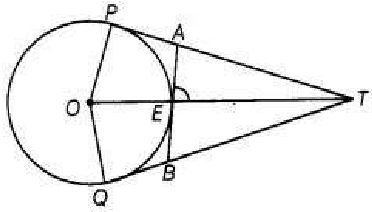
$\therefore$ $\angle AEO + \angle AED + \angle DEO' = 180^\circ$

So, OEO' is straight line.

Hence, O, E and O' are collinear. **Hence proved.**

Question 11:

In figure, 0 is the centre of a circle of radius 5 cm, T is a point such that OT = 13 and 0T intersects the circle at E, if AB is the tangent to the circle at E, find the length of AB.



Solution:

Given, OT = 13 cm and OP = 5 cm

Since, if we drawn a line from the centre to the tangent of the circle. It is always perpendicular to the tangent i.e., OP⊥PT.

In right angled ΔOPT, $OT^2 = OP^2 + PT^2$

[by Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$]

$\Rightarrow$ $PT^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$\Rightarrow$ $PT = 12$ cm

Since, the length of pair of tangents from an external point T is equal.

$\therefore$ $QT = 12$cm

Now, $TA = PT - PA$

$\Rightarrow$ $TA = 12 - PA$ ...(i)

and $TB = QT - QB$

$\Rightarrow$ $TB = 12 - QB$ ...(ii)

Again, using the property, length of pair of tangents from an external point is equal.

$\therefore$ $PA = AE$ and $QB = EB$ ...(iii)

$\therefore$ $OT = 13$cm

$\therefore$ $ET = OT - OE$ [$\because OE = 5$cm = radius]

$\Rightarrow$ $ET = 13 - 5$

$\Rightarrow$ $ET = 8$cm

Since, AB is a tangent and OE is the radius.

$\therefore$ $OE \perp AB$

$\Rightarrow$ $\angle OEA = 90°$

$\therefore$ $\angle AET = 180° - \angle OEA$ [linear pair]

$\Rightarrow$ $\angle AET = 90°$

Now, in right angled ΔAET,

$(AT)^2 = (AE)^2 + (ET)^2$ [by Pythagoras theorem]

$\Rightarrow$ $(PT - PA)^2 = (AE)^2 + (8)^2$

$\Rightarrow$ $(12 - PA)^2 = (PA)^2 + (8)^2$ [from Eq. (iii)]

$\Rightarrow$ $144 + (PA)^2 - 24 \cdot PA = (PA)^2 + 64$

$\Rightarrow$ $24 \cdot PA = 80$

$\Rightarrow$ $PA = \frac{10}{3}$cm

$\therefore$ $AE = \frac{10}{3}$ cm [from Eq. (iii)]

Join OQ.

Similarly $BE = \frac{10}{3}$cm

Hence, $AB = AE + EB$

$= \frac{10}{3} + \frac{10}{3}$

$= \frac{20}{3}$cm

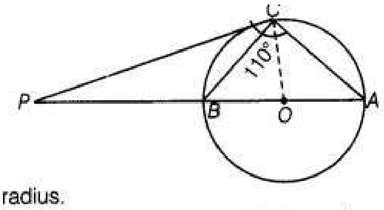
Hence, the required length AB is $\frac{20}{3}$cm.

Question 12:

The tangent at a point C of a circle and a diameter AB when extended intersect at P. If ∠PCA = 110°, find ∠CBA.

Solution:

Here, AB is a diameter of the circle from point C and a tangent is drawn which meets at a point P.



Join OC. Here, OC is radius.

Since, tangent at any point of a circle is perpendicular to the radius through point of contact circle.

$\therefore \quad OC \perp PC$

Now, $\quad \angle PCA = 110°$ [given]

$\Rightarrow \quad \angle PCO + \angle OCA = 110°$

$\Rightarrow \quad 90° + \angle OCA = 110°$

$\Rightarrow \quad \angle OCA = 20°$

$\therefore \quad OC = OA =$ Radius of circle

$\Rightarrow \quad \angle OCA = \angle OAC = 20°$

[since, two sides are equal, then their opposite angles are equal]

Since, PC is a tangent, so $\quad \angle BCP = \angle CAB = 20°$

[angles in a alternate segment are equal]

In ΔPBC, $\quad \angle P + \angle C + \angle A = 180°$

$\angle P = 180° - (\angle C + \angle A)$

$= 180° - (110° + 20°)$

$= 180° - 130° = 50°$

In ΔPBC, $\quad \angle BPC + \angle PCB + \angle PBC = 180°$

[sum of all interior angles of any triangle is 180°]

$\Rightarrow \quad 50° + 20° + \angle PBC = 180°$

$\Rightarrow \quad \angle PBC = 180° - 70°$

$\Rightarrow \quad \angle PBC = 110°$

Since, APB is a straight line.

$\therefore \quad \angle PBC + \angle CBA = 180°$

$\Rightarrow \quad \angle CBA = 180° - 110° = 70°$

Question 13:

If an isosceles ΔABC in which AB = AC = 6 cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.

Solution:

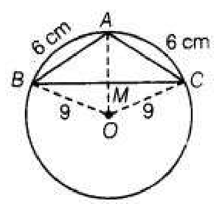
In a circle, ΔABC is inscribed.

Join OB, OC and OA.

Conside ΔABO and ΔACO

$AB = AC$ [given]

$BO = CO$ [radii of same circle]



AO is common.

$\therefore \quad \Delta ABO \cong \Delta ACO$ [by SSS congruence rule]

$\Rightarrow \quad \angle 1 = \angle 2$ [CPCT]

Now, in ΔABM and ΔACM,

$AB = AC$ [given]

$\angle 1 = \angle 2$ [proved above]

AM is common.

$\therefore \quad \Delta AMB \cong \Delta AMC$ [by SAS congruence rule]

$\Rightarrow \quad \angle AMB = \angle AMC$ [CPCT]

Also, $\quad \angle AMB + \angle AMC = 180°$ [linear pair]

$\Rightarrow \quad \angle AMB = 90°$

We know that a perpendicular from centre of circle bisects the chord.

So, OA is perpendicular bisector of BC.

Let $AM = x$, then $OM = 9 - x$ [$\because OA =$ radius $= 9$ cm]

In right angled ΔAMC, $\quad AC^2 = AM^2 + MC^2$ [by Pythagoras theorem]

i.e., $\quad (\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$\Rightarrow \quad MC^2 = 6^2 - x^2$ ...(i)

and in right ΔOMC, $\quad OC^2 = OM^2 + MC^2$ [by Pythagoras theorem]

$\Rightarrow \quad MC^2 = 9^2 - (9 - x)^2$ ...(ii)

From Eqs. (i) and (ii), $6^2 - x^2 = 9^2 - (9 - x)^2$

$\Rightarrow$ $36 - x^2 = 81 - (81 + x^2 - 18x)$

$\Rightarrow$ $36 = 18x \Rightarrow x = 2$

$\therefore$ $AM = x = 2$

In right angled ΔABM, $AB^2 = BM^2 + AM^2$ [by Pythagoras theorem]

$6^2 = BM^2 + 2^2$

$\Rightarrow$ $BM^2 = 36 - 4 = 32$

$\Rightarrow$ $BM = 4\sqrt{2}$

$\therefore$ $BC = 2\,BM = 8\sqrt{2}$ cm

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times BC \times AM$

$= \frac{1}{2} \times 8\sqrt{2} \times 2 = 8\sqrt{2}\ \text{cm}^2$

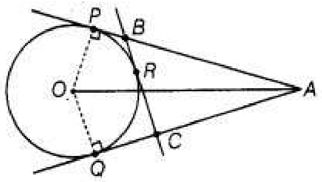
Hence, the required area of ΔABC is $8\sqrt{2}\ \text{cm}^2$.

Question 14:

A is a point at a distance 13 cm from the centre 0 of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC.

Solution:

Given Two tangents are drawn from an external point A to the circle with centre 0,



$OA = 13$ cm

Tangent BC is drawn at a point R. radius of circle equals 5cm.

To find perimeter of ΔABC.

Proof $\angle OPA = 90°$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + PA^2$ [by Pythagoras theorm]

$(13)^2 = 5^2 + PA^2$

$\Rightarrow$ $PA^2 = 144 = 12^2$

$\Rightarrow$ $PA = 12$ cm

Now, perimeter of $\Delta ABC = AB + BC + CA$

$= (AB + BR) + (RC + CA)$

$= AB + BP + CQ + CA$

[$\because BR = BP, RC = CQ$ tangents from internal point to a circle are equal]

$= AP + AQ$

$= 2\,AP$

$= 2(12)$

$= 24$cm

[$AP = AQ$ tangent from internal point to a circle are equal]

Hence, the perimeter of $\Delta ABC = 24$ cm.