

Differential Equations

Question 1.

If $(x + 2y^3) \frac{dy}{dx} = y$, then

(a) $\frac{x}{y} + y^2 = c$

(b) $\frac{y}{x} + x^2 = c$

(c) $\frac{x}{y} - y^2 = c$

(d) $\frac{y}{x} - x^2 = c$

Answer:

(c) $\frac{x}{y} - y^2 = c$

Question 2.

The solution of $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{\sqrt{1+x^2}}$ is

(a) $y = \frac{1+x^2}{x} + \frac{c}{x}$

(b) $y = \frac{\sqrt{1+x^2}}{x} + \frac{c}{x}$

(c) $y = \frac{x}{\sqrt{1+x^2}} + cx$

(d) none of these

Answer:

(b) $y = \frac{\sqrt{1+x^2}}{x} + \frac{c}{x}$

Question 3.

The solution of differential equation $\frac{dy}{dx} - 3y = \sin 2x$ is

(a) $y = e^{-3x} \left[\frac{\cos 2x + 3 \sin 2x}{13} \right] + c$

(b) $y = e^{-3x} \left(\frac{\cos 2x - 3 \sin 2x}{13} \right) + c$

(c) $ye^{-3x} = -e^{-3x} \frac{(2 \cos 2x + 3 \sin 2x)}{13} + c$

(d) none of these

Answer:

(c) $ye^{-3x} = -e^{-3x} \frac{(2 \cos 2x + 3 \sin 2x)}{13} + c$

Question 4.

The solution of the differential equation,

$x^2 \frac{dy}{dx} \cdot \cos \frac{1}{x} - y \sin \frac{1}{x} = -1$, where $y \rightarrow -1$ as $x \rightarrow \infty$, is

(a) $y = \sin \frac{1}{x} - \cos \frac{1}{x}$ (b) $y = \frac{x+1}{x \sin \frac{1}{x}}$

(c) $y = \cos \frac{1}{x} + \sin \frac{1}{x}$ (d) $y = \frac{x+1}{x \cos \frac{1}{x}}$

Answer:

(a) $y = \sin \frac{1}{x} - \cos \frac{1}{x}$

Question 5.

The degree of the differential equation

$\left(\frac{d^2 y}{dx^2} \right)^2 + \left(\frac{dy}{dx} \right)^2 = x \sin \left(\frac{dy}{dx} \right)$ is

(a) 1

(b) 2

(c) 3

(d) not defined

Answer:

(d) not defined

Question 6.

The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$ respectively are

(a) 2 and not defined

(b) 2 and 2

(c) 2 and 3

(d) 3 and 3

Answer:

(a) 2 and not defined

Question 7.

Integrating factor of the differential equation

$$(1-x^2)\frac{dy}{dx} - xy = 1 \text{ is}$$

(a) $-x$ (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}\log(1-x^2)$

Answer:

(c) $\sqrt{1-x^2}$

Question 8.

Integrating factor of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is

(a) $\cos x$

(b) $\sec x$

(c) $e^{\cos x}$

(d) $e^{\sec x}$

Answer:

(b) $\sec x$

Question 9.

If $(x+y)^2 \frac{dy}{dx} = a^2$, $y = 0$ when $x = 0$, then $y = a$ if $\frac{x}{a} =$

(a) 1

(b) $\tan 1$

(c) $\tan 1 + 1$

(d) $\tan 1 - 1$

Answer:

(d) $\tan 1 - 1$

Question 10.

If $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$, $y(0) = 0$, then

$$\tan \frac{x+y}{2} =$$

- (a) $e^x - 1$ (b) $\frac{e^x - 1}{2}$ (c) $2(e^x - 1)$ (d) $1 - e^x$

Answer:

- (a) $e^x - 1$

Question 11.

If $\sin x \frac{dy}{dx} + y \cos x = x \sin x$, then $(y - 1) \sin x =$

- (a) $c - x \sin x$
(b) $c + x \cos x$
(c) $c - x \cos x$
(d) $c + x \sin x$

Answer:

- (c) $c - x \cos x$

Question 12.

The solution of differential equation $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$ is

- (a) $(e^y + 1) \sin x = c$
(b) $e^x \sin x = c$
(c) $(e^x + 1) \cos x = c$
(d) none of these

Answer:

- (a) $(e^y + 1) \sin x = c$

Question 13.

The solution of the differential equation $\frac{dy}{dx} = \frac{x}{1+x^2}$ is

(a) $y = \frac{1}{2} \log|2+x^2| + c$ (b) $y = \frac{1}{2} \log(1+x) + c$

(c) $y = \log\left(\sqrt{1+x^2}\right) + c$ (d) none of these

Answer:

- (c) $y = \log(\sqrt{1+x^2}) + c$

Question 14.

If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$, when $x = 5$, then the value of x when $y = 3$ is

- (a) e^5 (b) $e^6 + 1$ (c) $\frac{e^6 + 9}{2}$ (d) $\log_e 6$

Answer:

(c) $\frac{e^6 + 9}{2}$

Question 15.

If $\frac{dy}{dx} = y \sin 2x$, $y(0) = 1$ then solution is

- (a) $y = e \sin^2 x$ (b) $y = \sin^2 x$
(c) $y = \cos^2 x$ (d) $y = e^{\cos^2 x}$

Answer:

(a) $y = e \sin^2 x$

Question 16.

The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$ is

- (a) $\frac{d^2x}{dt^2} + nx = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$
(c) $\frac{d^2x}{dt^2} - n^2x = 0$ (d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

Answer:

(b) $\frac{d^2x}{dt^2} + n^2x = 0$

Question 17.

The differential equation of all parabolas whose axes are parallel to y-axis is

- (a) $\frac{dy}{dx} = -\frac{c^2}{x^2}$ (b) $\frac{d^2x}{dy^2} = c$
(c) $\frac{d^3y}{dx^3} + \frac{d^2x}{dy^2} = 0$ (d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = c$

Answer:

(a) $\frac{dy}{dx} = -\frac{c^2}{x^2}$

Question 18.

The Solution of $\cos(x + y) dy = dx$ is

(a) $y = \tan\left(\frac{x+y}{2}\right) + C$ (b) $y = \cos^{-1}\left(\frac{y}{x}\right) + C$

(c) $y = x \sec\left(\frac{y}{x}\right) + C$ (d) none of these

Answer:

(a) $y = \tan\left(\frac{x+y}{2}\right) + C$

Question 19.

If $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 1$, then $y =$

(a) $x + \ln x$ (b) $x^2 + x \ln x$
(c) xe^{x-1} (d) $x + x \ln x$

Answer:

(d) $x + x \ln x$

Question 20.

If $(x^2 + y^2)dy = xy dx$, $y(1) = 1$, and $y(x_0) = e$, then $x_0 =$

(a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$
(c) $\sqrt{3}.e$ (d) $\sqrt{\frac{e^2 + 1}{2}}$

Answer:

(c) $\sqrt{3}e$

Question 21.

If $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$, $y(1) = \frac{\pi}{2}$, then $y\left(\frac{1}{2}\right) =$

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

Answer:

(d) $\frac{\pi}{12}$

Question 22.

If $\frac{dy}{dx} = \frac{y}{x} \left(\frac{x \cos \frac{y}{x} + y \sin \frac{y}{x}}{y \sin \frac{y}{x} - x \cos \frac{y}{x}} \right)$, then

- (a) $x \cos \frac{y}{x} = cy$ (b) $x \sec \frac{y}{x} = cy$
(c) $\cos \frac{y}{x} = cxy$ (d) $\sec \frac{y}{x} = cxy$

Answer:

(d) $\sec \frac{y}{x} = cxy$

Question 23.

If $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$, then

- (a) $\sqrt{\frac{x}{y}} = \ln cy$ (b) $-\sqrt{\frac{x}{y}} = \ln cy$
(c) $-2\sqrt{\frac{x}{y}} = \ln cy$ (d) $2\sqrt{\frac{x}{y}} = \ln cy$

Answer:

(c) $-2\sqrt{\frac{x}{y}} = \ln cy$

Question 24.

If $(1 + e^{x/y})dx + \left(1 - \frac{x}{y}\right)e^{x/y}dy = 0$, then

- (a) $x - ye^{x/y} = c$ (b) $y - xe^{x/y} = c$
(c) $x + ye^{x/y} = c$ (d) $y + xe^{x/y} = c$

Answer:

(c) $x + ye^{x/y} = c$

Question 25.

The solution curve of $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$, $y(-1) = 1$ is

- (a) a straight line (b) parabola
(c) circle (d) ellipse

Answer:

(c) Circle

Question 26.

The differential equation of all circles which pass through the origin and whose centre lies on y-axis is

- (a) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$ (b) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$
(c) $(x^2 - y^2)\frac{dy}{dx} - xy = 0$ (d) $(x^2 - y^2)\frac{dy}{dx} + xy = 0$

Answer:

(a) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

Question 27.

The differential equation of the family of circles touching the x-axis at origin is given by

- (a) $y'' = \frac{1}{x^2 - y^2}y'$ (b) $y' = \frac{2xy}{x^2 - y^2}$
(c) $y'' - y' = \frac{xy}{x^2 - y^2}$ (d) none of these

Answer:

(b) $y' = \frac{2xy}{x^2 - y^2}$

Question 28.

The differential equation representing the family of ellipses with centre at origin and foci on x-axis is given as

(a) $xy' + y = 0$

(b) $x^2y^2(y'')^2 + yy' = 0$

(c) $xyy'' + x(y')^2 - yy' = 0$

(d) None of these

Answer:

(b) $x^2y^2(y'')^2 + yy' = 0$

Question 29.

The differential equation of all parabolas whose axes are along x-axis is

(a) $y_2^2 + y_1 = 0$

(b) $y_1^2 + y_2 = 0$

(c) $y_1^2 + y_1y_2 = 0$

(d) $y_1^2 + yy_2 = 0$

Answer:

(d) $y_1^2 + yy_2 = 0$

Question 30.

The equation of family of curves for which the length of the normal is equal to the radius vector is

(a) $y^2 \mp x^2 = k^2$

(b) $y \pm x = k$

(c) $y^2 = kx$

(d) none of these

Answer:

(a) $y^2 \mp x^2 = k^2$

Question 31.

Given the differential equation $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$; $y(1) = \pi$

Mark out the correct statement.

(a) solution is $y^2 - \sin y = -2x^3 + C$

(b) solution is $y^2 + \sin y = 2x^3 + C$

(c) $C = \pi^2 + 2\sqrt{2}$

(d) $C = \pi^2 + 2$

Answer:

(b) solution is $y^2 + \sin y = 2x^3 + C$

Question 32.

The differential equation of all parabolas whose axis of symmetry is along the axis of the x-axis is of order

- (a) 3
- (b) 1
- (c) 2
- (d) none of these

Answer:

(c) 2

Question 33.

The degree of the equation satisfying the relation

$$\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda(\sqrt{1+y^2} - y\sqrt{1+x^2}) \text{ is}$$

- (a) 1
- (b) 2
- (c) 3
- (d) none of these

Answer:

(a) 1

Question 34.

The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{2/3} + 4 - \frac{3dy}{dx} = 0$ is

- (a) 2
- (b) 1
- (c) 3
- (d) none of these

Answer:

(a) 2

Question 35.

The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ is (a is a constant)

$$(a) \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = a^2 \frac{d^2 y}{dx^2}$$

$$(b) \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = a^2 \left(\frac{d^2 y}{dx^2} \right)^2$$

$$(c) \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = a^2 \left(\frac{d^2 y}{dx^2} \right)^2$$

(d) none of these

Answer:

$$(b) \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = a^2 \left(\frac{d^2 y}{dx^2} \right)^2$$

Question 36.

The differential equation satisfied by $y = \frac{A}{x} + B$ is (A, B are parameters)

$$(a) x^2 y_1 = y$$

$$(b) xy_1 + 2y_2 = 0$$

$$(c) xy_2 + 2y_1 = 0$$

(d) none of these

Answer:

$$(c) xy_2 + 2y_1 = 0$$

Question 37.

The solution of a differential equation is $y = c_1 e^{4x} + c_2 e^{3x}$, the differential equation is given by

$$(a) \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 7 = 0 \quad (b) \frac{d^2 y}{dx^2} + 7 \frac{dy}{dx} + 12y = 0$$

$$(c) \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 12y = 0 \quad (d) \text{ none of these}$$

Answer:

$$(c) \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 12y = 0$$

Question 38.

The differential equation satisfied by

$$\sqrt{1+x^2} + \sqrt{1+y^2}$$

$$= \lambda (x\sqrt{1+y^2} - y\sqrt{1+x^2}), \lambda \in \mathbb{R} \text{ is}$$

(a) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

(b) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

(c) $\frac{dy}{dx} = (1+x^2)(1+y^2)$ (d) none of these

Answer:

(b) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Question 39.

The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $y = \tan^{-1} x$

(b) $y - x = k(1 + xy)$

(c) $x = \tan^{-1} y$

(d) $\tan(xy) = k$

Answer:

(b) $y - x = k(1 + xy)$

Question 40.

The solution of the differential equation $\cos x \sin y \, dx + \sin x \cos y \, dy = 0$ is

(a) $\frac{\sin x}{\sin y} = c$

(b) $\sin x \sin y = c$

(c) $\sin x + \sin y = c$

(d) $\cos x \cos y = c$

Answer:

(b) $\sin x \sin y = c$

Question 41.

Which of the following is the general solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0 \text{ ?}$$

- (a) $y = (Ax + B)e^x$ (b) $y = (Ax + B)e^{-x}$
 (c) $y = Ae^x + Be^{-x}$ (d) $y = A\cos x + B\sin x$

Answer:

(a) $y = (Ax + B)e^x$

Question 42.

General solution of $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is

- (a) $y(1+x^2) = c + \tan^{-1} x$ (b) $\frac{y}{1+x^2} = c + \tan^{-1} x$
 (c) $y \log(1+x^2) = c + \tan^{-1} x$
 (d) $y(1+x^2) = c + \sin^{-1} x$

Answer:

(a) $y(1+x^2) = c + \tan^{-1} x$

Question 43.

If $\frac{xdy}{dx} - y = \sqrt{x^2 + y^2}$, then

- (a) $x + \sqrt{x^2 + y^2} = cy^2$ (b) $\sqrt{x^2 + y^2} - y = cx^2$
 (c) $\sqrt{x^2 + y^2} + y = cx^2$ (d) $\sqrt{x^2 + y^2} - x = cy^2$

Answer:

(c) $\sqrt{x^2 + y^2} + y = cx^2$

Question 44.

The solution of the differential equation $(x^2 + y^2) dx - 2xy dy = 0$ is

$$(a) \frac{y}{x^2 + y^2} = c$$

$$(b) \frac{x^2 + y^2}{x} = c$$

$$(c) \frac{y^2 - x^2}{y} = c$$

$$(d) \frac{x^2 - y^2}{x} = c$$

Answer:

$$(d) \frac{x^2 - y^2}{x} = c$$

Question 45.

The solution of the differential equation $x dy + (x + y) dx = 0$ is

$$(a) c = \frac{y^2}{2} + xy$$

$$(b) c = xy + \frac{x^2}{2}$$

$$(c) c = x + \frac{(xy)^2}{2}$$

(d) none of these

Answer:

$$(b) c = xy + \frac{x^2}{2}$$

Question 46.

The solution of differential equation $\frac{dy}{dx} = \frac{x-y}{x+y}$ is

$$(a) x^2 - y^2 + 2xy + c = 0$$

$$(b) x^2 - y^2 - xy + c = 0$$

$$(c) x^2 - y^2 + xy + c = 0$$

$$(d) x^2 - y^2 - 2xy + c = 0$$

Answer:

$$(d) x^2 - y^2 - 2xy + c = 0$$

Question 47.

The particular solution $\ln\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$ is

$$(a) e^{3x} + 3e^{-4y} = 4$$

$$(b) 4e^{3x} - 3e^{-4y} = 3$$

$$(c) 3e^{3x} + 4e^{4y} = 7$$

$$(d) 4e^{3x} + 3e^{-4y} = 7$$

Answer:

$$(d) 4e^{3x} + 3e^{-4y} = 7$$

Question 48.

The solution of the differential equation

$$\frac{x}{x^2 + y^2} dy = \left(\frac{y}{x^2 + y^2} - 1 \right) dx, \text{ is}$$

- (a) $y = x \cot(C - x)$ (b) $\cos^{-1} \frac{y}{x} = (-x + C)$
(c) $y = x \tan(C - x)$ (d) $\frac{y^2}{x^2} = x \tan(C - x)$

Answer:

(c) $y = x \tan(C - x)$

Question 49.

The solution of the differential equation

$$\left(\frac{x + y - 1}{x + y - 2} \right) \frac{dy}{dx} = \left(\frac{x + y + 1}{x + y + 2} \right), \text{ when } x = 1, y = 1, \text{ is}$$

- (a) $\log \left| \frac{(x - y)^2 - 2}{2} \right| = 2(x + y)$
(b) $\log \left| \frac{(x - y)^2 + 2}{2} \right| = 2(x - y)$
(c) $\log \left| \frac{(x + y)^2 + 2}{2} \right| = 2(x - y)$
(d) none of these

Answer:

(d) None of these

Question 50.

The solution of the differential equation

$$x dx + y dy + \frac{x dy - y dx}{x^2 + y^2} = 0, \text{ is}$$

(a) $y = x \tan\left(\frac{x^2 + y^2 + C}{2}\right)$

(b) $x = y \tan\left(\frac{x^2 + y^2 + C}{2}\right)$

(c) $y = x \tan\left(\frac{C - x^2 - y^2}{2}\right)$

(d) none of these

Answer:

(c) $y = x \tan\left(\frac{C - x^2 - y^2}{2}\right)$

Question 51.

If $\frac{dy}{dx} = \frac{2}{x + y}$, then $x + y + 2 =$

(a) ce^y (b) $ce^{y/2}$ (c) ce^{-y} (d) $ce^{-\frac{y}{2}}$

Answer:

(b) $ce^{y/2}$

Question 52.

The differential equation $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{y}}$ determines a family of circle with

- (a) variable radii and fixed centre (0, 1)
- (b) variable radii and fixed centre (0, -1)
- (c) fixed radius 1 and variable centre on x-axis
- (d) fixed radius 1 and variable centre on y-axis

Answer:

- (c) fixed radius 1 and variable centre on x-axis

Question 53.

If $y dx + y^2 dy = x dy$, $x \in \mathbb{R}$, $y > 0$ and $y(1) = 1$, then $y(-3) =$

- (a) 3

(b) 2

(c) 1

(d) 5

Answer:

(a) 3

Question 54.

The solution of $y \, dx + (x + x^2y) \, dy = 0$ is

(a) $-\frac{1}{xy} = c$

(b) $-\frac{1}{xy} + \ln y = c$

(c) $\frac{1}{xy} + \ln y = c$

(d) $\ln y = cx$

Answer:

(b) $-\frac{1}{xy} + \ln y = c$

Question 55.

If $\frac{xdy}{dx} + 2y = \ln x$, then $e^2 y(e) - y(1) =$

(a) $\frac{e^2 + 1}{2}$ (b) $\frac{e^2 + 1}{3}$ (c) $\frac{e^2 + 1}{4}$ (d) $e^2 + 1$

Answer:

(c) $\frac{e^2 + 1}{4}$

Question 56.

If $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$, then $4y(2) - y(1) =$

(a) 0 (b) 2 (c) 4 (d) 6

Answer:

(d) 6

Question 57.

If $x \ln x \frac{dy}{dx} + y = 2 \ln x$, $y(e) = 2$, then $y(e^2) =$

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

Answer:

(d) $\frac{5}{2}$

Question 58.

If $(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x, y(0) = 1$, then $y\left(\frac{\pi}{4}\right) =$

(a) $\frac{1}{e}$ (b) e (c) $2e$ (d) $\frac{2}{e}$

Answer:

(d) $\frac{2}{e}$