

Chapter 12 – Introduction to Three Dimensional Geometry

Page No 271:

Question 1:

A point is on the x-axis. What are its y-coordinates and z-coordinates?

Answer:

If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

Question 2:

A point is in the XZ-plane. What can you say about its y-coordinate?

Answer:

If a point is in the XZ plane, then its y-coordinate is zero.

Question 3:

Name the octants in which the following points lie:

$(1, 2, 3)$, $(4, -2, 3)$, $(4, -2, -5)$, $(4, 2, -5)$, $(-4, 2, -5)$, $(-4, 2, 5)$,
 $(-3, -1, 6)$, $(2, -4, -7)$

Answer:

The x-coordinate, y-coordinate, and z-coordinate of point $(1, 2, 3)$ are all positive. Therefore, this point lies in octant I.

The x-coordinate, y-coordinate, and z-coordinate of point $(4, -2, 3)$ are positive, negative, and positive respectively. Therefore, this point lies in octant IV.

The x-coordinate, y-coordinate, and z-coordinate of point $(4, -2, -5)$ are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

The x-coordinate, y-coordinate, and z-coordinate of point $(4, 2, -5)$ are positive, positive, and negative respectively. Therefore, this point lies in octant V.

The x-coordinate, y-coordinate, and z-coordinate of point $(-4, 2, -5)$ are negative, positive, and negative respectively. Therefore, this point lies in octant VI.

The x-coordinate, y-coordinate, and z-coordinate of point $(-4, 2, 5)$ are negative, positive, and positive respectively. Therefore, this point lies in octant II.

The x-coordinate, y-coordinate, and z-coordinate of point $(-3, -1, 6)$ are negative, negative, and positive respectively. Therefore, this point lies in octant III.

The x-coordinate, y-coordinate, and z-coordinate of point (2, -4, -7) are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

Question 4:

Fill in the blanks:

Answer:

- (i) The x-axis and y-axis taken together determine a plane known as XY – plane.
- (ii) The coordinates of points in the XY-plane are of the form (x, y, 0).
- (iii) Coordinate planes divide the space into eight octants.
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Page No 273:

Question 1:

Find the distance between the following pairs of points:

- (i) (2, 3, 5) and (4, 3, 1) (ii) (-3, 7, 2) and (2, 4, -1)
- (iii) (-1, 3, -4) and (1, -3, 4) (iv) (2, -1, 3) and (-2, 1, 3)

Answer:

The distance between points $P(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- (i) Distance between points (2, 3, 5) and (4, 3, 1)

$$\begin{aligned} &= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} \\ &= \sqrt{(2)^2 + (0)^2 + (-4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

- (ii) Distance between points (-3, 7, 2) and (2, 4, -1)

$$\begin{aligned} &= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} \\ &= \sqrt{(5)^2 + (-3)^2 + (-3)^2} \\ &= \sqrt{25+9+9} \\ &= \sqrt{43} \end{aligned}$$

(iii) Distance between points $(-1, 3, -4)$ and $(1, -3, 4)$

$$\begin{aligned} &= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2} \\ &= \sqrt{(2)^2 + (-6)^2 + (8)^2} \\ &= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26} \end{aligned}$$

(iv) Distance between points $(2, -1, 3)$ and $(-2, 1, 3)$

$$\begin{aligned} &= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2} \\ &= \sqrt{(-4)^2 + (2)^2 + (0)^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

Question 2:

Show that the points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Answer:

Let points $(-2, 3, 5)$, $(1, 2, 3)$, and $(7, 0, -1)$ be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$\begin{aligned}
 PQ &= \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} \\
 &= \sqrt{(3)^2 + (-1)^2 + (-2)^2} \\
 &= \sqrt{9+1+4} \\
 &= \sqrt{14}
 \end{aligned}$$

$$\begin{aligned}
 QR &= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} \\
 &= \sqrt{(6)^2 + (-2)^2 + (-4)^2} \\
 &= \sqrt{36+4+16} \\
 &= \sqrt{56} \\
 &= 2\sqrt{14}
 \end{aligned}$$

$$\begin{aligned}
 PR &= \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} \\
 &= \sqrt{(9)^2 + (-3)^2 + (-6)^2} \\
 &= \sqrt{81+9+36} \\
 &= \sqrt{126} \\
 &= 3\sqrt{14}
 \end{aligned}$$

$$\text{Here, } PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = PR$$

Hence, points P(-2, 3, 5), Q(1, 2, 3), and R(7, 0, -1) are collinear.

Question 3:

Verify the following:

- (i) (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an isosceles triangle.
- (ii) (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.
- (iii) (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram.

Answer:

- (i) Let points (0, 7, -10), (1, 6, -6), and (4, 9, -6) be denoted by A, B, and C respectively.

$$\begin{aligned}
 AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\
 &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\
 &= \sqrt{1+1+16} \\
 &= \sqrt{18} \\
 &= 3\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\
 &= \sqrt{(3)^2 + (3)^2} \\
 &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\
 &= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\
 &= \sqrt{16+4+16} = \sqrt{36} = 6
 \end{aligned}$$

Here, $AB = BC \neq CA$

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let $(0, 7, 10)$, $(-1, 6, 6)$, and $(-4, 9, 6)$ be denoted by A, B, and C respectively.

$$\begin{aligned}
 AB &= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} \\
 &= \sqrt{(-1)^2 + (-1)^2 + (-4)^2} \\
 &= \sqrt{1+1+16} = \sqrt{18} \\
 &= 3\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} \\
 &= \sqrt{(-3)^2 + (3)^2 + (0)^2} \\
 &= \sqrt{9+9} = \sqrt{18} \\
 &= 3\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 CA &= \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2} \\
 &= \sqrt{(4)^2 + (-2)^2 + (4)^2} \\
 &= \sqrt{16+4+16} \\
 &= \sqrt{36} \\
 &= 6
 \end{aligned}$$

$$\text{Now, } AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$, and $(2, -3, 4)$ be denoted by A, B, C, and D respectively.

$$\begin{aligned}AB &= \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2} \\&= \sqrt{4+16+16} \\&= \sqrt{36} \\&= 6\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2} \\&= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2} \\&= \sqrt{4+16+16} \\&= \sqrt{36} \\&= 6\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2} \\&= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

Here, $AB = CD = 6$, $BC = AD = \sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

Answer:

Let P (x, y, z) be the point that is equidistant from points A $(1, 2, 3)$ and B $(3, 2, -1)$.

Accordingly, $PA = PB$

$$\begin{aligned}\Rightarrow PA^2 &= PB^2 \\ \Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 &= (x-3)^2 + (y-2)^2 + (z+1)^2\end{aligned}$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

$$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$$

$$\Rightarrow -2x - 6z + 6x - 2z = 0$$

$$\Rightarrow 4x - 8z = 0$$

$$\Rightarrow x - 2z = 0$$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (-4, 0, 0) is equal to 10.

Answer:

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (-4, 0, 0) respectively.

It is given that $PA + PB = 10$.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$

$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$

$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$$

$$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

Question 1:

Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Answer:

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio $m:n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

Let R (x, y, z) be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ internally in the ratio 2:3

$$x = \frac{2(1) + 3(-2)}{2+3}, y = \frac{2(-4) + 3(3)}{2+3}, \text{ and } z = \frac{2(6) + 3(5)}{2+3}$$

$$\text{i.e., } x = \frac{-4}{5}, y = \frac{1}{5}, \text{ and } z = \frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5} \right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio $m:n$ are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$$

Let R (x, y, z) be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ externally in the ratio 2:3

$$x = \frac{2(1) - 3(-2)}{2-3}, y = \frac{2(-4) - 3(3)}{2-3}, \text{ and } z = \frac{2(6) - 3(5)}{2-3}$$

$$\text{i.e., } x = -8, y = 17, \text{ and } z = 3$$

Thus, the coordinates of the required point are $(-8, 17, 3)$.

Question 2:

Given that P $(3, 2, -4)$, Q $(5, 4, -6)$ and R $(9, 8, -10)$ are collinear. Find the ratio in which Q divides PR.

Answer:

Let point Q $(5, 4, -6)$ divide the line segment joining points P $(3, 2, -4)$ and R $(9, 8, -10)$ in the ratio $k:1$.

Therefore, by section formula,

$$(5, 4, -6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1} \right)$$

$$\Rightarrow \frac{9k+3}{k+1} = 5$$

$$\Rightarrow 9k+3 = 5k+5$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$.

Answer:

Let the YZ plane divide the line segment joining points $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1} \right)$$

On the YZ plane, the x-coordinate of any point is zero.

$$\frac{3k-2}{k+1} = 0$$

$$\Rightarrow 3k-2 = 0$$

$$\Rightarrow k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question 4:

Using section formula, show that the points A $(2, -3, 4)$, B $(-1, 2, 1)$ and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

Answer:

The given points are A $(2, -3, 4)$, B $(-1, 2, 1)$, and $C\left(0, \frac{1}{3}, 2\right)$.

Let P be a point that divides AB in the ratio $k:1$.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1} \right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1} = 0$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2 \right)$.

i.e., $C\left(0, \frac{1}{3}, 2 \right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

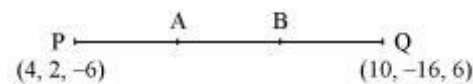
Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

Answer:

Let A and B be the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6)



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2} \right) = (6, -4, -2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)+1(6)}{2+1} \right) = (8, -10, 2)$$

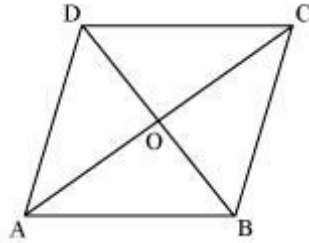
Thus, (6, -4, -2) and (8, -10, 2) are the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6).

Question 1:

Three vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4) and C (-1, 1, 2). Find the coordinates of the fourth vertex.

Answer:

The three vertices of a parallelogram ABCD are given as A (3, -1, 2), B (1, 2, -4), and C (-1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\Rightarrow (1, 0, 2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

$$\Rightarrow x = 1, y = -2, \text{ and } z = 8$$

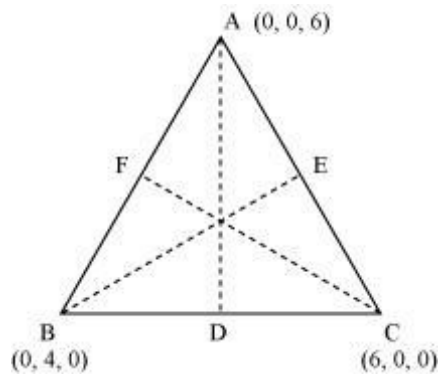
Thus, the coordinates of the fourth vertex are (1, -2, 8).

Question 2:

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Answer:

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

$$\therefore \text{Coordinates of point D} = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = (3, 2, 0)$$

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$$

Since BE is the median, E is the mid-point of AC.

$$\therefore \text{Coordinates of point E} = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2} \right) = (3, 0, 3)$$

$$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$$

Since CF is the median, F is the mid-point of AB.

$$\therefore \text{Coordinates of point F} = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right) = (0, 2, 3)$$

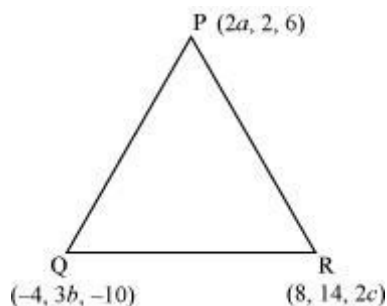
$$\text{Length of CF} = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

Thus, the lengths of the medians of $\triangle ABC$ are $7, \sqrt{34}$, and 7 .

Question 3:

If the origin is the centroid of the triangle PQR with vertices P $(2a, 2, 6)$, Q $(-4, 3b, -10)$ and R $(8, 14, 2c)$, then find the values of a, b and c.

Answer:



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1) ,

(x_2, y_2, z_2) and (x_3, y_3, z_3) , are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR

$$= \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR .

$$\begin{aligned}\therefore (0,0,0) &= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right) \\ \Rightarrow \frac{2a+4}{3} &= 0, \frac{3b+16}{3} = 0 \text{ and } \frac{2c-4}{3} = 0 \\ \Rightarrow a &= -2, b = -\frac{16}{3} \text{ and } c = 2\end{aligned}$$

Thus, the respective values of a, b, and c are $-2, -\frac{16}{3}, \text{ and } 2$.

Page No 279:

Question 4:

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, -2, 5).

Answer:

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, -2, 5).

Accordingly, $AP = 5\sqrt{2}$

$$\begin{aligned}\therefore AP^2 &= 50 \\ \Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 &= 50 \\ \Rightarrow 9 + 4 + b^2 + 4b + 25 &= 50 \\ \Rightarrow b^2 + 4b - 12 &= 0 \\ \Rightarrow b^2 + 6b - 2b - 12 &= 0 \\ \Rightarrow (b+6)(b-2) &= 0 \\ \Rightarrow b &= -6 \text{ or } 2\end{aligned}$$

Thus, the coordinates of the required points are (0, 2, 0) and (0, -6, 0).

Question 5:

A point R with x-coordinate 4 lies on the line segment joining the points P (2, -3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[Hint suppose R divides PQ in the ratio k: 1. The coordinates of the point R are given by

$$\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)]$$

Answer:

The coordinates of points P and Q are given as P (2, -3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1} \right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)$$

It is given that the x-coordinate of point R is 4.

$$\begin{aligned} \therefore \frac{8k+2}{k+1} &= 4 \\ \Rightarrow 8k+2 &= 4k+4 \\ \Rightarrow 4k &= 2 \\ \Rightarrow k &= \frac{1}{2} \end{aligned}$$

$$\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} \right) = (4, -2, 6)$$

Therefore, the coordinates of point R are

Question 6:

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Answer:

The coordinates of points A and B are given as (3, 4, 5) and (-1, 3, -7) respectively.

Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$\begin{aligned}
 PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\
 &= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\
 &= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50 \\
 PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\
 &= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59
 \end{aligned}$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\begin{aligned}
 (x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) &= k^2 \\
 \Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 &= k^2 \\
 \Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) &= k^2 - 109 \\
 \Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z &= \frac{k^2 - 109}{2}
 \end{aligned}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$.
