

Chapter 2 – Relations and Functions

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Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Answer:

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore,

$$\frac{x}{3}+1=\frac{5}{3} \text{ and } y-\frac{2}{3}=\frac{1}{3}.$$

$$\begin{aligned}\frac{x}{3}+1 &= \frac{5}{3} \\ \Rightarrow \frac{x}{3} &= \frac{5}{3}-1 \\ \Rightarrow \frac{x}{3} &= \frac{2}{3} & y-\frac{2}{3} &= \frac{1}{3} \\ \Rightarrow x &= 2 & \Rightarrow y &= \frac{1}{3}+\frac{2}{3} \\ & & \Rightarrow y &= 1\end{aligned}$$

$\therefore x = 2$ and $y = 1$

Question 2:

If the set A has 3 elements and the set $B = \{3, 4, 5\}$, then find the number of elements in $(A \times B)$?

Answer:

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

$\Rightarrow$ Number of elements in set B = 3

Number of elements in $(A \times B)$

= (Number of elements in A) $\times$ (Number of elements in B)

= $3 \times 3 = 9$

Thus, the number of elements in $(A \times B)$ is 9.

Question 3:

If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$, find $G \times H$ and $H \times G$.

Answer:

$$G = \{7, 8\} \text{ and } H = \{5, 4, 2\}$$

We know that the Cartesian product $P \times Q$ of two non-empty sets P and Q is defined as

$$P \times Q = \{(p, q): p \in P, q \in Q\}$$

$$\therefore G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$$

$$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$$

Question 4:

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m, n), (n, m)\}$.

(ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.

(iii) If $A = \{1, 2\}$, $B = \{3, 4\}$, then $A \times (B \cap \Phi) = \Phi$.

Answer:

(i) False

If $P = \{m, n\}$ and $Q = \{n, m\}$, then

$$P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$$

(ii) True

(iii) True

Question 5:

If $A = \{-1, 1\}$, find $A \times A \times A$.

Answer:

It is known that for any non-empty set A , $A \times A \times A$ is defined as

$$A \times A \times A = \{(a, b, c): a, b, c \in A\}$$

It is given that $A = \{-1, 1\}$

$$\therefore A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),$$

$$(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$$

Question 6:

If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

Answer:

It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as $P \times Q = \{(p, q): p \in P, q \in Q\}$

$\therefore$ A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

Question 7:

Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times C$ is a subset of $B \times D$

Answer:

(i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

$\therefore$ L.H.S. = $A \times (B \cap C) = A \times \Phi = \Phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\therefore$ R.H.S. = $(A \times B) \cap (A \times C) = \Phi$

$\therefore$ L.H.S. = R.H.S

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) To verify: $A \times C$ is a subset of $B \times D$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$.

Therefore, $A \times C$ is a subset of $B \times D$.

Question 8:

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

Answer:

$$A = \{1, 2\} \text{ and } B = \{3, 4\}$$

$$\therefore A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$\Rightarrow n(A \times B) = 4$$

We know that if C is a set with $n(C) = m$, then $n[P(C)] = 2^m$.

Therefore, the set $A \times B$ has $2^4 = 16$ subsets. These are

$$\Phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\}, \{(1, 3), (1, 4)\}, \{(1, 3), (2, 3)\},$$

$$\{(1, 3), (2, 4)\}, \{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3), (2, 4)\},$$

$$\{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4), (2, 4)\}, \{(1, 3), (2, 3), (2, 4)\},$$

$$\{(1, 4), (2, 3), (2, 4)\}, \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

Question 9:

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B , where x, y and z are distinct elements.

Answer:

It is given that $n(A) = 3$ and $n(B) = 2$; and $(x, 1), (y, 2), (z, 1)$ are in $A \times B$.

We know that A = Set of first elements of the ordered pair elements of $A \times B$

B = Set of second elements of the ordered pair elements of $A \times B$.

$\therefore x, y$, and z are the elements of A ; and 1 and 2 are the elements of B .

Since $n(A) = 3$ and $n(B) = 2$, it is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

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Question 10:

The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

Answer:

We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

$$\therefore n(A \times A) = n(A) \times n(A)$$

It is given that $n(A \times A) = 9$

$$\therefore n(A) \times n(A) = 9$$

$$\Rightarrow n(A) = 3$$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.

We know that $A \times A = \{(a, a): a \in A\}$. Therefore, $-1, 0$, and 1 are elements of A .

Since $n(A) = 3$, it is clear that $A = \{-1, 0, 1\}$.

The remaining elements of set $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0)$, and $(1, 1)$

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Question 1:

Let $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$. Write down its domain, codomain and range.

Answer:

The relation R from A to A is given as

$$R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$$

$$\text{i.e., } R = \{(x, y): 3x = y, \text{ where } x, y \in A\}$$

$$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$$\therefore \text{Domain of } R = \{1, 2, 3, 4\}$$

The whole set A is the codomain of the relation R .

$$\therefore \text{Codomain of } R = A = \{1, 2, 3, \dots, 14\}$$

The range of R is the set of all second elements of the ordered pairs in the relation.

$$\therefore \text{Range of } R = \{3, 6, 9, 12\}$$

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Question 2:

Define a relation R on the set N of natural numbers by $R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4; x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

Answer:

$$R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbb{N}\}$$

The natural numbers less than 4 are 1, 2, and 3.

$$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$$\therefore \text{Domain of } R = \{1, 2, 3\}$$

The range of R is the set of all second elements of the ordered pairs in the relation.

$$\therefore \text{Range of } R = \{6, 7, 8\}$$

Question 3:

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$. Write R in roster form.

Answer:

$$A = \{1, 2, 3, 5\} \text{ and } B = \{4, 6, 9\}$$

$$R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$$

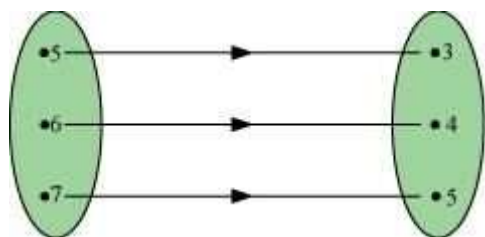
$$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

Question 4:

The given figure shows a relationship between the sets P and Q. write this relation

(i) in set-builder form (ii) in roster form.

What is its domain and range?



Answer:

$$\text{According to the given figure, } P = \{5, 6, 7\}, Q = \{3, 4, 5\}$$

$$(i) R = \{(x, y): y = x - 2; x \in P\} \text{ or } R = \{(x, y): y = x - 2 \text{ for } x = 5, 6, 7\}$$

$$(ii) R = \{(5, 3), (6, 4), (7, 5)\}$$

$$\text{Domain of } R = \{5, 6, 7\}$$

$$\text{Range of } R = \{3, 4, 5\}$$

Question 5:

Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by

$\{(a, b): a, b \in A, b \text{ is exactly divisible by } a\}$.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R .

Answer:

$A = \{1, 2, 3, 4, 6\}$, $R = \{(a, b): a, b \in A, b \text{ is exactly divisible by } a\}$

(i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

(ii) Domain of $R = \{1, 2, 3, 4, 6\}$

(iii) Range of $R = \{1, 2, 3, 4, 6\}$

Question 6:

Determine the domain and range of the relation R defined by $R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$.

Answer:

$R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

$\therefore R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

$\therefore$ Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Range of $R = \{5, 6, 7, 8, 9, 10\}$

Question 7:

Write the relation $R = \{(x, x^3): x \text{ is a prime number less than } 10\}$ in roster form.

Answer:

$R = \{(x, x^3): x \text{ is a prime number less than } 10\}$

The prime numbers less than 10 are 2, 3, 5, and 7.

$\therefore R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

Question 8:

Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B .

Answer:

It is given that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

$$\therefore A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$$

Since $n(A \times B) = 6$, the number of subsets of $A \times B$ is 2^6 .

Therefore, the number of relations from A to B is 2^6 .

Question 9:

Let R be the relation on Z defined by $R = \{(a, b): a, b \in Z, a - b \text{ is an integer}\}$. Find the domain and range of R .

Answer:

$$R = \{(a, b): a, b \in Z, a - b \text{ is an integer}\}$$

It is known that the difference between any two integers is always an integer.

$$\therefore \text{Domain of } R = Z$$

$$\text{Range of } R = Z$$

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Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

$$(i) \{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$$

$$(ii) \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$$

$$(iii) \{(1, 3), (1, 5), (2, 5)\}$$

Answer:

$$(i) \{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$$

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

$$\text{Here, domain} = \{2, 5, 8, 11, 14, 17\} \text{ and range} = \{1\}$$

$$(ii) \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$$

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

$$\text{Here, domain} = \{2, 4, 6, 8, 10, 12, 14\} \text{ and range} = \{1, 2, 3, 4, 5, 6, 7\}$$

(iii) $\{(1, 3), (1, 5), (2, 5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:

Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9-x^2}$

Answer:

(i) $f(x) = -|x|, x \in \mathbb{R}$

We know that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

Since $f(x)$ is defined for $x \in \mathbb{R}$, the domain of f is $\mathbb{R}$.

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

$\therefore$ The range of f is $(-\infty, 0]$.

(ii) $f(x) = \sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3 , the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3 .

$\therefore$ The range of $f(x)$ is $\{x : 0 \leq x \leq 3\}$ or $[0, 3]$.

Question 3:

A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

Answer:

The given function is $f(x) = 2x - 5$.

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

$$(ii) f(7) = 2 \times 7 - 5 = 14 - 5 = 9$$

$$(iii) f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$$

Question 4:

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is

defined by $t(C) = \frac{9C}{5} + 32$.

Find (i) t (0) (ii) t (28) (iii) t (-10) (iv) The value of C, when t(C) = 212

Answer:

The given function is $t(C) = \frac{9C}{5} + 32$.

Therefore,

$$(i) \quad t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$$

$$(ii) \quad t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$$

$$(iii) \quad t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$$

(iv) It is given that t(C) = 212

$$\begin{aligned} \therefore 212 &= \frac{9C}{5} + 32 \\ \Rightarrow \frac{9C}{5} &= 212 - 32 \\ \Rightarrow \frac{9C}{5} &= 180 \\ \Rightarrow 9C &= 180 \times 5 \\ \Rightarrow C &= \frac{180 \times 5}{9} = 100 \end{aligned}$$

Thus, the value of t, when t(C) = 212, is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x$, $x \in \mathbb{R}$, $x > 0$.

(ii) $f(x) = x^2 + 2$, x, is a real number.

(iii) $f(x) = x$, x is a real number

Answer:

(i) $f(x) = 2 - 3x$, $x \in \mathbb{R}$, $x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	...
f(x)	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	...

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.

i.e., range of $f = (-\infty, 2)$

Alter:

Let $x > 0$

$$\Rightarrow 3x > 0$$

$$\Rightarrow 2 - 3x < 2$$

$$\Rightarrow f(x) < 2$$

$\therefore$ Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2$, x , is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	± 0.3	± 0.8	± 1	± 2	± 3		...
f(x)	2	2.09	2.64	3	6	11		

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

i.e., range of $f = [2, \infty)$

Alter:

Let x be any real number.

Accordingly,

$$x^2 \geq 0$$

$$\Rightarrow x^2 + 2 \geq 0 + 2$$

$$\Rightarrow x^2 + 2 \geq 2$$

$$\Rightarrow f(x) \geq 2$$

∴ Range of $f = [2, \infty)$

(iii) $f(x) = x$, x is a real number

It is clear that the range of f is the set of all real numbers.

∴ Range of $f = \mathbb{R}$

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Question 1:

The relation f is defined by
$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

The relation g is defined by
$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

Show that f is a function and g is not a function.

Answer:

The relation f is defined as
$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

It is observed that for

$$0 \leq x < 3, f(x) = x^2$$

$$3 < x \leq 10, f(x) = 3x$$

Also, at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$

Therefore, for $0 \leq x \leq 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation g is defined as
$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

It can be observed that for $x = 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

Question 2:

If $f(x) = x^2$, find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$.

Answer:

$$f(x) = x^2$$
$$\therefore \frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Question 3:

Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Answer:

The given function is $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It can be seen that function f is defined for all real numbers except at $x = 6$ and $x = 2$.

Hence, the domain of f is $\mathbb{R} - \{2, 6\}$.

Question 4:

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x - 1)}$.

Answer:

The given real function is $f(x) = \sqrt{x - 1}$.

It can be seen that $\sqrt{x - 1}$ is defined for $(x - 1) \geq 0$.

i.e.,

$$f(x) = \sqrt{(x - 1)} \text{ is defined for } x \geq 1.$$

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f = [1, \infty)$.

As $x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow$

$$\sqrt{x - 1} \geq 0$$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f = [0, \infty)$.

Question 5:

Find the domain and the range of the real function f defined by $f(x) = |x - 1|$.

Answer:

The given real function is $f(x) = |x - 1|$.

It is clear that $|x - 1|$ is defined for all real numbers.

$\therefore$ Domain of $f = \mathbb{R}$

Also, for $x \in \mathbb{R}$, $|x - 1|$ assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

Question 6:

Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbb{R} \right\}$ be a function from $\mathbb{R}$ into $\mathbb{R}$. Determine the range of f .

Answer:

$$f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbb{R} \right\}$$
$$= \left\{ (0, 0), \left(\pm 0.5, \frac{1}{5} \right), \left(\pm 1, \frac{1}{2} \right), \left(\pm 1.5, \frac{9}{13} \right), \left(\pm 2, \frac{4}{5} \right), \left(3, \frac{9}{10} \right), \left(4, \frac{16}{17} \right), \dots \right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of $f = [0, 1)$

Question 7:

Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$.

Answer:

$f, g: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = x + 1$, $g(x) = 2x - 3$

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$$\therefore (f + g)(x) = 3x - 2$$

$$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$$

$$\therefore (f - g)(x) = -x + 4$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, 2x-3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$$

Question 8:

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from $\mathbf{Z}$ to $\mathbf{Z}$ defined by $f(x) = ax + b$, for some integers a, b . Determine a, b .

Answer:

$$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$$

$$f(x) = ax + b$$

$$(1, 1) \in f$$

$$\Rightarrow f(1) = 1$$

$$\Rightarrow a \times 1 + b = 1$$

$$\Rightarrow a + b = 1$$

$$(0, -1) \in f$$

$$\Rightarrow f(0) = -1$$

$$\Rightarrow a \times 0 + b = -1$$

$$\Rightarrow b = -1$$

On substituting $b = -1$ in $a + b = 1$, we obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$.

Thus, the respective values of a and b are 2 and -1 .

Question 9:

Let R be a relation from $\mathbf{N}$ to $\mathbf{N}$ defined by $R = \{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in \mathbf{N}$

(ii) $(a, b) \in R$, implies $(b, a) \in R$

(iii) $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

Answer:

$$R = \{(a, b): a, b \in \mathbb{N} \text{ and } a = b^2\}$$

(i) It can be seen that $2 \in \mathbb{N}$; however, $2 \neq 2^2 = 4$.

Therefore, the statement " $(a, a) \in R$, for all $a \in \mathbb{N}$ " is not true.

(ii) It can be seen that $(9, 3) \in R$ because $9, 3 \in \mathbb{N}$ and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin R$

Therefore, the statement " $(a, b) \in R$, implies $(b, a) \in R$ " is not true.

(iii) It can be seen that $(16, 4) \in R, (4, 2) \in R$ because $16, 4, 2 \in \mathbb{N}$ and $16 = 4^2$ and $4 = 2^2$.

Now, $16 \neq 2^2 = 4$; therefore, $(16, 2) \notin R$

Therefore, the statement " $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$ " is not true.

Question 10:

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B .

Justify your answer in each case.

Answer:

$$A = \{1, 2, 3, 4\} \text{ and } B = \{1, 5, 9, 11, 15, 16\}$$

$$\therefore A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$$

It is given that $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

It is observed that f is a subset of $A \times B$.

Thus, f is a relation from A to B .

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Question 11:

Let f be the subset of $\mathbb{Z} \times \mathbb{Z}$ defined by $f = \{(ab, a + b) : a, b \in \mathbb{Z}\}$. Is f a function from $\mathbb{Z}$ to $\mathbb{Z}$: justify your answer.

Answer:

The relation f is defined as $f = \{(ab, a + b) : a, b \in \mathbb{Z}\}$

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B .

Since $2, 6, -2, -6 \in \mathbb{Z}$, $(2 \times 6, 2 + 6), (-2 \times -6, -2 + (-6)) \in f$

i.e., $(12, 8), (12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8 . Thus, relation f is not a function.

Question 12:

Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \rightarrow \mathbb{N}$ be defined by $f(n) =$ the highest prime factor of n . Find the range of f .

Answer:

$A = \{9, 10, 11, 12, 13\}$

$f: A \rightarrow \mathbb{N}$ is defined as

$f(n) =$ The highest prime factor of n

Prime factor of $9 = 3$

Prime factors of $10 = 2, 5$

Prime factor of $11 = 11$

Prime factors of $12 = 2, 3$

Prime factor of $13 = 13$

$\therefore f(9) =$ The highest prime factor of $9 = 3$

$f(10) =$ The highest prime factor of $10 = 5$

$f(11) =$ The highest prime factor of $11 = 11$

$f(12) =$ The highest prime factor of $12 = 3$

$f(13) =$ The highest prime factor of $13 = 13$

The range of f is the set of all $f(n)$, where $n \in A$.

$\therefore$ Range of $f = \{3, 5, 11, 13\}$

