Question 1:

Express the given complex number in the form *a* + *ib*: $(5i)\left(-\frac{3}{5}i\right)$

Answer:

$$(5i)\left(\frac{-3}{5}i\right)=-5\times\frac{3}{5}\times i\times i$$
$$=-3i^2$$
$$=-3(-1) \qquad \left[i^2=-1\right]$$
$$=3$$

Question 2:

Express the given complex number in the form *a* + *ib*: $i^9 + i^{19}$

Answer:

$$i^9+i^{19}=i^{4\times2+1}+i^{4\times4+3}$$
$$=\left(i^4\right)^2\cdot i+\left(i^4\right)^4\cdot i^3$$
$$=1\times i+1\times(-i) \qquad \left[i^4=1,\ i^3=-i\right]$$
$$=i+(-i)$$
$$=0$$

Question 3:

Express the given complex number in the form *a* + *ib*: i^{-39}

Answer:

$$i^{-39}=i^{-4\times9-3}=\left(i^4\right)^{-9}\cdot i^{-3}$$
$$=(1)^{-9}\cdot i^{-3} \qquad \left[i^4=1\right]$$
$$=\frac{1}{i^3}=\frac{1}{-i} \qquad \left[i^3=-i\right]$$
$$=\frac{-1}{i}\times\frac{i}{i}$$
$$=\frac{-i}{i^2}=\frac{-i}{-1}=i \qquad \left[i^2=-1\right]$$

Question 4:

Express the given complex number in the form $a + ib$: 3(7 + i7) + i(7 + i7)

Answer:

$$\begin{aligned} 3(7+i7)+i(7+i7) &= 21+21i+7i+7i^2 \\ &= 21+28i+7\times(-1) \qquad [\because i^2=-1] \\ &= 14+28i \end{aligned}$$

Question 5:

Express the given complex number in the form $a + ib$: (1 − i) − (−1 + i6)

Answer:

$$\begin{aligned} (1-i)-(-1+i6) &= 1-i+1-6i \\ &= 2-7i \end{aligned}$$

Question 6:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Question 7:

Express the given complex number in the form $a + ib$: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Answer:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Question 8:

Express the given complex number in the form $a + ib$: $(1 - \mathrm{i})^4$

Answer:

$$\begin{aligned}(1-i)^4 &= \left[(1-i)^2\right]^2\\ &= \left[1^2+i^2-2i\right]^2\\ &= \left[1-1-2i\right]^2\\ &= (-2i)^2\\ &= (-2i)\times(-2i)\\ &= 4i^2=-4 \qquad \left[i^2=-1\right]\end{aligned}$$

Question 9:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{3}+3i\right)^3$

Answer:

$$\begin{aligned}\left(\frac{1}{3}+3i\right)^3 &= \left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)\\ &= \frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)\\ &= \frac{1}{27}+27(-i)+i+9i^2 \qquad \left[i^3=-i\right]\\ &= \frac{1}{27}-27i+i-9 \qquad \left[i^2=-1\right]\\ &= \left(\frac{1}{27}-9\right)+i(-27+1)\\ &= \frac{-242}{27}-26i\end{aligned}$$

Question 10:

Express the given complex number in the form $a + ib$: $\left(-2-\frac{1}{3}i\right)^3$

Answer:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number 4 – 3i

Answer:

Let z = 4 – 3i

Then,

$\bar{z}$ = 4 + 3i and $|z|^2=4^2+(-3)^2=16+9=25$

Therefore, the multiplicative inverse of 4 – 3i is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Answer:

Let z = $\sqrt{5}+3i$

Then, $\bar{z}=\sqrt{5}-3i$ and $|z|^2=\left(\sqrt{5}\right)^2+3^2=5+9=14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number –i

Answer:

Let z = –i

Then, $\bar{z}=i$ and $|z|^2=1^2=1$

Therefore, the multiplicative inverse of –i is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}=i$$

Question 14:

Express the following expression in the form of a + ib.

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Answer:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Page No 108:

Question 1:

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

Answer:

$z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+\left(-\sqrt{3}\right)^2$$

$$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+3$$

$$\Rightarrow r^2=4 \qquad \left[\cos^2\theta+\sin^2\theta=1\right]$$

$$\Rightarrow r=\sqrt{4}=2 \qquad [\text{Conventionally, } r>0]$$

$\therefore$ Modulus $=2$

$\therefore\ 2\cos\theta=-1$ and $2\sin\theta=-\sqrt{3}$

$$\Rightarrow \cos\theta=\frac{-1}{2} \text{ and } \sin\theta=\frac{-\sqrt{3}}{2}$$

Since both the values of sin θ and cos θ are negative and sinθ and cosθ are negative in III quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$

Answer:

$z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$$

$$\Rightarrow r^2 = 3 + 1 = 4 \qquad \left[\cos^2\theta + \sin^2\theta = 1\right]$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad \left[\text{Conventionally, } r > 0\right]$$

$$\therefore \text{Modulus } = 2$$

$$\therefore\ 2\cos\theta = -\sqrt{3} \text{ and } 2\sin\theta = 1$$

$$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \qquad \left[\text{As } \theta \text{ lies in the II quadrant}\right]$$

Thus, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question 3:

Convert the given complex number in polar form: 1 – i

Answer:

1 – i

Let r cos θ = 1 and r sin θ = –1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = 1 \text{ and } \sqrt{2}\sin\theta = -1$$
$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

$\therefore 1 - i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$ This is the required polar form.

Question 4:

Convert the given complex number in polar form: – 1 + i

Answer:

– 1 + i

Let r cos θ = –1 and r sin θ = 1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$
$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1 + i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: – 1 – i

Answer:

– 1 – i

Let r cos θ = –1 and r sin θ = –1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad \left[\text{Conventionally, } r > 0\right]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$
$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad \left[\text{As } \theta \text{ lies in the III quadrant}\right]$$

$$\therefore -1 - i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4} = \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$ This is the required polar form.

Question 6:

Convert the given complex number in polar form: –3

Answer:

–3

Let r cos θ = –3 and r sin θ = 0

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 9$$
$$\Rightarrow r^2 = 9$$
$$\Rightarrow r = \sqrt{9} = 3 \qquad \left[\text{Conventionally, } r > 0\right]$$
$$\therefore 3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$
$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$
$$\therefore \theta = \pi$$
$$\therefore -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3\left(\cos\pi + i\sin\pi\right)$$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3}+i$

Answer:

$\sqrt{3}+i$

Let r cos θ = $\sqrt{3}$ and r sin θ = 1

On squaring and adding, we obtain

$$
\begin{aligned}
& r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2 \\
& \Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 3+1 \\
& \Rightarrow r^2 = 4 \\
& \Rightarrow r = \sqrt{4} = 2 \qquad \left[\text{Conventionally, } r>0\right] \\
& \therefore 2\cos\theta = \sqrt{3} \text{ and } 2\sin\theta = 1 \\
& \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2} \\
& \therefore \theta = \frac{\pi}{6} \qquad \left[\text{As } \theta \text{ lies in the I quadrant}\right] \\
& \therefore \sqrt{3}+i = r\cos\theta + i\,r\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)
\end{aligned}
$$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: i

Answer:

i

Let r cosθ = 0 and r sin θ = 1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$$
$$\Rightarrow r^2 = 1$$
$$\Rightarrow r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$
$$\therefore \theta = \frac{\pi}{2}$$
$$\therefore i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

Question 1:

Solve the equation $x^2 + 3 = 0$

Answer:

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = 1, b = 0, and c = 3

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2\times 1} = \frac{\pm\sqrt{12}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$
$$= \frac{\pm 2\sqrt{3}\,i}{2} = \pm\sqrt{3}\,i$$

Question 2:

Solve the equation $2x^2 + x + 1 = 0$

Answer:

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain a = 2, b = 1, and c = 1

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times 2} = \frac{-1 \pm \sqrt{7}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$

Question 3:

Solve the equation $x^2 + 3x + 9 = 0$

Answer:

The given quadratic equation is $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = 1, b = 3, and c = 9

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Question 4:

Solve the equation $-x^2 + x - 2 = 0$

Answer:

The given quadratic equation is $-x^2 + x - 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = –1, b = 1, and c = –2

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm \sqrt{7}\, i}{-2} \qquad \left[\sqrt{-1} = i\right]$$

Question 5:

Solve the equation $x^2 + 3x + 5 = 0$

Answer:

The given quadratic equation is $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 3$, and $c = 5$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Question 6:

Solve the equation $x^2 - x + 2 = 0$

Answer:

The given quadratic equation is $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = -1$, and $c = 2$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1} = \frac{1 \pm \sqrt{7}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Question 7:

Solve the equation $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Answer:

The given quadratic equation is $\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = $\sqrt{2}$, b = 1, and c = $\sqrt{2}$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4\times\sqrt{2}\times\sqrt{2} = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-1\pm\sqrt{-7}}{2\times\sqrt{2}} = \frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

Question 8:

Solve the equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Answer:

The given quadratic equation is $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = $\sqrt{3}$, b = $-\sqrt{2}$, and c = $3\sqrt{3}$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = \left(-\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(3\sqrt{3}\right) = 2 - 36 = -34$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}} = \frac{\sqrt{2}\pm\sqrt{34}\,i}{2\sqrt{3}} \qquad \left[\sqrt{-1} = i\right]$$

Question 9:

Solve the equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Answer:

The given quadratic equation is $x^2 + x + \frac{1}{\sqrt{2}} = 0$

This equation can also be written as $\sqrt{2}x^2+\sqrt{2}x+1=0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = \sqrt{2}$, and $c = 1$

$\therefore \text{Discriminant } (D) = b^2 - 4ac = \left(\sqrt{2}\right)^2 - 4\times\left(\sqrt{2}\right)\times 1 = 2-4\sqrt{2}$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}} = \frac{-\sqrt{2}\pm\sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$= \left(\frac{-\sqrt{2}\pm\sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right) \qquad \left[\sqrt{-1}=i\right]$$

$$= \frac{-1\pm\left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

Question 10:

Solve the equation $x^2+\frac{x}{\sqrt{2}}+1=0$

Answer:

The given quadratic equation is $x^2+\frac{x}{\sqrt{2}}+1=0$

This equation can also be written as $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$

$\therefore \text{Discriminant } (D) = b^2-4ac = 1^2-4\times\sqrt{2}\times\sqrt{2} = 1-8 = -7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-1\pm\sqrt{-7}}{2\sqrt{2}} = \frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1}=i\right]$$

Page No 112:

Question 1:

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Answer:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$
$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$
$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$
$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$
$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$
$$=\left[-1+\frac{i}{i^2}\right]^3$$
$$=\left[-1-i\right]^3$$
$$=(-1)^3\left[1+i\right]^3$$
$$=-\left[1^3+i^3+3\cdot1\cdot i(1+i)\right]$$
$$=-\left[1+i^3+3i+3i^2\right]$$
$$=-\left[1-i+3i-3\right]$$
$$=-\left[-2+2i\right]$$
$$=2-2i$$

Question 2:

For any two complex numbers z_1 and z_2, prove that

$\text{Re}\,(z_1z_2) = \text{Re}\,z_1\,\text{Re}\,z_2 - \text{Im}\,z_1\,\text{Im}\,z_2$

Answer:

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2 y_1y_2$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad \left[i^2 = -1\right]$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

Question 3:

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Answer:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$
$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad \left[\text{On multiplying numerator and denominator by } (14 + 5i)\right]$$
$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$
$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

This is the required standard form.

Question 4:

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Answer:

$$x - iy = \sqrt{\frac{a - ib}{c - id}}$$

$$= \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}} \left[\text{On multiplying numerator and denominator by } (c + id)\right]$$

$$= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}}$$

$$\therefore (x - iy)^2 = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

$$\Rightarrow x^2 - y^2 - 2ixy = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2},\ -2xy = \frac{ad - bc}{c^2 + d^2} \qquad (1)$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \qquad [\text{Using } (1)]$$

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}$$

$$= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}$$

$$= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}$$

$$= \frac{a^2 + b^2}{c^2 + d^2}$$

Hence, proved.

Question 5:

Convert the following in the polar form:

(i) $\frac{1 + 7i}{(2 - i)^2}$, (ii) $\frac{1 + 3i}{1 - 2i}$

Answer:

(i) Here, $z = \frac{1 + 7i}{(2 - i)^2}$

$$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$$

$$= -1+i$$

Let r cos θ = –1 and r sin θ = 1

On squaring and adding, we obtain

$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$

$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2 \Rightarrow r^2 = 2$ $[\cos^2 \theta + \sin^2 \theta = 1]$

$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$

$$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in II quadrant}]$$

$\therefore z = r \cos \theta + i\, r \sin \theta$

$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

(ii) Here, $z = \frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i+3i-6}{1+4}$$

$$= \frac{-5+5i}{5} = -1+i$$

Let r cos θ = –1 and r sin θ = 1

On squaring and adding, we obtain

$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2$

$\Rightarrow r^2 = 2$ $[\cos^2 \theta + \sin^2 \theta = 1]$

$\Rightarrow r = \sqrt{2}$ $\qquad$ $[\text{Conventionally, } r > 0]$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\qquad$ $[\text{As } \theta \text{ lies in II quadrant}]$

$\therefore z = r\cos\theta + i\, r\sin\theta$

$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question 6:

Solve the equation $3x^2 - 4x + \frac{20}{3} = 0$

Answer:

The given quadratic equation is $3x^2 - 4x + \frac{20}{3} = 0$

This equation can also be written as $9x^2 - 12x + 20 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

a = 9, b = –12, and c = 20

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3}i$$

Question 7:

Solve the equation $x^2 - 2x + \frac{3}{2} = 0$

Answer:

The given quadratic equation is $x^2 - 2x + \frac{3}{2} = 0$

This equation can also be written as $2x^2 - 4x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

a = 2, b = –4, and c = 3

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}\,i}{4} \qquad \left[\sqrt{-1} = i\right]$$
$$= \frac{2 \pm \sqrt{2}\,i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

Question 8:

Solve the equation $27x^2 - 10x + 1 = 0$

Answer:

The given quadratic equation is $27x^2 - 10x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = 27, b = –10, and c = 1

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}\,i}{54} \qquad \left[\sqrt{-1} = i\right]$$
$$= \frac{5 \pm \sqrt{2}\,i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27} i$$

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Question 9:

Solve the equation $21x^2 - 28x + 10 = 0$

Answer:

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 21$, $b = -28$, and $c = 10$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}\,i}{42}$$
$$= \frac{28 \pm 2\sqrt{14}\,i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42} i = \frac{2}{3} \pm \frac{\sqrt{14}}{21} i$$

Question 10:

If $z_1 = 2 - i,\ z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$.

Answer:

$$z_1 = 2 - i,\ z_2 = 1 + i$$
$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$
$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$
$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$
$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$
$$= \left|\frac{2(1+i)}{2}\right|$$
$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Thus, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

Question 11:

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x+1)^2}$

Answer:

$$a+ib=\frac{(x+i)^2}{2x^2+1}$$
$$=\frac{x^2+i^2+2xi}{2x^2+1}$$
$$=\frac{x^2-1+i2x}{2x^2+1}$$
$$=\frac{x^2-1}{2x^2+1}+i\left(\frac{2x}{2x^2+1}\right)$$

On comparing real and imaginary parts, we obtain

$$a=\frac{x^2-1}{2x^2+1} \text{ and } b=\frac{2x}{2x^2+1}$$
$$\therefore a^2+b^2=\left(\frac{x^2-1}{2x^2+1}\right)^2+\left(\frac{2x}{2x^2+1}\right)^2$$
$$=\frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$$
$$=\frac{x^4+1+2x^2}{(2x^2+1)^2}$$
$$=\frac{(x^2+1)^2}{(2x^2+1)^2}$$
$$\therefore a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence, proved.

Question 12:

Let $z_1 = 2-i,\ z_2 = -2+i$. Find

(i) $\operatorname{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right)$, (ii) $\operatorname{Im}\left(\frac{1}{z_1\overline{z}_1}\right)$

Answer:

$z_1 = 2-i,\ z_2 = -2+i$

(i) $z_1z_2 = (2-i)(-2+i) = -4+2i+2i-i^2 = -4+4i-(-1) = -3+4i$

$$\bar{z}_1 = 2+i$$
$$\therefore \frac{z_1z_2}{\bar{z}_1} = \frac{-3+4i}{2+i}$$

On multiplying numerator and denominator by (2 – i), we obtain

$$\frac{z_1z_2}{\bar{z}_1} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$
$$= \frac{-2+11i}{5} = \frac{-2}{5}+\frac{11}{5}i$$

On comparing real parts, we obtain

$$\mathrm{Re}\left(\frac{z_1z_2}{\bar{z}_1}\right) = \frac{-2}{5}$$

(ii) $\frac{1}{z_1\bar{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2+(1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$$\mathrm{Im}\left(\frac{1}{z_1\bar{z}_1}\right) = 0$$

Question 13:

Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

Answer:

Let $z = \frac{1+2i}{1-3i}$, then

$$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$$
$$= \frac{-5+5i}{10} = \frac{-5}{10}+\frac{5i}{10} = \frac{-1}{2}+\frac{1}{2}i$$

Let $z = r\cos\theta + ir\sin\theta$

i.e., $r\cos\theta = \frac{-1}{2}$ and $r\sin\theta = \frac{1}{2}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta+\sin^2\theta\right)=\left(\frac{-1}{2}\right)^2+\left(\frac{1}{2}\right)^2$$
$$\Rightarrow r^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$
$$\Rightarrow r=\frac{1}{\sqrt{2}} \qquad \left[\text{Conventionally, } r>0\right]$$
$$\therefore \frac{1}{\sqrt{2}}\cos\theta=\frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2}$$
$$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}} \text{ and } \sin\theta=\frac{1}{\sqrt{2}}$$
$$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4} \qquad \left[\text{As } \theta \text{ lies in the II quadrant}\right]$$

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

Question 14:

Find the real numbers x and y if (x – iy) (3 + 5i) is the conjugate of –6 – 24i.

Answer:

Let $z=(x-iy)(3+5i)$

$$z=3x+5xi-3yi-5yi^2=3x+5xi-3yi+5y=(3x+5y)+i(5x-3y)$$
$$\therefore \overline{z}=(3x+5y)-i(5x-3y)$$

It is given that, $\overline{z}=-6-24i$

$$\therefore (3x+5y)-i(5x-3y)=-6-24i$$

Equating real and imaginary parts, we obtain

$$3x+5y=-6 \qquad \ldots \text{(i)}$$
$$5x-3y=24 \qquad \ldots \text{(ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$\begin{array}{r} 9x+15y=-18 \\ \underline{25x-15y=120} \\ 34x \quad =102 \end{array}$$
$$\therefore x=\frac{102}{34}=3$$

Putting the value of x in equation (i), we obtain

$3(3)+5y=-6$
$\Rightarrow 5y=-6-9=-15$
$\Rightarrow y=-3$

Thus, the values of x and y are 3 and –3 respectively.

Question 15:

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Answer:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$
$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$
$$=\frac{4i}{2}=2i$$
$$\therefore \left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

Question 16:

If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$.

Answer:

$$(x+iy)^3=u+iv$$
$$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$$
$$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$$
$$\Rightarrow x^3-iy^3+3x^2yi-3xy^2=u+iv$$
$$\Rightarrow \left(x^3-3xy^2\right)+i\left(3x^2y-y^3\right)=u+iv$$

On equating real and imaginary parts, we obtain

$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$

$$\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$
$$= \frac{x\left(x^2 - 3y^2\right)}{x} + \frac{y\left(3x^2 - y^2\right)}{y}$$
$$= x^2 - 3y^2 + 3x^2 - y^2$$
$$= 4x^2 - 4y^2$$
$$= 4\left(x^2 - y^2\right)$$
$$\therefore \frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$$

Hence, proved.

Question 17:

If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$.

Answer:

Let $\alpha = a + ib$ and $\beta = x + iy$

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2 + y^2} = 1$$
$$\Rightarrow x^2 + y^2 = 1 \qquad \text{... (i)}$$

$$\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right| = \left|\frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)}\right|$$
$$= \left|\frac{(x - a) + i(y - b)}{1 - (ax + aiy - ibx + by)}\right|$$
$$= \left|\frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)}\right|$$
$$= \frac{\left|(x - a) + i(y - b)\right|}{\left|(1 - ax - by) + i(bx - ay)\right|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$
$$= \frac{\sqrt{(x - a)^2 + (y - b)^2}}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}}$$
$$= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}}$$

$$=\frac{\sqrt{\left(x^2+y^2\right)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2\left(x^2+y^2\right)+b^2\left(y^2+x^2\right)-2ax-2by}}$$

$$=\frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad \left[\text{Using (1)}\right]$$

$$=1$$

$$\therefore\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

Question 18:

Find the number of non-zero integral solutions of the equation $\left|1-i\right|^x=2^x$.

Answer:

$$\left|1-i\right|^x=2^x$$

$$\Rightarrow\left(\sqrt{1^2+(-1)^2}\right)^x=2^x$$

$$\Rightarrow\left(\sqrt{2}\right)^x=2^x$$

$$\Rightarrow 2^{\frac{x}{2}}=2^x$$

$$\Rightarrow\frac{x}{2}=x$$

$$\Rightarrow x=2x$$

$$\Rightarrow 2x-x=0$$

$$\Rightarrow x=0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Question 19:

If (a + ib) (c + id) (e + if) (g + ih) = A + iB, then show that

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Answer:

$$(a+ib)(c+id)(e+if)(g+ih)=\mathrm{A}+i\mathrm{B}$$

$$\therefore\left|(a+ib)(c+id)(e+if)(g+ih)\right|=\left|\mathrm{A}+i\mathrm{B}\right|$$

$$\Rightarrow\left|(a+ib)\right|\times\left|(c+id)\right|\times\left|(e+if)\right|\times\left|(g+ih)\right|=\left|\mathrm{A}+i\mathrm{B}\right| \qquad \left[\left|z_1z_2\right|=\left|z_1\right|\left|z_2\right|\right]$$

$$\Rightarrow\sqrt{a^2+b^2}\times\sqrt{c^2+d^2}\times\sqrt{e^2+f^2}\times\sqrt{g^2+h^2}=\sqrt{\mathrm{A}^2+\mathrm{B}^2}$$

On squaring both sides, we obtain

$(a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = A^2 + B^2$

Hence, proved.

Question 20:

If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

Answer:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$\therefore m = 4k$, where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of m is 4 (= 4 × 1).
