EXERCISE 6.3

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Solution:

We have to find the number of 3-digit numbers that can be formed by using the digits 1 to 9 if no digit is repeated.

The given number of digits = 9 (from 1 to 9).
No. of ways of choosing first digit = 9
Since no digit should be repeated,
No. of ways of choosing second digit = 8
No. of ways of choosing to third digit = 7

Using the fundamental principle of counting,
Total possible numbers of ways = 9×8×7 = 504

2. How many 4-digit numbers are there with no digit repeated?

Solution:

We have to find the number of 4-digit numbers with no digits repeated.

The total number of digits = 10 (0 to 9)
No. of ways of choosing first digit = 9 (as 0 cannot be the first digit)
No. of ways of choosing second digit = 9
No. of ways of choosing third digit = 8
No. of ways of choosing fourth digit = 7

Using the fundamental principle of counting,
Total possible number of ways = 9×9×8×7 = 4,536

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Solution:

We have to find the number of 3-digit even numbers that can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated.

The total number of digits available = 6 (which are 1, 2, 3, 4, 6, 7).
For the number to be even the last digit must be 2, 4, or 6.

So no. of ways of choosing last digit = 3
As no digit should be repeated,
No. of ways of choosing first digit = 5
No. of ways of choosing second digit = 4

Using the fundamental principle of counting,
Total possible number of ways = 3×5×4 = 60

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

Solution:

We have to find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated.

Total number of digits available = 5 (from 1 to 5)
No. of ways of choosing first digit = 5
Since no digit should be repeated,
No. of ways of choosing second digit = 4
No. of ways of choosing third digit = 3
No. of ways of choosing fourth digit = 2
Using the fundamental principle of counting,
Total possible number of ways = 5×4×3×2 = 120

For the number to be even the last digit must be either 2 or 4.
No. of ways of choosing last digit = 2
Since no digit should be repeated,
No. of ways of choosing first digit = 4
No. of ways of choosing second digit = 3
No. of ways of choosing third digit = 2
Using the fundamental principle of counting,
Total possible number of ways that will be even number =2×4×3×2=48

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person can not hold more than one position?

Solution:

Given, a committee of 8 persons:
No. of ways of choosing a chairman = 8
Since one person can not hold more than one position,
No. of ways of choosing a vice-chairman = 7

Using the fundamental principle of counting,
Total possible number of ways = 8×7 = 56

6. Find n if $^{n-1}P_3 : {^nP_4} = 1 : 9$.

Solution:

Given that, $^{n-1}P_3 : {^nP_4} = 1 : 9$

$^{n-1}P_3 / {^nP_4} = 1 / 9$

Cross multiplying,

$9 \times {^{n-1}P_3} = 1 \times {^nP_4}$

Using nPr formula,

$9(n - 1)!/[(n - 1) - 3]! = n!/(n - 4)!$

$9(n - 1)!/(n - 4)! = n!/(n - 4)!$

$9(n - 1)! = n(n - 1)!$

$9 = n$

Thus, $n = 9$

7. Find r if (i) $^5P_r = 2\ {^6P_{r-1}}$ (ii) $^5P_r = {^6P_{r-1}}$

Solution:

(i) Given that, $^5P_r = 2\ {^6P_{r-1}}$

Using nPr formula,

$5!/(5 - r)! = 2 \times 6!/[6 - (r - 1)]!$

$5!/(5 - r)! = 2 \times 6 \times 5!/(7 - r)!$

$5!/(5 - r)! = 2 \times 6 \times 5!/[(7 - r)(6 - r)(5 - r)!]$

$(7 - r)(6 - r) = 12$

$42 - 7r - 6r + r^2 = 12$

$r^2 - 13r + 30 = 0$

$(r - 3)(r - 10) = 0$

$r = 3, 10.$

Since we are given with ^5P_r and $^6P_{r-1}$ in the problem, r ≤ 5 and r-1 ≤ 6. Thus, r cannot be 10.

Thus, r = 3.

(ii) Given that, $^5P_r = {}^6P_{r-1}$.

Using nPr formula,

$5!/(5 - r)! = 6!/[6 - (r - 1)]!$

$5!/(5 - r)! = 6 \times 5!/(7 - r)!$

$5!/(5 - r)! = 6 \times 5!/[(7 - r)(6 - r)(5 - r)!]$

$(7 - r)(6 - r) = 6$

$42 - 7r - 6r + r^2 = 6$

$r^2 - 13r + 36 = 0$

$(r - 4)(r - 9) = 0$

r = 4, 9.

Since we are given with ^5P_r and $^6P_{r-1}$ in the problem, r ≤ 5 and r-1 ≤ 6. Thus, r cannot be 9.

Thus, r = 4

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution:

No. of letters in the word "EQUATION" = 8 = n.
No.of letters in each word should be, r = 8.

Since arrangement matters in forming a word, we use the permutation. i.e., we use the nPr formula to find the required number of words.
Required no. of words = $^8P_8$
= 8!/(8 - 8)!
= 8!/(0)!
= 8×7×6×5×4×3×2×1
= 40,320

9. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.
(i) 4 letters are used at a time,
(ii) all letters are used at a time,
(iii) all letters are used but first letter is a vowel?

Solution:

The total no. of letters in the word MONDAY = 6 = n.

We know that the number of ways of selecting and arranging r different things from n different things is a permutation and we calculate it using the nPr formula.

(i) Number of 4-letter words that can be formed from the letters of the word MONDAY, without repetition of letters
= $^6P_4$
= 6!/(6 - 4)!
= 6!/2!
= 360.

(ii) Number of words that can be formed from the letters of the word MONDAY, if all letters are used at a time
= $^6P_6$
= 6!/(6 - 6)!
= 6!/0!
= 720.

(iii) Number of words that can be formed from the letters of the word MONDAY, if all letters are used but the first letter is a vowel
= 2 × 5 × 4 × 3 × 2 × 1
= 240

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Solution:

We know that the number of arrangements (permutations) that can be made out of n things out of which there are p, q, r, ... number of repetitions = n! / [p! q! r! ...].

In the given word MISSISSIPPI,

No. of M's = 1
No. of I's = 4
No. of S's = 4

No. of P's = 2
Total number of letters = 11.

This, the total number of arrangements possible with the given alphabets = 11! / (4! 4! 2!) = 34,650.

If we consider all the four I's one unit, then we get

M, S, S, S, S, P, P, IIII.

Then total number of units is 8 among which S is repeated 4 times and P is repeated twice.

Then the total number of arrangements (permutations) possible with the given alphabets = 8! / (4! 2!) = 840.

The distinct number of permutations of the letters in MISSISSIPPI that do not have the four I's together = 34,650 - 840 = 33,810

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the
(i) words start with P and end with S,
(ii) vowels are all together,
(iii) there are always 4 letters between P and S?

Solution:

We know that the number of arrangements (permutations) that can be made out of n things out of which there are p, q, r, ... number of repetitions = n! / [p! q! r! ...].

In the given word PERMUTATIONS,

No. of T's = 2
Total number of letters = 12.

(i) It is given that words start with P and end with S. So the letters in the first and last positions are fixed. The middle 10 positions have to be filled with the remaining 10 letters among which there are 2 Ts (which are repeated).

No. of words = 10!/2! = 18,14,400.

Permutation Formula

$$^{n}P_{r} \text{ (or) } P(n, r) = (n!) / (n - r)!$$

(ii) vowels are all together,

We know that there 5 vowels in the given letter. Then it becomes P, R, M, T, T, N, S, EUAIO, so there are 8 units among which there are 2 Ts'.

Also, vowels can be interchanged within themselves in 5! ways.

Thus, the number of words = 8!/2!× 5! = 24,19,200.

(iii) It is given that there are always 4 letters between P and S. So P and S can take the following positions respectively.

P	S
1st	6th
2nd	7th
3rd	8th
4th	9th
5th	10th
6th	11th
7th	12th

There are in total 7 ways in which there are 4 letters between P and S.

If we interchange P and S in the above table, we get 7 more ways of placing P and S.

Thus, total number of ways in which P and S can be placed = 7 + 7 = 14.

Now, the remaining 10 positions have to be filled with the remaining 10 letters among which there are 2 Ts. So

No. of ways for filling the remaining 10 positions = 10!/2!.

Total no. of ways = 14×10!/2! = 2,54,01,600