Class 11
Ncert
Maths
Chapter 4
COMPLEX NUMBERS
AND
QUADRATIC EQUATIONS
EXERCISE 4.1

Express each of the complex number given in the Exercises 1 to 10 in the form a + ib :
1. (5i)(-3/5i)

Solution:

A complex number is the sum of a real number and an imaginary number. A complex number is of the form a + ib and is usually represented by z. Here both a and b are real numbers.

The value 'a' is called the real part which is denoted by Re(z), and 'b' is called the imaginary part Im(z). Also, ib is called an imaginary number.

The given complex number is,

(5i)(-3/5i) = 5i x -3/5 x i

= $-3i^2$ [$\because i^2 = -1$]

= - 3 x (- 1)

= 3

= 3 + 0i

2. $i^9 + i^{19}$

Solution:

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The value 'a' is called the real part which is denoted by Re(z), and 'b' is called the imaginary part Im(z). Also, ib is called an imaginary number.

The given complex number is,

$i^9 + i^{19} = i^{4 \times 2 + 1} + i^{4 \times 4 + 3}$

$= (i^4)^2 \times i + (i^4)^4 \times i^3$

$= 1 \times i + 1 \times (- i)$ [$\because i^4 = 1, i^3 = - i$]

$= i + (- i)$

$= 0$

$= 0 + i0$

3. i^{-39}

Solution:

The given complex number is,

$i^{- 39} = i^{4 \times (- 9) - 3}$

$= (i^4)^{- 9} \times i^{- 3}$

$= (1)^{- 9} \times i^{- 3}$ [$\because i^4 = 1$]

$= 1/i^3$

$= 1/ - i$ [$\because i^3 = - i$]

$= 1/ - i \times i/i$

$= - i/i^2$

$= - i/- 1$ [$\because i^2 = - 1$]

$= i$

= 0 + i

4. 3(7 + i7) + i (7 + i7)

Solution:

The given complex number is,

$3(7 + i7) + i(7 + i7) = 21 + 21i + 7i + 7i^2$

$= 21 + 28i + 7 \times (-1)$ [$\because i^2 = -1$]

$= 14 + 28i$

5. (1 - i) - (- 1 + i6)

Solution: The given complex number is,

$(1 - i) - (-1 + i6) = 1 - i + 1 - 6i$

$= 2 - i7$

6. (1/5 + i2/5) - (4 + i5/2)

Solution:

The given complex number is,

$(1/5 + i2/5) - (4 + i5/2) = 1/5 + 2/5i - 4 - 5/2i$

$= (1/5 - 4) + i(2/5 - 5/2)$

$= (-19/5) + i(-21/10)$

$= -19/5 - i(21/10)$

7. [(1/3 + i 7/3) + (4 + i 1/3)] - (- 4/3 + i)

Soution: The given complex number is,

$[(1/3 + i\,7/3) + (4 + i\,1/3)] - (-4/3 + i)$

$= 1/3 + 7/3\,i + 4 + 1/3\,i + 4/3 - i$

$= (1/3 + 4 + 4/3) + i\,(7/3 + 1/3 - 1)$

$= 17/3 + i\,5/3$

8. **$(1 - i)^4$**

Solution:

The given complex number is,

$(1 - i)^4 = [(1 - i)^2]^2$

$= [1^2 + i^2 - 2i]^2$

$= [1 - 1 - 2i]^2$ [$\because i^2 = -1$]

$= [-2i]^2$

$= 4i^2$

$= -4$ [$\because i^2 = -1$]

9. **$(1/3 + 3i)^3$**

Solution:

The given complex number is,

$(1/3 + 3i)^3$

$= (1/3)^3 + (3i)^3 + 3(1/3)(3i)(1/3 + 3i)$ [$\because (a + b)^3 = a^3 + b^3 + 3ab(a + b)$]

$= 1/27 + 27i^3 + (3i)(1/3 + 3i)$

$= 1/27 + 27(-i) + i + 9i^2$ [$\because i^3 = -i$]

$= 1/27 - 27i + i - 9$ [$\because i^2 = -1$]

$= (1/27 - 9) - 26i$

$= -242/27 - i26$

10. **$(-2 - 1/3i)^3$**

Solution:

The given complex number is,

$(-2 - 1/3i)^3 = (-1)^3 (2 + 1/3i)^3$

$= -[2^3 + (i/3)^3 + 3(2)(i/3)(2 + i/3)]$

$= -[8 + i^3/27 + 2i(2 + i/3)]$

$= -[8 - i/27 + 4i + 2/3\, i^2]$ [$\because i^3 = -i$]

$= - [8 - i/27 + 4i - 2/3]$ [$\because i^2 = - 1$]

$= - [22/3 + 107/27\ i]$

$= - 22/3 - i\ (107/27)$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13 :

11. 4 - 3i

Solution:

The given complex number is,

$z = 4 - 3i$

Then, $z = 4 + 3i$ and

$|z|^2 = 4^2 + (- 3)^2$

$= 16 + 9$

$= 25$

Therefore, the multiplicative inverse of 4 - 3i is given by

$z^{-1} = z/|z|^2$

$= (4 + 3i)/25$

$= 4/25 + i\ (3/25)$

12. √5 + 3i

Solution:

The given complex number is,

$z = \sqrt{5} + 3i$

Then, $z = \sqrt{5} - 3i$ and

$|z|^2 = (\sqrt{5})^2 + (-3)^2$

$= 5 + 9$

$= 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by:

$z^{-1} = z/|z|^2$

$= (\sqrt{5} - 3i)/14$

$= \sqrt{5}/14 - (3/14)i$

13. Express the following expression in the form a + ib :
$(3 + i\sqrt{5})(3 - i\sqrt{5})/[(\sqrt{3} + \sqrt{2}\,i)-(\sqrt{3} - i\sqrt{2})]$

Solution:

The given complex number is,

$(3 + i\sqrt{5})(3 - i\sqrt{5})/[(\sqrt{3} + i\sqrt{2})(\sqrt{3} - i\sqrt{2})]$

$= (3)^2 - (i\sqrt{5})^2/ [\sqrt{3} + i\sqrt{2}\ \sqrt{3} + i\sqrt{2}]$ [$\because$ Using algebraic identity $(a + b)(a - b) = a^2 - b^2$].

$= (9 - 5i^2)/2\, i\sqrt{2}$

$= 9 - 5(- 1) / 2\, i\sqrt{2}$ [$\because i^2 = - 1$]

$= (9 + 5) / 2i\sqrt{2}$

$= 14/2i\sqrt{2} x\ i/i$

$= 7i/\sqrt{2}i^2$

$= 7i/\sqrt{2} (- 1)$ [$\because i^2 = - 1$]

$= - 7i/\sqrt{2} \times \sqrt{2}/\sqrt{2}$

$= - 7i\sqrt{2}/2$

$= 0 + i (- 7\sqrt{2}i/2)$