Class 11th

Maths

Chapter 8

SEQUENCES AND SERIES

Exersice8.2

1. Find the 20th and nth terms of the G.P 5/2, 5/4, 5/8 ....

Solution:

The given G.P. is

5/2, 5/4, 5/8 ....

Hence, a = 5/2 and r = (5/4)/(5/2)

= 1/2

Therefore,

$a_{20} = ar^{20-1} = 5/2\ (1/2)^{19} = 5/[(2)\ (2)^{19}]$

$= 5/(2)^{20}$

$a_n = ar^{n-1} = 5/2\ (1/2)^{n-1} = 5/[(2)\ (2)^{n-1}]$

$= 5/(2)^n$

2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2

Solution:

Let a be the first term of the G.P

It is given that common ratio, r = 2

Eighth term of the G.P, $a_8 = 192$

Therefore,

$\Rightarrow a_8 = ar^{8-1} = ar^7$

$\Rightarrow ar^7 = 192$

$\Rightarrow a(2)^7 = 192$

$\Rightarrow a(2)^7 = (2)^6(3)$

$\Rightarrow a = [(2)^6(3)]/(2)^7 = 3/2$

Hence,

$a_{12} = ar^{12-1}$

$= ar^{11}$

$= 3/2(2)^{11}$

$= (3)(2)^{10}$

$a_{12} = 3072$

3. The 5th, 8th and 11th terms of a G.P are p, q and s respectively. Show that $q^2 = ps$

Solution:

Let a be the first term and r be the common ratio of the G.P.

According to the question,

$a_5 = ar^{5-1} = ar^4 = p$ ....(1)

$a_8 = ar^{8-1} = ar^7 = q$ ....(2)

$a_{11} = ar^{11-1} = ar^{10}$ ....(3)

Dividing (2) by (1) , we obtain

$ar^7/ar^4 = q/p$

$r^3 = q/p$ ....(4)

Dividing (3) by (2) , we obtain

$ar^{10}/ar^{7} = s/q$

$r^3 = s/q$ ....(5)

Equating the values of r^3 obtained in (4) and (5), we obtain

$\Rightarrow q/p = s/q$

$\Rightarrow q^2 = ps$

Hence proved

4. The 4^{th} term of a G.P is square of its second term, and the first term is - 3. Determine its 7^{th} term

Solution:

Let a be the first term and r be the common ratio of the G.P.

It is known that $a = ar^{n-1}$

Therefore,

$a_2 = ar^1 = (-3)\, r$

$a_4 = ar^3 = (-3)\, r^3$

According to the question,

$\Rightarrow (-3)\, r^3 = [(-3)\, r]^2$

$\Rightarrow -3r^3 = 9r^2$

$\Rightarrow r = -3$

Hence,

$a = ar^{7-1}$

$= ar^6$

$= (-3)(-3)^6$

$= (-3)^7$

$= -2187$

Thus, the seventh term of the G.P is - 2187

5. Which term of the following sequences: (a) 2, 2√2, 4, ... is 128? (b) √3,3,3√3, ... is 729? (c) 1/3, 1/9, 1/27, ... is 1/19683?

Solution:

(a) The given sequence is 2, 2√2, 4,...

Here, a = 2 and r = 2√2/2 = √2

Let the n^{th} term of the sequence be 128

⇒ $a = ar^{n-1} = 128$

⇒ $(2)(\sqrt{2})^{n-1} = 128$

⇒ $(2)(2)^{(n-1)/2} = (2)^7$

⇒ $(2)(2)^{(n-1)/2+1} = (2)^7$

Hence,

⇒ (n - 1)/2 + 1 = 7

⇒ (n - 1)/2 = 6

⇒ n - 1 = 12

⇒ n = 13

Thus, the 13^{th} term of the sequence be 128.

(b) The given sequence is √3,3,3√3,...

Here, a = √3 and r = 3/√3 = √3

Let the nth term of the sequence be 729.

⇒ $a = ar^{n-1} = 729$

⇒ $(\sqrt{3})(\sqrt{3})^{n-1} = 729$

⇒ $a = (3)^{1/2}(\sqrt{3})^{(n-1)/2} = (3)^6$

⇒ $(3)^{1/2+(n-1)/2} = (3)^6$

Hence,

⇒ 1/2 + (n - 1)/2 = 6

⇒ $(1 + n - 1)/2 = 6$

⇒ $n = 12$

Thus, the 12th term of the sequence is 729.

(c) The given sequence is 1/3, 1/9, 1/27,...

Here, $a = 1/3$ and $r = 1/9/1/3 = 1/3$

Let the nth term of the sequence be 1/19683

⇒ $a = ar^{n - 1} = 1/19683$

⇒ $(1/3)(1/3)^{n - 1} = 1/19683$

⇒ $(1/3)^n = (1/3)^9$

⇒ $n = 9$

Thus, the 9th term of the sequence be 1/19683

6. For what values of x, the numbers - 2/7, x, 2/7 are in G.P.?

Solution:

The given numbers are - 2/7, x, 2/7

Hence,

Common ratio $r = x/ - 2/7 = - 7x/2$

Also, Common ratio $r = (- 7/2)/x = - 7/2x$

Therefore,

⇒ $- 7x/2 = - 7/2x$

⇒ $14x^2 = 14$

⇒ $x^2 = 1$

⇒ $x = \pm \sqrt{1}$

⇒ $x = \pm 1$

Thus, for $x = \pm 1$, the given numbers will be in G.P

Find the sum up to 20 terms in the G.P

7. 0.15, 0.015, 0.0015 .....

Solution:

The given G.P is

0.15, 0.015, 0.0015 .....

Here,

a = 0.15 and r = 0.015/0.15 = 0.1

It is known that

$S_n = a (1 - r^n)/(1 - r)$

Therefore,

Substituting n = 20,we get

$S_{20} = 0.15 (1 - (0.1)^{20})/(1 - 0.1)$

$S_{20} = 0.15/0.9 [(1 - (0.1)^{20})]$

$= 1/6 [(1 - (0.1)^{20})]$

8 Find the sum of n terms in the G.P √7, √21, 3, √7, ...

Solution:

The given G.P is

√7, √21, 3, √7, ...

Here, a = √7 and r = √21/√7 = √3

It is known that $S_n = a (1 - r^n)/(1 - r)$

Therefore,

$S_n = \sqrt{7} (1 - (\sqrt{3})^n)/(1 - \sqrt{3})$

$= [\sqrt{7} (1 - (\sqrt{3})^n)/(1 - \sqrt{3})] \times [(1 + \sqrt{3})/(1 + \sqrt{3})]$

$= \sqrt{7}(1 + \sqrt{3}) [(1 - (\sqrt{3})^n)]/(1 - 3)$

$= - \sqrt{7}(1 + \sqrt{3}) [(1 - (\sqrt{3})^n)]/2$

$= \sqrt{7}(1 + \sqrt{3})/2\ [(3)^{n/2} - 1]$

9. 1, -a, a^2, -a^3 …. (if a ≠ -1)

Solution:

The given G.P is

$1, -a, a^2, -a^3, ...$

Here, $a_1 = 1$ and $r = -a$

It is known that $S_n = a(1 - r^n)/(1 - r)$

Therefore,

$S_n = 1(1 - (-a)^n)/(1 - (-a))$

$= [1 - (-a)^n]/(1 + a)$

10. x^3, x^5, x^7, ... (if x ≠ ± 1)

Solution:

The given G.P is

$x^3, x^5, x^7, ...$

Here, $a = x^3$ and $r = x^2$

It is known that

$S_n = a(1 - r^n)/(1 - r)$

Therefore,

$S_n = x^3(1 - (x^2)^n)/(1 - x^2)$

$= x^3(1 - x^{2n})/(1 - x^2)$

11. Evaluate $\sum^{11}_{k=1}(2 + 3^k)$

Solution:

$\sum^{11}_{k=1}(2 + 3^k) = {}^{11}\sum_{k=1}(2) + {}^{11}\sum_{k=1}(3^k)$

$= 22 + {}^{11}\sum_{k=1}(3^k)$ ....(1)

Now,

$^{11}\sum_{k=1}(3^k) = 3^1 + 3^2 + 3^3 + ... + 3^{11}$

The terms of this sequence 3, 3^2, 3^3 .... forms a G.P

Therefore,

$S_n = 3(3^{11} - 1)/(3 - 1)$

$= 3/2(3^{11} - 1)$

Substituting this value in (1) , we obtain

$\sum^{11}_{k=1}(2 + 3^k) = 22 + 3/2(3^{11} - 1)$

12. The sum of first three terms of a G.P is 39/10 and their product is 1. Find the common ratio and the terms

Solution:

Let a/r, a, ar be the first three terms of the G.P.

It is given that a/r + a + ar = 39/10 ....(1)

(a/r) (a) (ar) = 1 ....(2)

From (2), we obtain

$a^3 = 1$

⇒ a = 1 (considering real roots)

Substituting a = 1 in (1), we obtain

⇒ 1/r + 1 + r = 39/10

⇒ $1 + r + r^2 = 39/10\ r$

⇒ $10 + 10r + 10r^2 - 39r = 0$

⇒ $10r^2 - 29r + 10 = 0$

⇒ $10r^2 - 25r - 4r + 10 = 0$

⇒ 5r (2r - 5) - 2 (2r - 5) = 0

⇒ (5r - 2)(2r - 5) = 0

r = 2/5, 5/2

Thus, the three terms of the G.P are 2/5, 5/2

13. **How many terms of the G.P 3, 3^2, 3^3, ... are needed to give the sum 120?**

Solution:

The given G.P is 3, 3^2, 3^3,...

Let n terms of this G.P be required to obtain a sum as 120.

Here, a = 3 and r = 3

Therefore,

$S_n = 3 (3^n - 1)/(3 - 1)$

$= 3 (3^n - 1)/2 = 120$

⇒ (120 x 2)/3 = 3n - 1

⇒ $3^n - 1 = 80$

⇒ $3^n = 81$

⇒ $3^n = 3^4$

⇒ n = 4

Thus, four terms of the given G.P are required to obtain the sum of 120

14. **The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P**

Solution:

Let the G.P be a, ar, ar^2, ar^3, ...

According to the given question

$a + ar + ar^2 = 16$

and $ar^3 + ar^4 + ar^5 = 128$

Therefore,

$a (1 + r + r^2) = 16$ ....(1)

$ar^3 (1 + r + r^2) = 128$ ....(2)

Dividing (2) by (1), we obtain

$\Rightarrow [ar^3 (1+ r + r^2)] / [a (1+ r + r^2)]$

$= 128/16$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 2$

Substituting $r = 2$ in (1) , we obtain

$\Rightarrow a (1 + 2 + 4) = 16$

$\Rightarrow a (7) = 16$

$\Rightarrow a = 16/7$

Hence,

$S_n = a (1 - r^n)/(1 - r)$

$= 16/7 (2^n - 1)/(2 - 1)$

$= 16/7 (2^n - 1)$

Thus, the first term, $a = 16/7$, common ratio, $r = 2$ and sum to n terms, $S_n = 16/7 (2^n - 1)$

15. Given a G.P with a = 729 and 7th term 64, determine S_7

Solution:

It is given that

$a = 729$ and $a_7 = 64$

Let r be the common ratio of the G.P.

It is known that Therefore, $a = ar^{n - 1}$

$\Rightarrow a = ar^{7 - 1} = ar^6$

$\Rightarrow 64 = 729r^6$

$r^6 = (2/3)^6$

$r = 2/3$

Also,

$S_n = a (1 - r^n)/(1 - r)$

$= 729 (1 - (2/3)^7)/(1 - 2/3)$

$= 3 \times 729 (1 - (2/3)^7)$

$(3)^7 [(3)^7 - (2)^7]/(3)^7$

$(3)^7 - (2)^7$

$= 2187 - 128$

$= 2059$

16. Find a G.P. for which sum of the first two terms is – 4 and the fifth term is 4 times the third term

Solution:

Let a be the first term and r be the common ratio of the G.P.

According to the given conditions,

$a_5 = 4 (a_3)$

$ar^4 = 4ar^2$

$r^2 = 4$

$r = \pm 2$

Also,

$S_2 = a (1 - r^2)/(1 - r)$

Case I : for r = 2

$\Rightarrow a (1 - 2^2)/(1 - 2)$

$\Rightarrow a (1 - 4)/(- 1) = - 4$

$\Rightarrow 3a - 4$

$\Rightarrow a = - 4/3$

Case II: for r = - 2

⇒ a $(1 - (-2)^2)/(1 - (-2))$

⇒ a (1 - 4)/(1 + 2) = - 4

⇒ - 3a/3 = - 4

⇒ a = 4

Thus, the required G.P is - 4/3, - 8/3, - 16/3, ... or 4, - 8, 16, ...

17. If the 4th, 10th and 16th terms of a G.P are x, y and z respectively. Prove that x, y, z are in G.P

Solution:

Let a be the first term and r be the common ratio of the G.P.

According to the given statement,

$a = ar^3 = x$ ....(1)

$a_{10} = ar^9 = y$ ....(2)

$a_{16} = ar^{15} = z$ ....(3)

Dividing (2) by (1), we obtain

⇒ $y/x = ar^9/ar^3$

⇒ $y/x = r^6$

Dividing (3) by (2), we obtain

⇒ $z/y = ar^{15}/ar^9$

⇒ $z/y = r^6$

Hence,

y/x = z/y

⇒ $y^2 = xz$

⇒ $y = \sqrt{xz}$

Thus, x, y, z are in G.P, proved

18. Find the sum to n terms of the given sequence is 8, 88, 888, ...

Solution:

The given sequence is 8, 88, 888, ... n terms .

S_n = 8 + 88 + 888 + ... n terms

= 8 [9 + 99 + 999 + ... terms]

= 8/9 [(10 - 1) + (100 - 1) + (1000 - 1) + ... n terms]

= 8/9 [(10 + 10^2 + 10^3 + ... n terms) - (1 + 1 + 1 + ... n terms)]

= 8/9 [10 (10^n - 1)/(10 - 1) - n]

= 8/9 [10 (10^n - 1)/9 - n]

= 80/81 (10^n - 1) - 8/9 n

19. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, 1/2

Solution:

The given sequences are

2, 4, 8, 16, 32 and 128, 32, 8, 2, 1/2.

Accordingly, the required sum,

S = 2 x 128 + 4 x 32 + 8 x 8 + 16 x 2 + 32 x $1/2^2$

64 [4 + 2 + 1 + 1/2 + $1/2^2$ ....(1)

It can be seen 4, 2, (1 + 1/2), $1/2^2$ is a G.P.

Here, a = 4 and r = 1/2

It is known that S_n = a (1 - r^n)/(1 - r)

Therefore,

S_5 = a (4 - $(1/2)^n$)/(1 - 1/2)

= 4 [1 - 1/32]/(1/2)

= 31/4

Hence,

$[4 + 2 + 1 + 1/2 + 1/2^2] = 31/4$

Putting this value in (1) , we obtain

S = 64 x 31/4

= 16 x 31

= 496

Thus, the required sum is 496

20 Show that the products of the corresponding terms of the sequences a, ar, ar^2, …ar^{n-1} and A, AR, AR^2, … AR^{n-1} form a G.P, and find the common ratio

Solution:

The given sequences are a, ar, ar^2, ... ar^{n-1} and A, AR, AR^2, AR^{n-1}

We need to prove that the sequence: aA, arAR, ar^2AR^2, ..., $ar^{n-1}AR^{n-1}$ form a G.P.

Let us find the ratio of the sequence

$\Rightarrow a_2 / a_1 = arAR / aA$

= rR

$\Rightarrow a_3 / a_2 = ar^2AR^2 / arAR$

= rR

Thus, the above sequence forms a G.P with a common ratio rR

21. Find four numbers forming a G.P in which the third term is greater than the first term by 9, the second term is greater than the 4th by 18

Solution:

Let a be the first term and r be the common ratio of the G.P. Hence,

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

According to the given condition,

⇒ $a_3 = a_1 + 9$

⇒ $ar^2 = a + 9$

⇒ $ar^2 - a = 9$

⇒ $a (r^2 - 1) = 9$ ....(1)

⇒ $a_2 = a_4 + 18$

⇒ $ar = ar^3 + 18$

⇒ $ar^3 - ar = - 18$

⇒ $ar (r^2 - 1) = -18$ ....(2)

Dividing (2) by (1) , we obtain

⇒ $ar (r^2 - 1)/a (r^2 - 1) = - 18/9$

⇒ $r = - 2$

Substituting r = - 2 in (1), we obtain

⇒ a [(- 2)2 - 1] = 9

⇒ a [4 - 1] = 9

⇒ 3a = 9

⇒ a = 9/3

⇒ a = 3

Thus, the first four numbers of the G.P. are 3, 3(- 2), $3(- 2)^{2,}$ and $3(- 2)^3$

i.e., 3, - 6, 12, - 24

22.If p^{th} , q^{th} and r^{th} terms of a G.P are a, b and c respectively. Prove that $a^{q - r} \times b^{r - p} \times c^{p - q} = 1$

Solution:

Let A be the first term and R be the common ratio of the G.P.

According to the given condition,

$AR^{p - 1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

Therefore,

$a^{q-r} \times b^{r-p} \times c^{p-q} = A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Hence proved

23. If the first and n^{th} term of the G.P is a and b respectively, if P is the product of n terms, prove that $P^2 = (ab)^n$

Solution:

It is given that the first term of the G.P is a and the last term is b .

Let the common ratio be r

Hence, the G.P is

$a, ar, ar^2, ar^3, ..., ar^{n-1}$ and $b = ar^{n-1}$....(1)

Now, the product of n terms

$P = (a) \times (ar) \times (ar^2) \times .... \times (ar^{n-1})$

$= (a \times a \times a .... n \text{ times}) (r \times r^2 \times .... \times r^{n-1})$

$= anr^{1+2+....+(n-1)}$ ....(2)

Here, 1, 2, ..., (n - 1) is an A.P.

$1 + 2 + ... + (n - 1) = (n - 1)/2 [2 + (n - 1 - 1) \times 1]$

$= (n - 1)/2 [2 + n - 2]$

$= n (n - 1)/2$

Substituting this value in (2) , we obtain

$P = a^n r^{n(n-1)/2}$

$P^2 = a^{2n}r^{n(n - 1)}$

$= [a^2r^{(n - 1)}]^n$

$= [a \times ar^{(n - 1)}]^n$ ....[Using (1)]

$P^2 = (ab)^n$

24. Show that the ratio of the sum of first n terms of a G.P to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $1/r^n$

Solution:

Let a be the first term and r be the common ratio of the G.P.

Sum of first n terms,

$S = [a (1 - r^n)/(1 - r)]$

Since, there are n terms from $(n + 1)^{th}$ to $(2n)^{th}$ term.

Hence, sum of the n terms from $(n + 1)^{th}$ to $(2n)^{th}$ terms

$S_n = a_{n + 1} (1 - r^n)/(1 - r)$

It is known that $a_n = ar^{n - 1}$

Therefore,

$a_{n + 1} = ar^{n + 1 - 1}$

$= ar^n$

Now,

$S_n = ar^n (1 - r^n)/(1 - r)$

Thus, the required ratio

$S/S_n = [a (1 - r^n)/(1 - r)]/[ar^n (1 - r^n)/(1 - r)]$

$= [a (1 - r^n)/(1 - r)] \times [(1 - r)/ar^n (1 - r^n)]$

$= 1/r^n$

Hence Proved

25. If a, b, c an d are in G.P. Show that: $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$

Solution:

If a, b, c and d are in G.P.

Therefore,

$bc = ad$ ....(1)

$b^2 = ac$ ....(2)

$c^2 = bd$ ....(3)

We need to prove that, $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$

Since,

RHS = $(ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d (a + c)]^2$

$= a^2b^2 + 2abd (a + c) + d^2 (a + c)2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2 (a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

Using (2) and (3) and rearranging the terms

RHS = $a^2 (b^2 + c^2 + d^2) + b^2 (b^2 + c^2 + d^2) + c^2 (b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

= LHS

Thus, $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$ Proved

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P

Solution:

Let G_1 and G_2 be two numbers between 3 and 81 such that the series 3, G_1, G_2, 81 forms a G.P.

Let a be the first term and r be the common ratio of the G.P.

Therefore, a = 3 and a4 = 81

$\Rightarrow 3r^3 = 81$

$\Rightarrow r^3 = 27$

$\Rightarrow$ r = 3 (considering real roots only)

Hence,

$G_1 = ar = 3 \times 3 = 9$

$G_2 = ar^2 = 3 \times (3)^2 = 27$

Thus, the required two numbers are 9 and 27

27. Find the value of n so that $a^{n+1} + b^{n+1}/a^n + b^n$ may be the geometric mean between a and b

Solution:

It is known that G.M. of a and b is $\sqrt{ab}$

By the given condition

$a^{n+1} + b^{n+1}/a^n + b^n = \sqrt{ab}$

By squaring both sides, we obtain

$(a^{n+1} + b^{n+1})^2/(a^n + b^n)^2 = ab$

$\Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2}$

$= (ab)(a^{2n} + 2a^nb^n + b^{2n})$

$\Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2}$

$= a^{2n+1}b + 2a^{n+1}b^{n+1} + ab^{2n+1}$

$\Rightarrow a^{2n+2} + b^{2n+2} = a^{2n+1}b + ab^{2n+1}$

$\Rightarrow a^{2n+2} - ab^{2n+1} = ab^{2n+1} - b^{2n+2}$

$\Rightarrow a^{2n+1} (a - b) = b^{2n+1} (a - b)$

$\Rightarrow (a/b)^{2n+1} = 1 = (a/b)^0$

$\Rightarrow 2n + 1 = 0$

$\Rightarrow n = - 1/2$

Thus, the value of n = - 1/2

28. The sum of two numbers is 6 times their G.M, show that numbers are in the ratio (3 + 2√2) : (3 - 2√2)

Solution:

Let the two numbers be a and b.

Then its G.M. = $\sqrt{ab}$

According to the given condition,

$\Rightarrow a + b = 6\sqrt{ab}$ ....(1)

$\Rightarrow (a + b)^2 = 36(ab)$

Also,

$(a - b)^2 = (a + b)^2 - 4ab$

$= 36ab - 4ab$

$= 32ab$

$a - b = \sqrt{32}\,\sqrt{ab}$

$= 4\sqrt{2}\,\sqrt{ab}$ ...(2)

Adding (1) and (2) , we obtain

$2a = (6 + 4\sqrt{2})\,\sqrt{ab}$

$a = (3 + 2\sqrt{2})\,\sqrt{ab}$

Substituting the value of a in (1) , we obtain

$b = 6\sqrt{ab} - (3 + 2\sqrt{2})\,\sqrt{ab}$

$= (3 - 2\sqrt{2})\,\sqrt{ab}$

Hence the ratio of the numbers is

$a/b = [(3 + 2\sqrt{2})\sqrt{ab}] / [(3 - 2\sqrt{2})\sqrt{ab}]$

$= (3 + 2\sqrt{2}) / (3 - 2\sqrt{2})$

Thus, the required ratio is $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$

29. If A and G be A.M and G.M, respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A + G)(A - G}$

Solution:

It is given that A and G are A.M and G.M, respectively between two positive numbers.

Let these two positive numbers be a and b.

Therefore,

$A.M = A = (a + b) / 2$

$\Rightarrow a + b = 2A$ ....(1)

And,

$G.M = G = \sqrt{ab}$

$\Rightarrow ab = G^2$ ....(2)

Since,

$(a - b)^2 = (a + b)^2 - 4ab$

$= 4A^2 - 4G^2$ [Using (1) and (2)]

$= 4(A^2 - G^2)$

$= 4(A + G)(A - G)$

$(a - b) = 2\sqrt{(A + G)(A - G)}$ ....(3)

By adding (1) and (3), we obtain

$\Rightarrow 2a = 2A + 2\sqrt{(A + G)(A - G)}$

$\Rightarrow a = A + \sqrt{(A + G)(A - G)}$

Substituting the value of a in (1) , we obtain b = 2 A - (A + √(A + G)(A - G))

= A - √(A + G)(A - G))

Thus, the two numbers are A ± √(A + G)(A - G))

30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

Solution:

It is given that the number of bacteria doubles every hour.

Hence, the number of bacteria after every hour will form a G.P with a = 30 and r = 2

Therefore, the number of bacteria at the end of 2^{nd} hour will be

$a = ar^2$

$= 30 \times (2)^2$

= 120

The number of bacteria at the end of 4th hour will be

$a = ar^4$

$= 30 \times (2)^4$

= 480

The number of bacteria at the end of the n^{th} hour will be:

$a_{n+1} = ar^n$

$= 30 \times (2)^n$

31. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Solution:

The amount deposited in the bank is ₹ 500.

At the end of the first year,

the amount in ₹ = 500 (1 + 1/10)

= 500 (1.1)

At the end of second year, amount in ₹

= 500 (1.1)(1.1)

At the end of third year, amount in ₹

= 500 (1.1)(1.1)(1.1) and so on…

Therefore, at the end of 10 years, amount in ₹

= 500(1.1)(1.1)(1.1) .... 10 times

= $500(1.1)^{10}$

32. If A.M and G.M are roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation

Solution:

Let the roots of the quadratic equations be a and b .

According to the condition,

A.M = (a + b)/2 = 8

⇒ a + b = 16 ....(1)

G.M = $\sqrt{ab}$ = 5

⇒ ab = 25 ....(2)

The quadratic equation is given by,

x^2 - x (Sum of roots) + (Product of roots) = 0

x^2 - x (a + b) + (ab) = 0

x^2 - 16x + 25 = 0 [Using (1) and (2)]

Thus, the required quadratic equation is x^2 -16x + 25 = 0