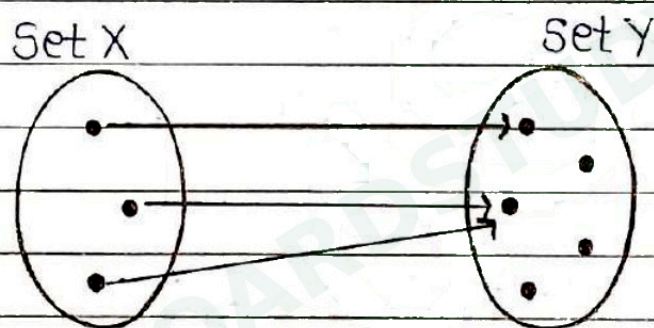


- $y = f(x)$

- x is the Pre-image.
- y is the Image.

Key Terms

1. **Domain** : The complete set X .
2. **Codomain** : The complete set Y .
3. **Range** : The set of actual outputs $\{f(x) : x \in X\}$.
(Range is a subset of codomain).



- **Rule** : Every dot in X must shoot exactly one arrow. No dot in X can be left without an arrow, and no dot in X can shoot two arrows.

6. Types of Functions

1. One - One (Injective)

Distinct (different) elements of X have distinct images in Y . No two inputs give the same output.

• General Form :

$$A = [a_{ij}]_{m \times n}$$

Where i represents the row number and j represents the column number.

- Example : a_{23} is the element in the 2nd row and 3rd column.
- Number of Elements : The total number of elements in an $m \times n$ matrix is the product mn .

2. Types of Matrices

- Diagonal : The line of elements extending from the top-left corner to the bottom right corner.

Type	Definition	Order	Example
Column Matrix	A matrix having only one column.	$m \times 1$	$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
Row Matrix	A matrix having only one row.	$1 \times n$	$[1 \ 2 \ 3]$
Square Matrix	Number of rows equals number of columns	$m \times m$	$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ (2×2)

B. Equation of a Line

To find the line passing through (x_1, y_1) and (x_2, y_2) , assume a third point (x, y) on the line and set area to zero:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

5. Minors and Cofactors

Minors (M_{ij})

The determinant obtained by deleting the i -th row and j -th column.

- Example: In $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, to find Minor of 1 (M_{11}), hide Row 1 and col 1:

$$\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix}$$

Cofactors (A_{ij})

The minor multiplied by a sign factor based on position.

- Formula: $A_{ij} = (-1)^{i+j} M_{ij}$
- If $i+j$ is Even: cofactor = Minor.
- If $i+j$ is Odd: cofactor = $-1 \times$ Minor.

$$\bullet \frac{d}{dx} (\sin x) = \cos x$$

$$\bullet \frac{d}{dx} (\cos x) = -\sin x$$

$$\bullet \frac{d}{dx} (\tan x) = \sec^2 x$$

$$\bullet \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$\bullet \frac{d}{dx} (\sec x) = \sec x \tan x$$

$$\bullet \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

Inverse Trigonometric Functions :

$$\bullet \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\bullet \frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\bullet \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\bullet \frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\bullet \frac{d}{dx} (\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\bullet \frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

Equations of Lines

Using the point-slope form ($y - y_1 = m(x - x_1)$):

1. Equation of Tangent :

$$y - y_0 = f'(x_0)(x - x_0)$$

2. Equation of Normal :

$$y - y_0 = \frac{-1}{f'(x_0)}(x - x_0)$$

Special Cases

- If tangent is parallel to X-axis : Slope $(dy/dx) = 0$.
- If tangent is parallel to Y-axis (perpendicular to x-axis) : Slope (dy/dx) is not defined (denominator is 0) or $dx/dy = 0$.
- If two lines are parallel, their slopes are equal ($m_1 = m_2$).
- If two lines are perpendicular, the product of their slopes is -1 ($m_1 \cdot m_2 = -1$).

5. Approximations (Differentials)

We can use derivative to find the approximate value of a function. Let $y = f(x)$. Let Δx be a small change in x and Δy be the

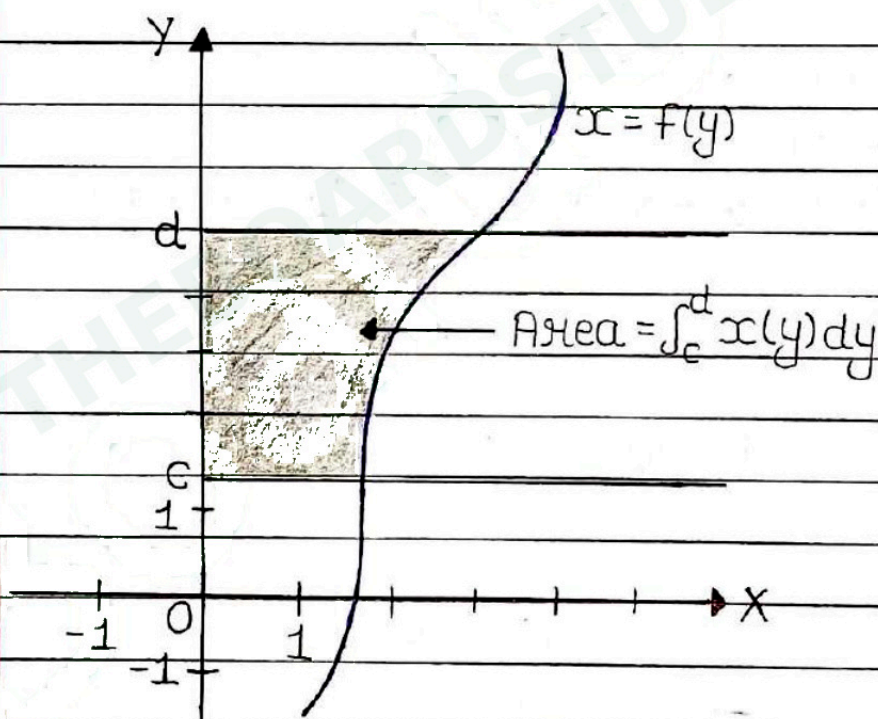
Case B: Area against the Y-axis

If you want to find the area of a region bounded by :

1. The curve $x = g(y)$ (or $x = \phi(y)$)
2. The Y-axis
3. The lines $y = c$ and $y = d$ (where $d > c$)

Formula :

$$\text{Area} = \int_c^d x \, dy = \int_c^d g(y) \, dy$$



⚠ Important Note on Negative Area

The value of an integral (the answer you get after calculating) can be negative if the curve is below the x-axis. However,

4. Formation of Different Equations

How to create a D.E. when given a normal equation.

Steps :

1. Count the number of arbitrary constants (say 'n').
2. Differentiate the given equation 'n' times with respect to x .
3. Eliminate the arbitrary constants using the original and differentiated equations.

Example : Form D.E. for $y = ae^{3x} + be^{-2x}$ (constants : a, b)

1. Differentiate 1 : $y' = 3ae^{3x} - 2be^{-2x}$
2. Differentiate 2 : $y'' = 9ae^{3x} + 4be^{-2x}$
3. Eliminate : Subtract equations to remove a and b .

• Result : $y'' - y' - 6y = 0$

5. Methods to solve Differential Equations

There are three main methods depending on the type of equation.

Method 1 : Variable Separable Form

6. Vector Identities and Formulas

Based on the dot and cross products, the following identities hold :

$$1. |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$2. |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$3. |\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$$

$$4. (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$$

Triangle Projection Formulae : For a triangle with sides a, b, c and angles A, B, C :

- $a = b \cos C + c \cos B$
- $b = c \cos A + a \cos C$
- $c = a \cos B + b \cos A$

7. Vector (cross) Product

The vector product of two non-zero vectors $\vec{a}$ and $\vec{b}$ is a vector $\vec{c}$ perpendicular to both.

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$.

The general equation of first degree in x, y, z represents a plane :

$$Ax + By + Cz + D = 0$$

- **Important :** The coefficients A, B, C are the Direction Ratios of the normal to the plane.

B. Equation of a plane in Normal Form

- **Vector Form :** $\vec{r} \cdot \hat{n} = d$
 - $\hat{n}$: Unit Vector normal to the plane from origin.
 - d : Perpendicular distance from origin.
- **Cartesian Form :** $lx + my + nz = d$
 - l, m, n : Direction cosines of the normal.
- **Foot of perpendicular :** If l, m, n are direction cosines of the normal and d is the distance from the origin, the coordinates of the foot of the perpendicular drawn from the origin to the plane are :

$$(ld, md, nd)$$

Bayes' Theorem (Reverse Probability)

If $E_1, E_2, \dots, E_n$ are mutually exclusive and exhaustive events, and A is any event with non-zero probability, then the probability of a specific event E_i happening given that A has happened is:

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{j=1}^n P(E_j) \cdot P(A|E_j)}$$

Structure for Solving Bayes' Problems:

1. Identify Partition: Split the sample space into E_1, E_2 (e.g., Factor A vs Factor B).
2. Identify Event A: The outcome that has already occurred (e.g., A defective item is found).
3. Calculate: Find $P(E_i)$ and $P(A|E_i)$ for all i .
4. Apply Formula: Numerator is the path you want; Denominator is the sum of all paths to A .
5. Random Variables and Probability Distribution.

Definition: A Random Variable is a real